

- Final Exam, Thursday 12/10 @1:30 pm  
Remote, via Zoom
- Final homework due Friday.
- HW 10-01 – mag. dipole approx. (eqn 5.88)
- After today can do problems 1–9 and 14.
- Self Inductance
- Mutual Inductance

## Faraday's Law

$$\varepsilon = - \frac{\partial \Phi_B}{\partial t}$$

$$\oint \vec{E} \cdot d\vec{l} = - \frac{\partial \Phi_B}{\partial t}$$

$$\varepsilon = - \frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\oint \vec{E} \cdot d\vec{l} = - \frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

## Electric Dipole

$$\vec{p} = q \vec{d}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$$

$$\vec{E} = -\nabla V$$

$$\vec{E} = \frac{p}{4\pi\epsilon_0} \frac{(2\cos\theta\hat{r} + \sin\theta\hat{\theta})}{r^3}$$

## Magnetic Dipole

$$\vec{m} = I \vec{a}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{(2\cos\theta\hat{r} + \sin\theta\hat{\theta})}{r^3}$$

## Mutual Inductance:

- Change the current in coil 2 and you create a changing EMF on coil 1

# Self Inductance:

- Change the current in a coil and it creates its own EMF (Voltage) to try to stop you.

## Electric

$$Q_1 = C_1 V_1$$

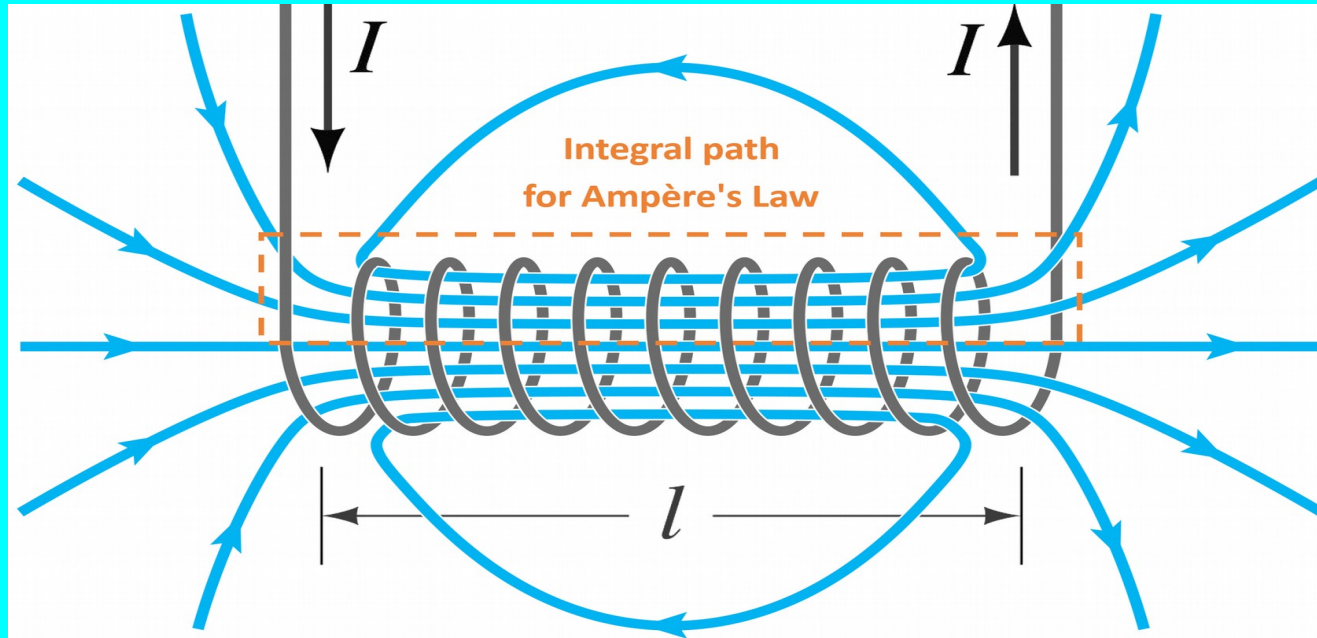
$$Q_1 = C_{21} V_2$$

## Magnetic

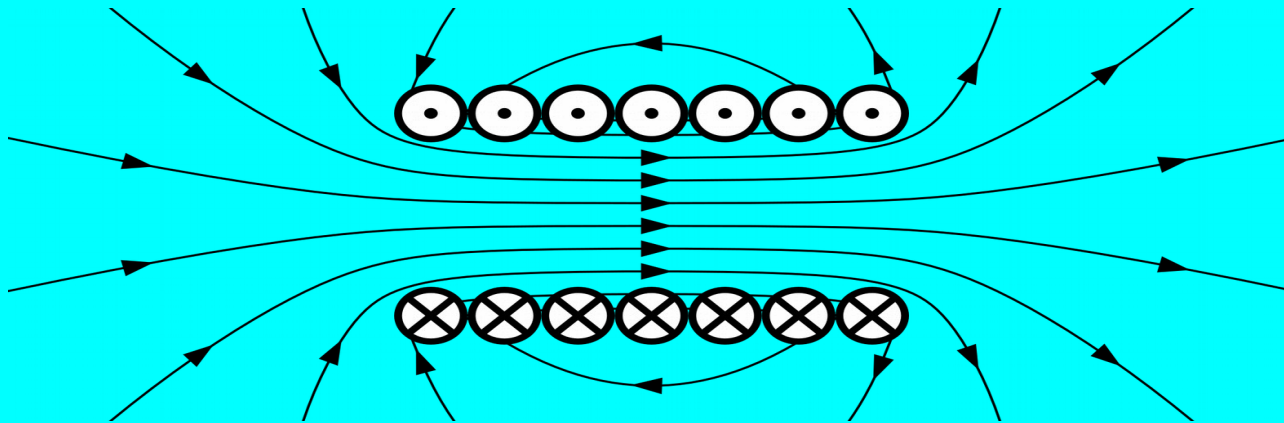
$$\frac{dI}{dt} = -\frac{\varepsilon}{L} \quad I = -\frac{\Phi_B}{L}$$

$$\frac{dI_1}{dt} = -\frac{\varepsilon_2}{M_{21}} \quad I_1 = -\frac{\Phi_{B_2}}{M_{21}}$$

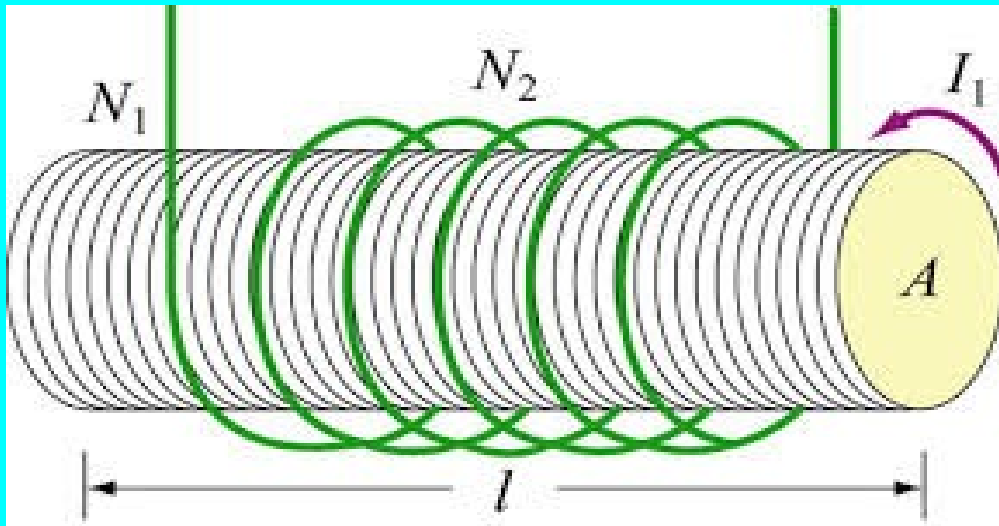
## Self Inductance of a solenoid (problem 8):



$$\varepsilon = - \frac{\partial \Phi_B}{\partial t}$$



## Mutual Inductance of two solenoids:





## Lenz's Law “Back EMF”:

- Lenz's law is the minus sign in Faraday's law
- If you try to increase  $B$  through a loop, a current in the loop tries to prevent it from increasing
- This induced current also produces an opposing magnetic field, as we saw with eddy currents and the magnet that would not fall onto the copper plate

# Mutual Inductance

## Ampere's Law

(with displacement current)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \Phi_E$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$$

$$\int \nabla \times \vec{B} \cdot d\vec{a} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

## Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \Phi_B$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\int \nabla \times \vec{E} \cdot d\vec{a} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$