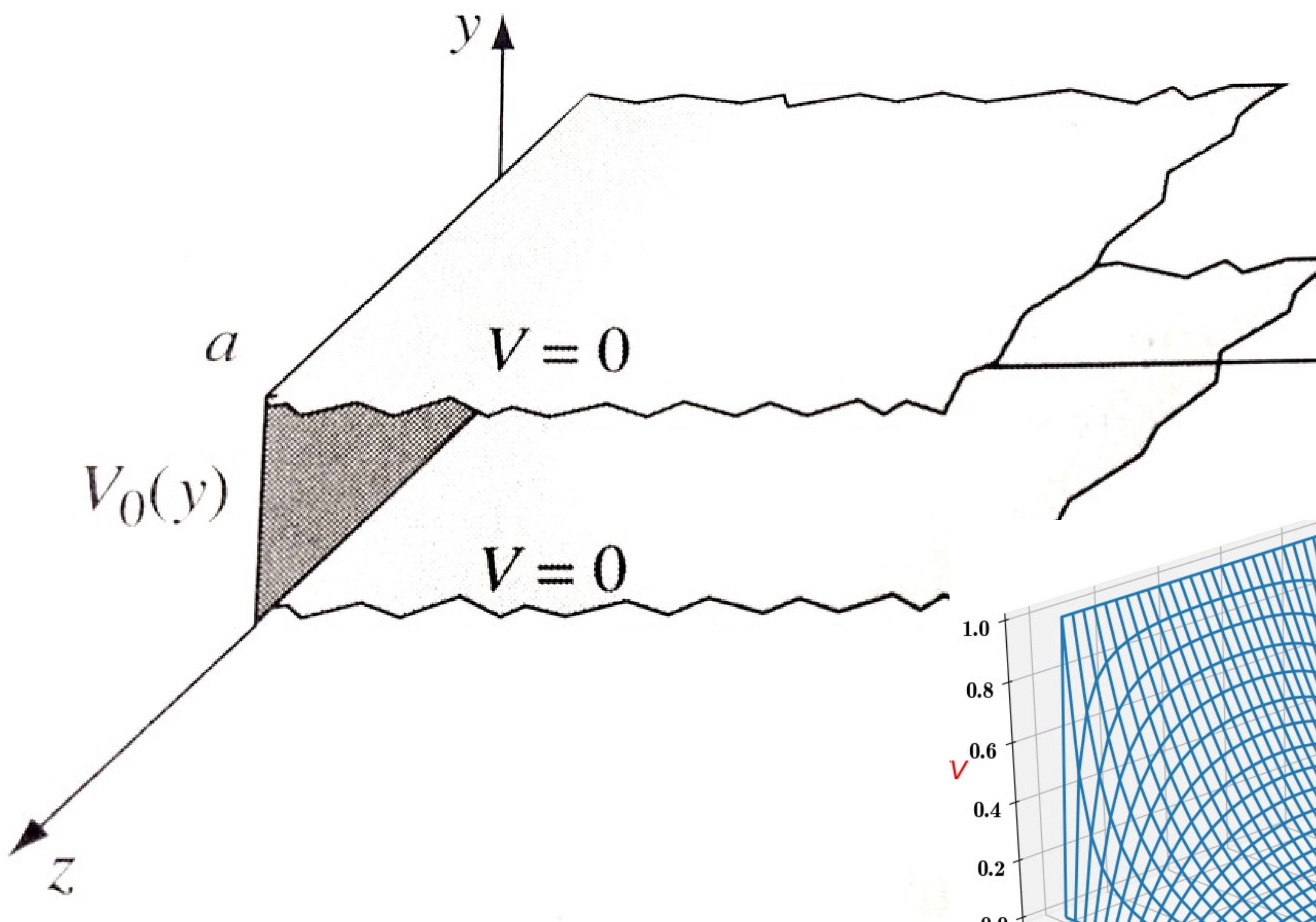
Lecture 21 outline:

- Homework 5
- Separation of Variables
  - Cartesian Coords
  - Spherical Coords
    - Legendre Polynomials

Example 3-3



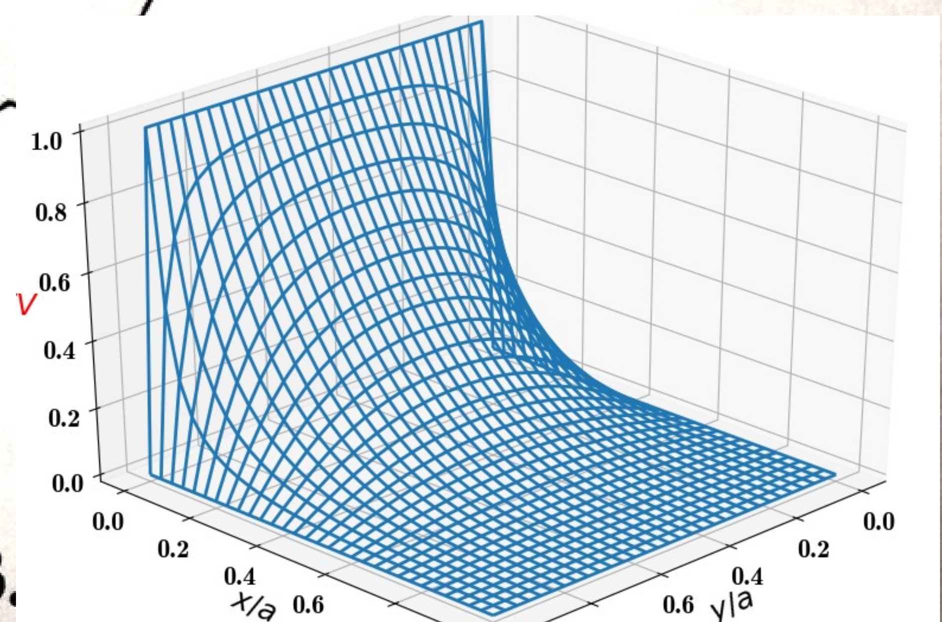


FIGURE 3.

Example 3-3

$(i)\ V=0 \text{ at } y=0$

$(ii)\ V=0 \text{ at } y=a$

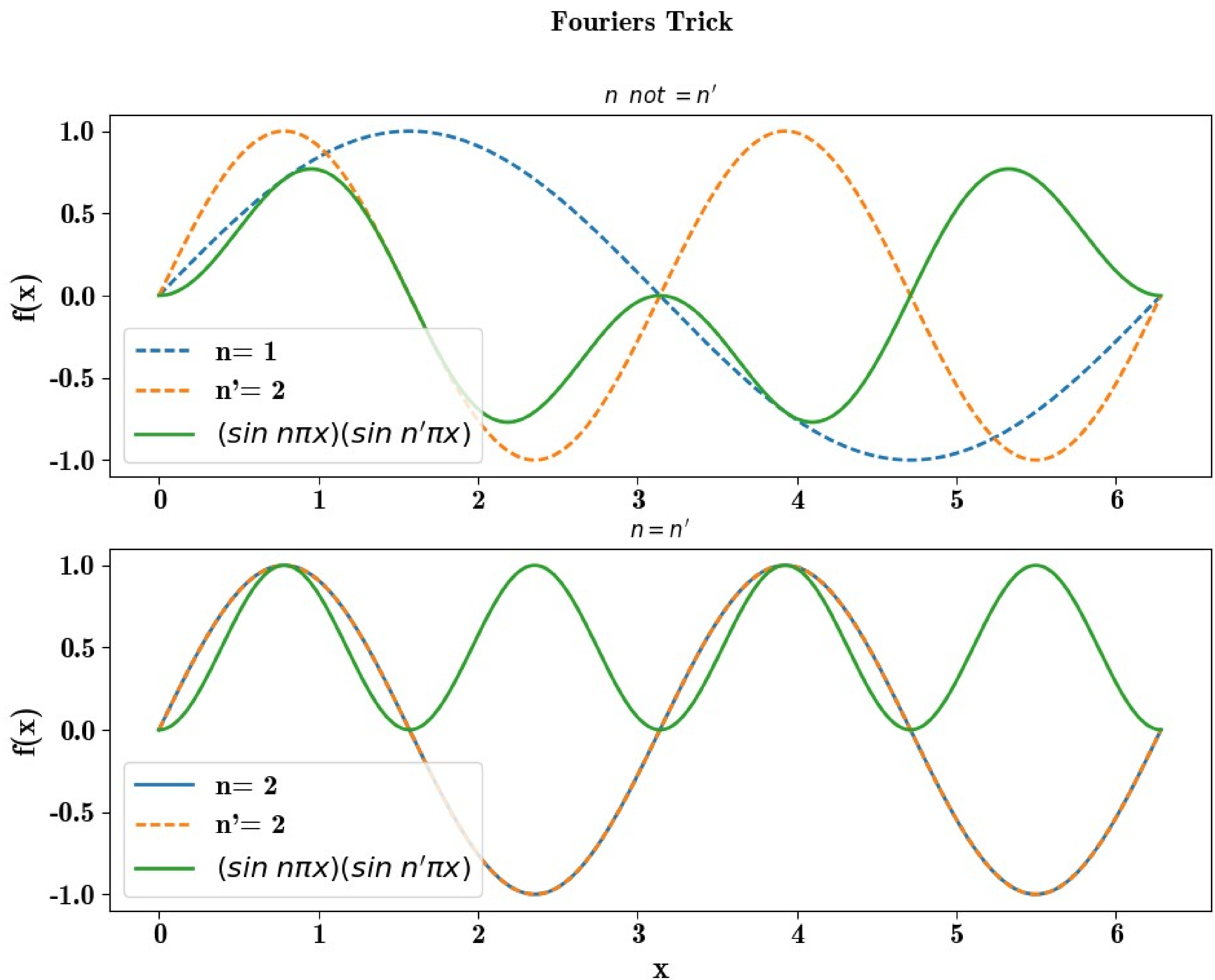
$(iii)\ V=V_0(y) \text{ at } x=0$

$(iv)\ V \rightarrow 0 \text{ at } x \rightarrow \infty$

How to find the Cn?

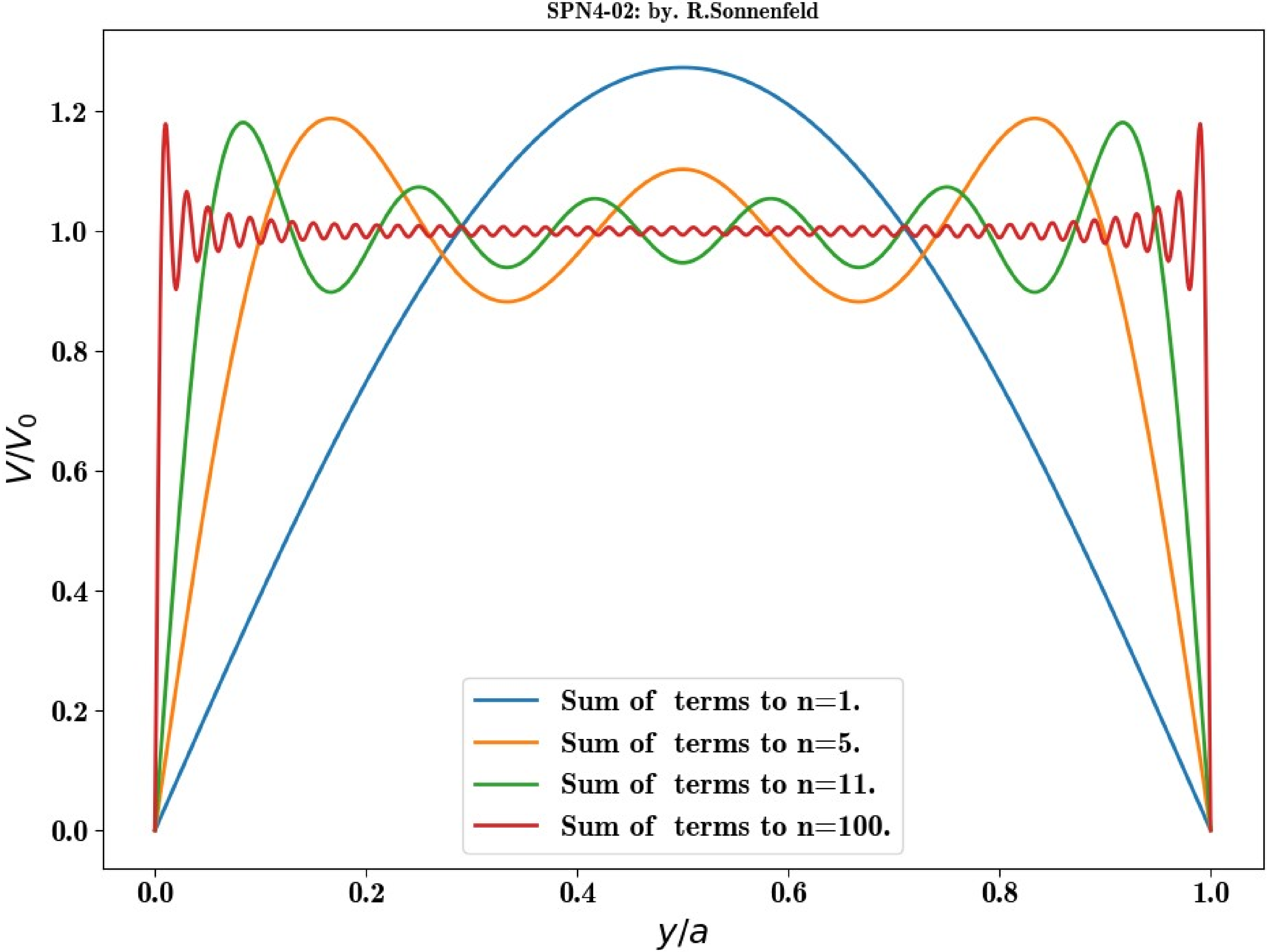
$$V(x,y)=\sum_n C_n e^{(-n\pi x/a)} \sin\frac{n\pi}{a}y$$

$$\int_0^a V_0(y)\sin(n'\pi y/a)\,dy=\sum_n C_n \int_0^a \sin\frac{n\pi}{a}y \sin\frac{n'\pi}{a}y\,dy$$



Fouriers Trick
$n\ \ not\ = n'$
f(x)
1.0
0.5
0.0
-0.5
-1.0
0
1
2
3
4
5
6
n= 1
n'= 2
$(sin\ n\pi x)(sin\ n'\pi x)$
$n = n'$
f(x)
1.0
0.5
0.0
-0.5
-1.0
0
1
2
3
4
5
6
x
n= 2
n'= 2
$(sin\ n\pi x)(sin\ n'\pi x)$

Griffiths Figure 3.19



What is the general solution to Laplace in spherical coordinates?

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) = 0$$

Legendre Polynomials

$$P_0(x) = 1 \qquad P_0(\cos\theta) = 1$$

$$P_1(x) = x \qquad P_1(\cos\theta) = \cos\theta$$

$$P_2(x) = \frac{(3x^2 - 1)}{2} \qquad P_2(\cos\theta) = \frac{(3\cos^2\theta - 1)}{2}$$

$$P_3(x) = \frac{(5x^3 - 3x)}{2} \qquad P_3(\cos\theta) = \frac{(5\cos^3\theta - 3\cos\theta)}{2}$$

$$P_4(x) = \frac{(35x^4 - 30x^2 + 3)}{8}$$