

- Final Exam, Thursday 12/10 @1:30 pm
Remote, via Zoom
- Faraday's Law
- Inductance/Capacitance/Resistance
- Mutual Inductance

Maxwell's Equations

Gauss's Law

$$\oiint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$$
$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

The “no monopole” equation

$$\oiint \vec{B} \cdot d\vec{a} = \mu_0 Q_{\text{monopole}} = 0$$
$$\nabla \cdot \vec{B} = 0$$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$
$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Faraday's Law

$$\varepsilon = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a}$$

$$\int \nabla \times \vec{B} \cdot d\vec{a} = \mu_0 \int \vec{J} \cdot d\vec{a}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \Phi_B$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\int \nabla \times \vec{E} \cdot d\vec{a} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's Law

$$\begin{aligned} \oint \vec{E} \cdot d\vec{l} &= -\frac{\partial}{\partial t} \Phi_B \\ \varepsilon &= -\frac{\partial}{\partial t} \Phi_B \rightarrow \\ \oint \vec{E} \cdot d\vec{l} &= -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a} \\ \varepsilon &= -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a} \rightarrow \end{aligned}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's Law:

- You get a current in a loop IF
- B changes magnitude
- Loop area changes
- Angle between B and area changes

Lenz's Law “Back EMF”:

- Lenz's law is the minus sign in Faraday's law
- If you try to increase B through a loop, a current in the loop tries to prevent it from increasing
- This induced current also produces an opposing magnetic field, as we saw with eddy currents and the magnet that would not fall onto the copper plate

Example 1: Change magnitude of B

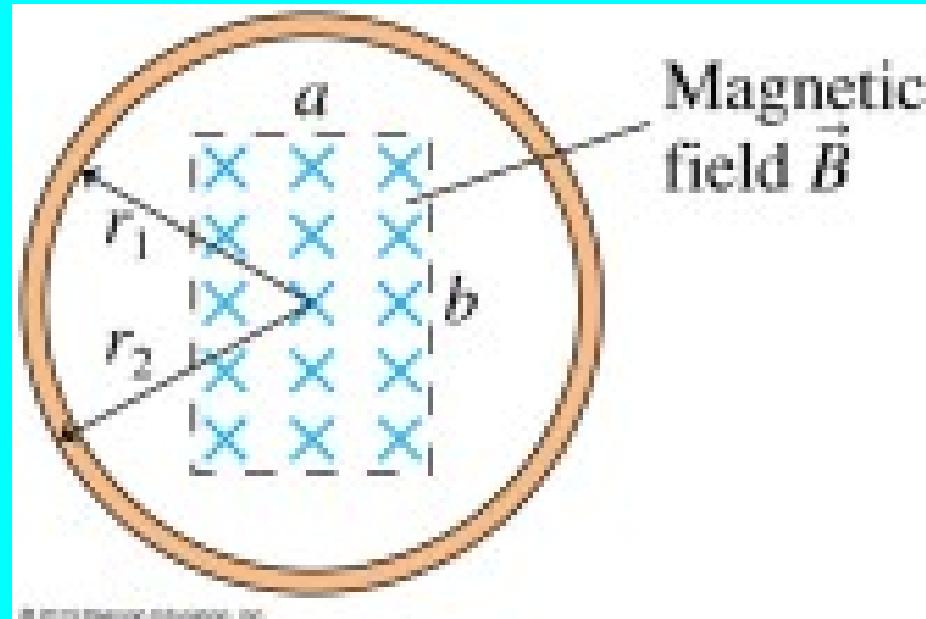
What is the EMF if $\vec{B} = k t^3 \hat{z}$

A) $\varepsilon = k t^3 \pi r_1^2$

B) $\varepsilon = 3 k t^2 \pi r_1^2$

C) $\varepsilon = k t^3 \pi r_2^2$

D) $\varepsilon = 3 k t^2 a b$



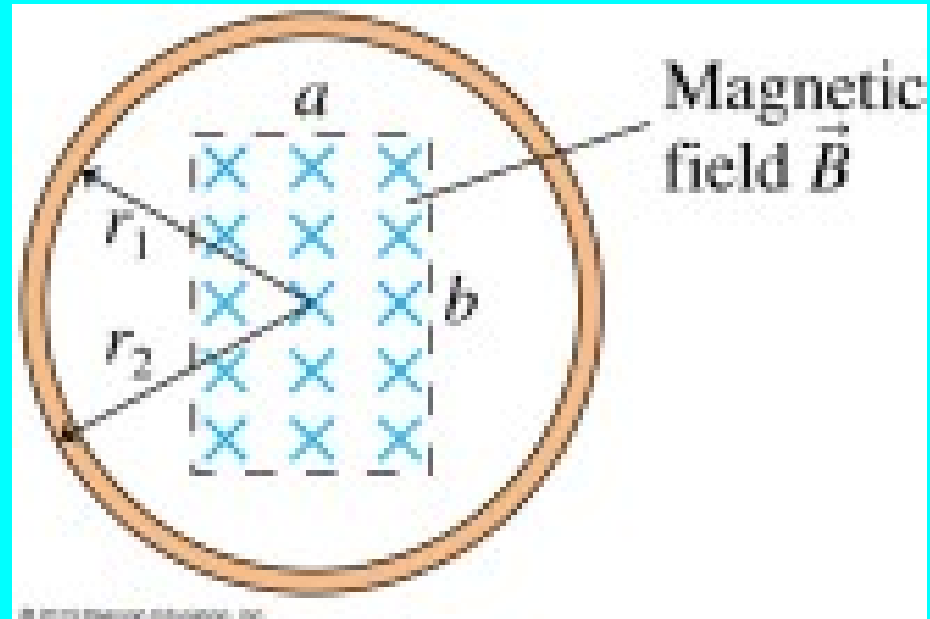
What direction does

The induced current flow if $\vec{B} = -k t^3 \hat{z}$?

A) Clockwise

B) CCW

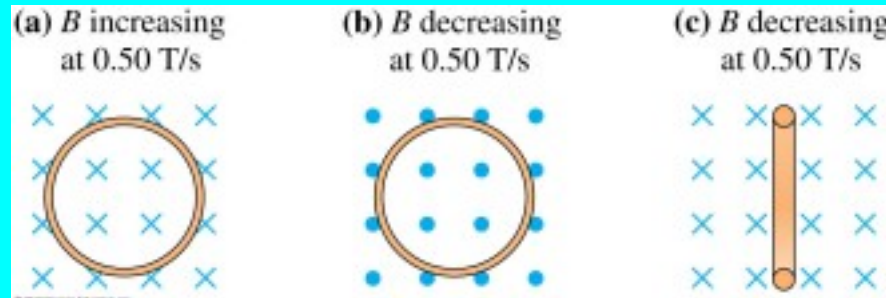
C) It's zero



What direction does

The induced current flow in each case?

- A) CCW, CW, CCW
- B) CCW, CCW, CCW
- C) CCW, CW, Zero
- D) CCW, CCW, Zero



C) It's zero

Example 2: Change magnitude of A

Example 3: Change angle between B and A

Inductance, Capacitance, Resistance

Mutual Inductance

Ampere's Law

(with displacement current)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \Phi_E$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$$

$$\int \nabla \times \vec{B} \cdot d\vec{a} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \Phi_B$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\int \nabla \times \vec{E} \cdot d\vec{a} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$