Lecture 25 outline: 10/14/2020

- No classes FRIDAY!

- Multipole expansion
  - Vector form of dipole term
  - Potential and field of dipole in spherical coords

- Application
  - Conducting sphere in uniform E-field
  - Induced dipoles

Show that the dipole potential may be written

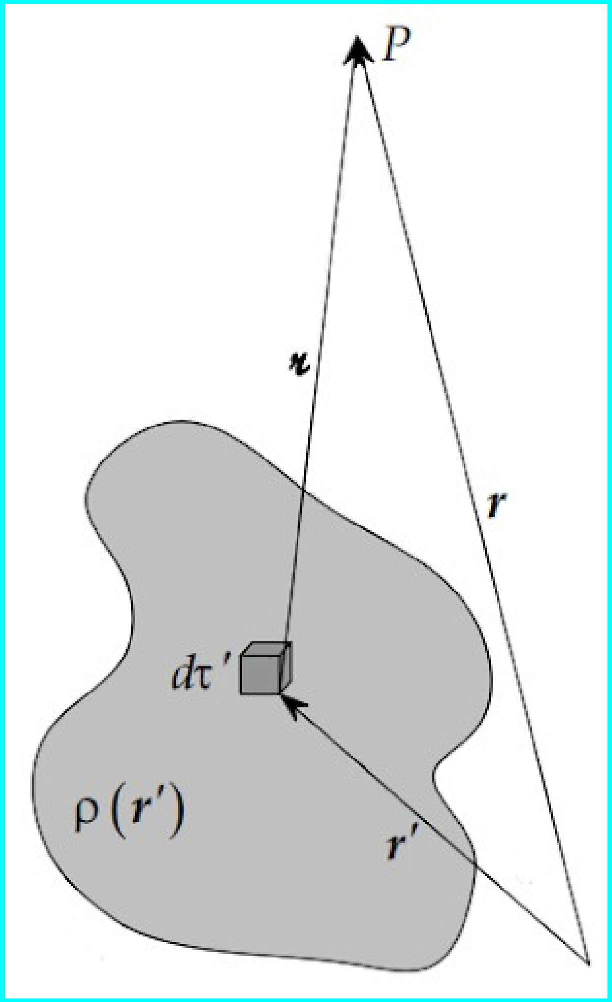
$$V\left(\vec{r}_T\right)=\frac{1}{4\pi\epsilon_0 r_T^2}\hat{r}_T\cdot\vec{p}$$

What is the electric field of a dipole in spherical coordinates?

$$V(r,\theta)=\frac{p\cos\theta}{4\pi\epsilon_0 r^2}$$

Multipole expansion

$$V=\frac{1}{4\pi\epsilon_0}\int\frac{\rho(r')}{\mathscr{r}}d\tau' \quad \frac{1}{\mathscr{r}}=\frac{1}{r_T}\left[1-\frac{\epsilon}{2}+\frac{3}{8}\epsilon^2-\frac{5}{16}\frac{\epsilon^3}{3!}\right]$$



$$\epsilon\stackrel{\text{def}}{=}\Delta^2-2\Delta\cos\alpha$$

$$\Delta\stackrel{\text{def}}{=}\frac{r_S}{r_T}$$

Legendre Polynomials

$$P_0(\cos\theta) = 1$$

$$P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{(3\cos^2\theta - 1)}{2}$$

$$P_3(\cos\theta) = \frac{(5\cos^3\theta - 3\cos\theta)}{2}$$

HW6-05 A sphere of radius R has charge for r<R defined as:

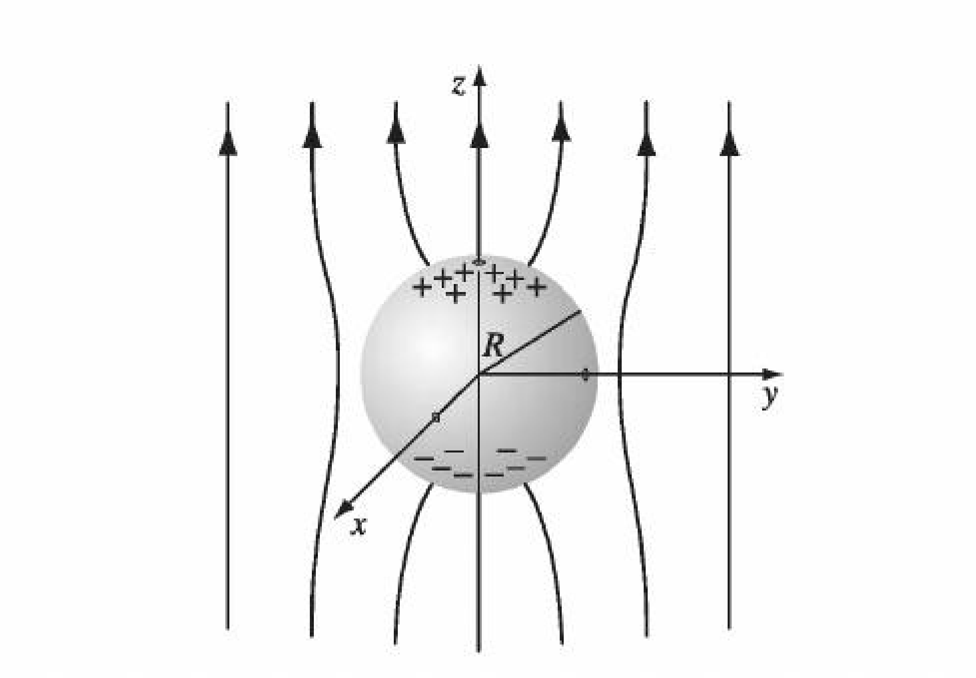
$$\rho(r,\theta)=a\frac{R}{r^2}(R-2r)\sin\theta$$

$$\vec{p}=\int\vec{r}\,\rho\,d\tau$$

Calculate the potential for z>>R.

HW6-07 A conducting sphere of radius R is placed in a uniform electric field $\vec{E}=E_0\hat{z}$.

What is the potential inside and outside the sphere?



$$V(r,\theta)=\left(Ar^L+\frac{B}{r^{L+1}}\right)\sum_{L=0}^{L=\infty} P_L(\cos\theta)$$

What is the general solution to Laplace in spherical coordinates?

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) = 0$$

$$V(r,\theta) = \left(Ar^L + \frac{B}{r^{L+1}}\right) \sum_{L=0}^{L=\infty} P_L(\cos\theta)$$