Lecture 23 outline:

- Separation of Variables in Spherical Coords
  - Legendre Polynomials
  - Applying boundary conditions

What is the general solution to Laplace in spherical coordinates?

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) = 0$$

$$V(r,\theta) = \left(Ar^L + \frac{B}{r^{L+1}}\right)\sum_{L=0}^{L=\infty} P_L(\cos\theta)$$

Legendre Polynomials

$$P_0(x)=1 \qquad P_0(\cos\theta)=1$$

$$P_1(x)=x \qquad P_1(\cos\theta)=\cos\theta$$

$$P_2(x)=\frac{(3x^2-1)}{2} \qquad P_2(\cos\theta)=\frac{(3\cos^2\theta-1)}{2}$$

$$P_3(x)=\frac{(5x^3-3x)}{2} \qquad P_3(\cos\theta)=\frac{(5\cos^3\theta-3\cos\theta)}{2}$$

$$P_4(x)=\frac{(35x^4-30x^2+3)}{8}$$

$$\int_{-1}^{1} P_L(x)P_{L'}(x)dx=\frac{2}{2L+1}\delta_{LL'}$$

On the surface of a sphere of radius R,
the potential is $V(\theta)=\cos(3\theta)$
What is the potential inside and outside the sphere?

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$\cos(3\theta)=4\cos^3(\theta)-3\cos\theta$

On the surface of a sphere or radius R,
the potential is $V(\theta)=70\cos^4\theta+10\cos\theta-14$
What is the potential inside and outside the sphere? [skipped]