Lecture 22 outline:

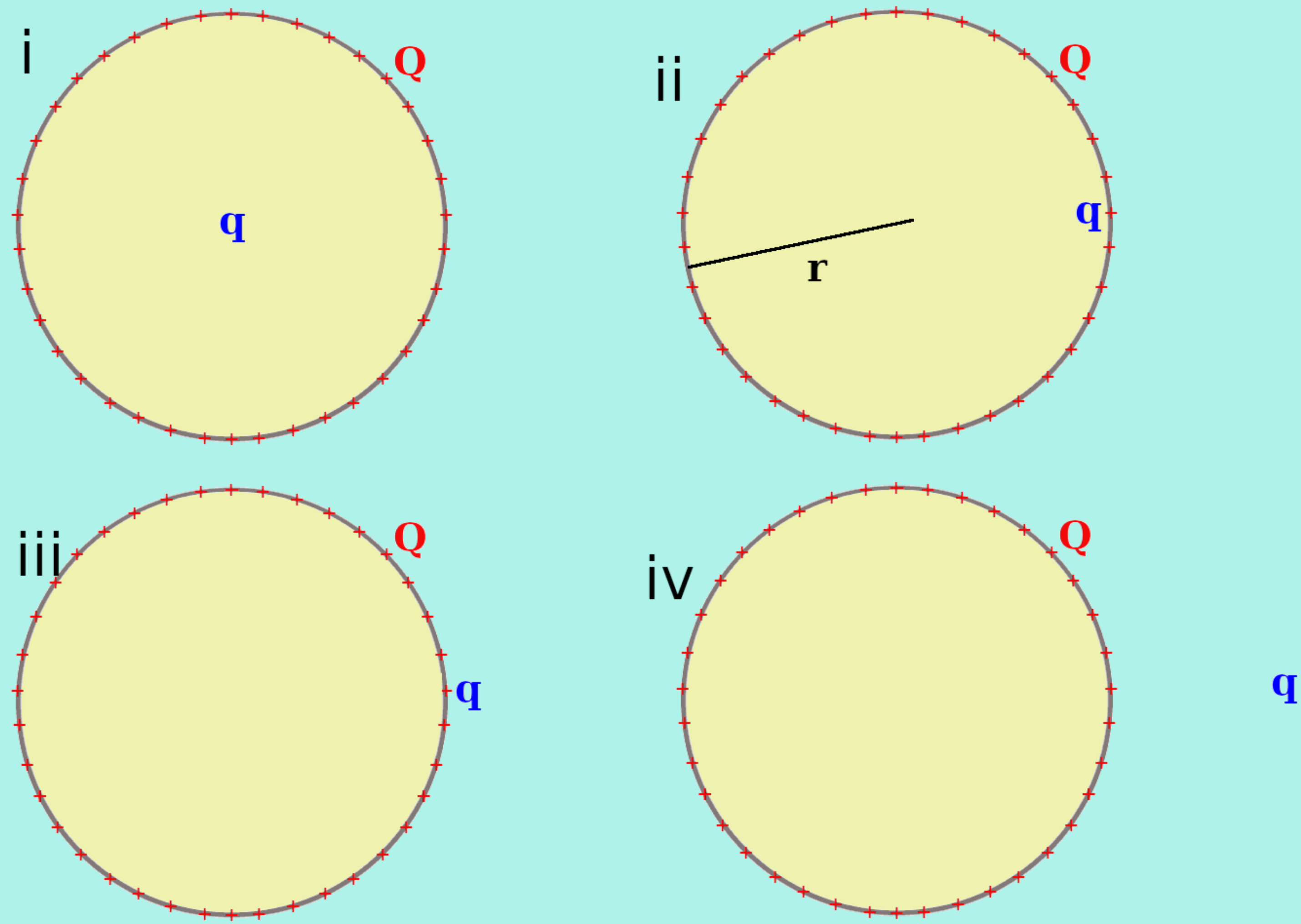
- Separation of Variables
  - Spherical Coords
  - Legendre Polynomials
  - Applying boundary conditions

- Application
  - Conducting sphere in uniform E-field

- Multipole expansion

Hollow Insulating Sphere

- Uniform charge stuck outside
- Free q inside



(A) $i: F=0, ii: F\sim 0, iii: F\sim 0, iv: F=k\,qQ/r^2$

(B) $i: F=0, ii: F=0, iii: F\sim 0, iv: F=k\,qQ/r^2$

(C) $i: F=0, ii: F=0, iii: F=0, iv: F=k\,qQ/r^2$

(D) $i: F=0, ii: F=0, iii: F=k\,qQ/r^2, iv: F=k\,qQ/r^2$

What is the general solution to Laplace in spherical coordinates?

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) = 0$$

Legendre Polynomials are a complete orthogonal basis set like Sines and Cosines

$$\int_0^a \sin\frac{n\pi}{a}x \sin\frac{n'\pi}{a}x\, dx = \frac{a}{2}\delta_{nn'}$$

$$\int_{-1}^{1} P_L(x) P_{L'}(x)\, dx = \frac{2}{2L+1}\delta_{LL'}$$

Legendre Polynomials

$$P_0(x) = 1 \qquad P_0(\cos\theta) = 1$$

$$P_1(x) = x \qquad P_1(\cos\theta) = \cos\theta$$

$$P_2(x) = \frac{(3x^2 - 1)}{2} \qquad P_2(\cos\theta) = \frac{(3\cos^2\theta - 1)}{2}$$

$$P_3(x) = \frac{(5x^3 - 3x)}{2} \qquad P_3(\cos\theta) = \frac{(5\cos^3\theta - 3\cos\theta)}{2}$$

$$P_4(x) = \frac{(35x^4 - 30x^2 + 3)}{8}$$

On the surface of a sphere of radius R,

the potential is $V(\theta) = \cos(3\theta)$

What is the potential inside and outside the sphere?

On the surface of a sphere of radius R,

the potential is $V(\theta) = \cos(3\theta)$

$$\cos(3\theta) = 4\cos^3(\theta) - 3\cos\theta$$

On the surface of a sphere, the potential is $V(\theta)=\cos(3\theta)$
What is the potential inside and outside the sphere?

$$\cos(3\theta)=4\cos^3(\theta)-3\cos\theta$$

On the surface of a sphere or radius R,

the potential is $V(\theta) = 70\cos^4\theta + 10\cos\theta - 14$

What is the potential inside and outside the sphere?

A conducting sphere of radius R is placed in a uniform electric field $\vec{E} = E_0 \hat{z}$.

What is the potential inside and outside the sphere?

What is the potential at P which is “far” outside of an arbitrary continuous charge distribution?

Does distance make the problem simpler?