Lecture 15 outline:

- Exam protocol
- Laplace's Equation
  - Extrema are at edges (boundaries)
  - Every point has the average value of its neighborhood
- Relaxation method
- First Uniqueness Theorem
  - If you specify the potential on any set of conductors and Laplace is solved between, there is exactly one solution.
- Method of Images
- Second Uniqueness Theorem
  - If you specify the charge on any set of conductors and Laplace is solved between, there is exactly one solution.

Laplace's Equation (General Properties)

- Extrema are at edges (boundaries)
- Every point has the average value of its neighborhood

Laplace's Equation (3D)

- Every point has the average value of its neighborhood
- Is this surprising? Imagine a charge Q 1 meter from a sphere with R=3.
- $$V_{AVG}(\vec{r}) \stackrel{def}{=} \frac{1}{4\pi R^2} \int V(R,\theta,\phi)\, da$$

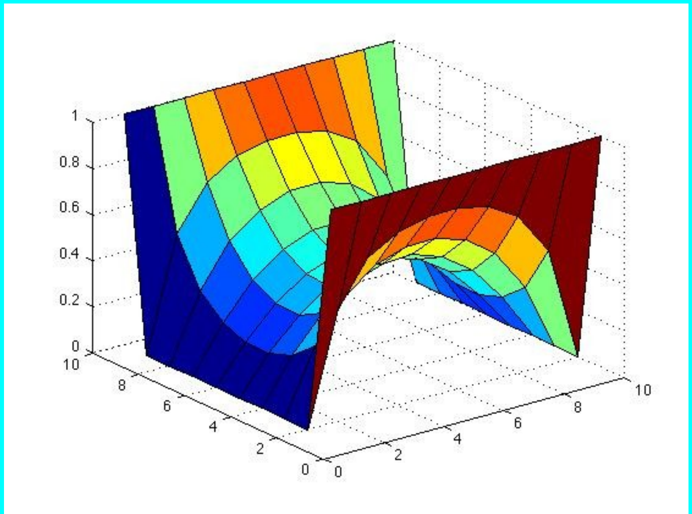
Proof: Every point has the average value of its neighborhood

$$V_{AVG}(\vec{r}) \stackrel{def}{=} \frac{1}{4\pi R^2} \int V(R, \theta, \phi)\, da$$

$$\text{Prove} \ldots \frac{dV_{AVG}}{dR} = 0 \rightarrow V_{AVG} \text{ indep of } R$$

Relaxation Method

- Works in 2D or 3D (or 4D)
- Specify V at boundaries
- Guess V elsewhere.
- Replace V by the average of V.
- Repeat until converged.



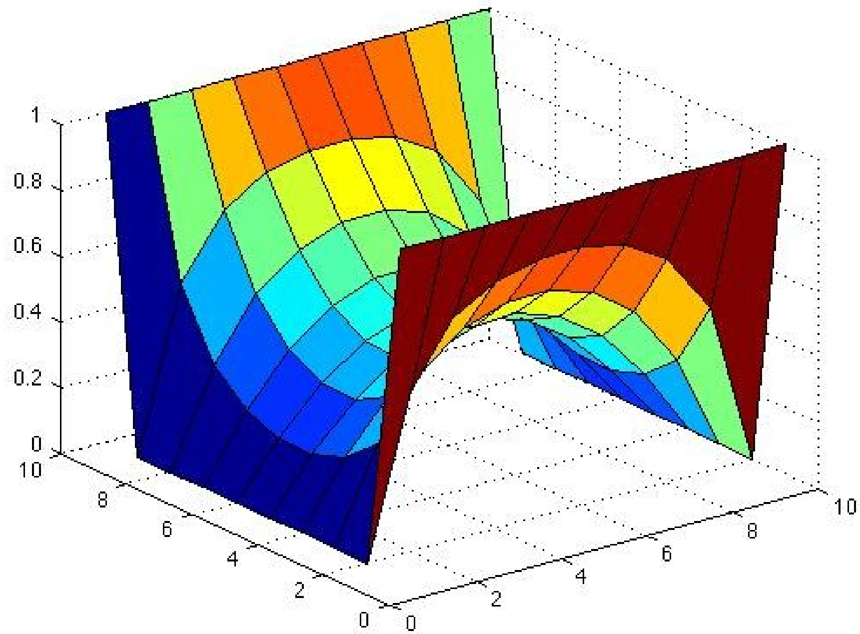
When V is properly calculated then

$$V(i,j)=\frac{1}{4}[V(i+1,j)+V(i-1,j)+V(i,j+1)+V(i,j-1)]$$

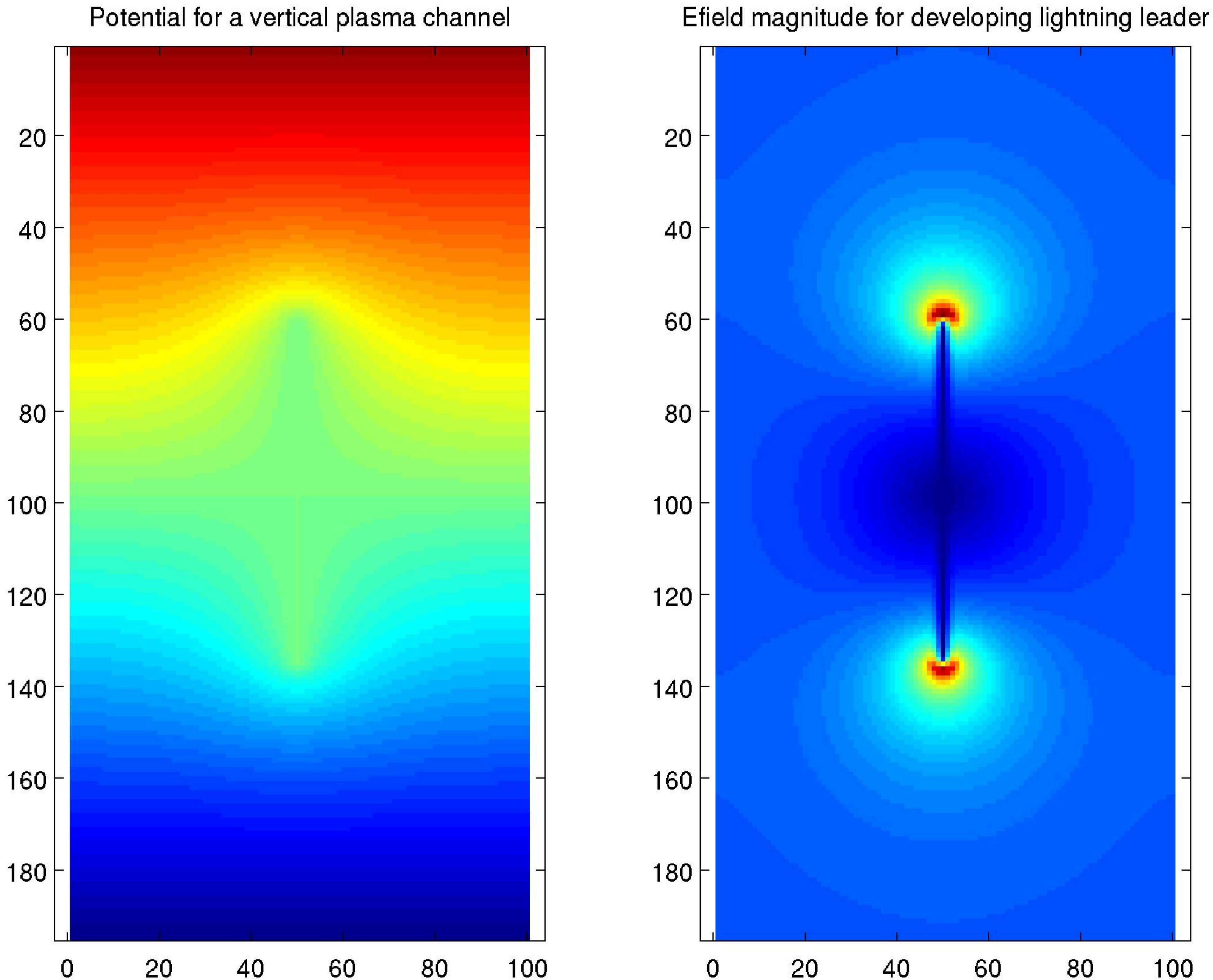
```
A = zeros(10,10);
A(1,:) = 1;
A(10,:)=1;
Aold = 1;
Anew = 0;
k=1;
surf(A)

while (abs(Aold-Anew) > 0.01)
    Aold = A(5,5);
    for i = 2:9
        for j = 2:9
            A(i,j) = (A(i-1,j)+A(i+1,j)+A(i,j-1)+A(i,j+1))/4;
        end
    end
    Anew = A(5,5);
    figure
    surf(A)
end

%surf(A)
```

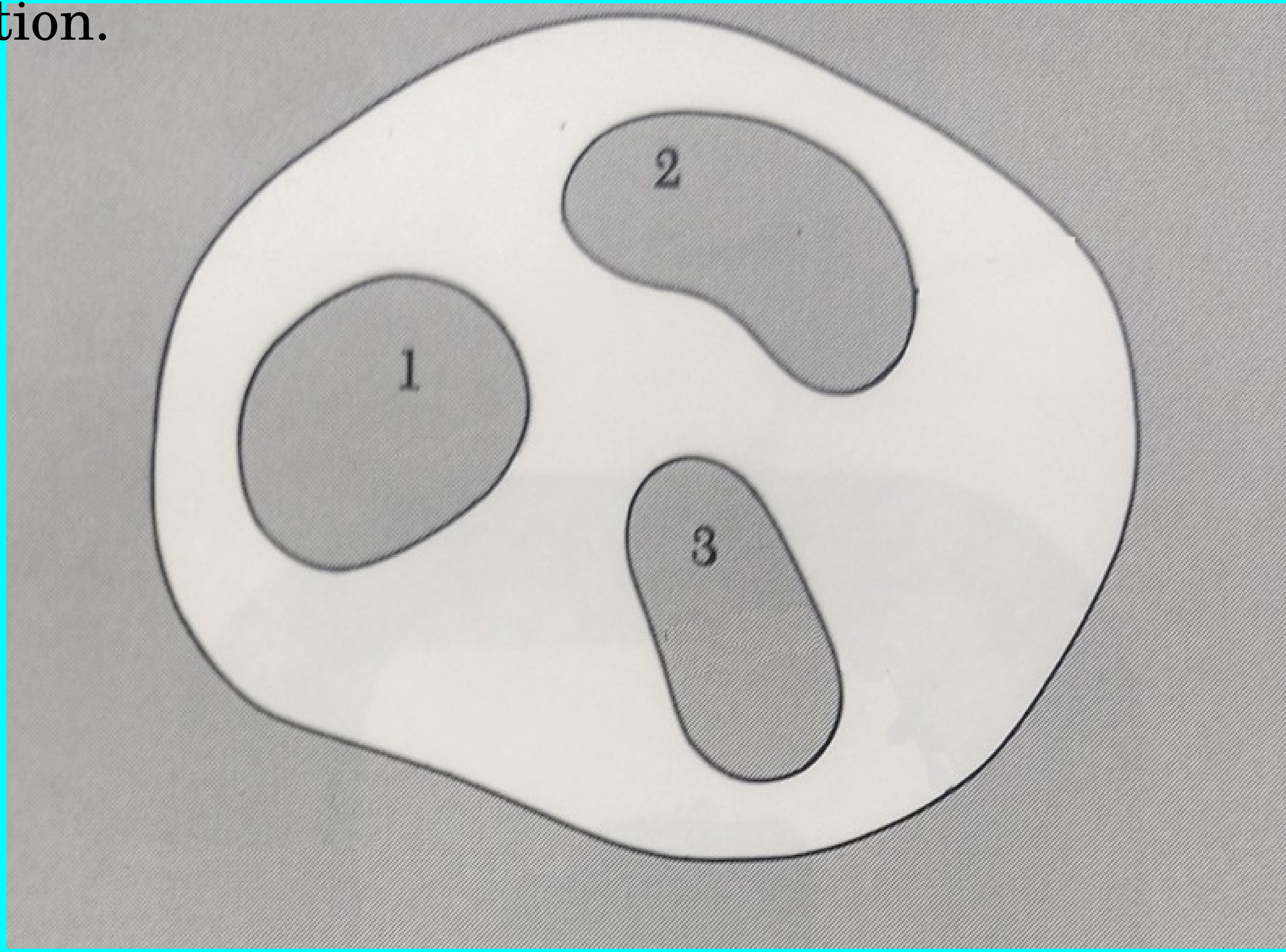


Potential and field of a wire in a constant field



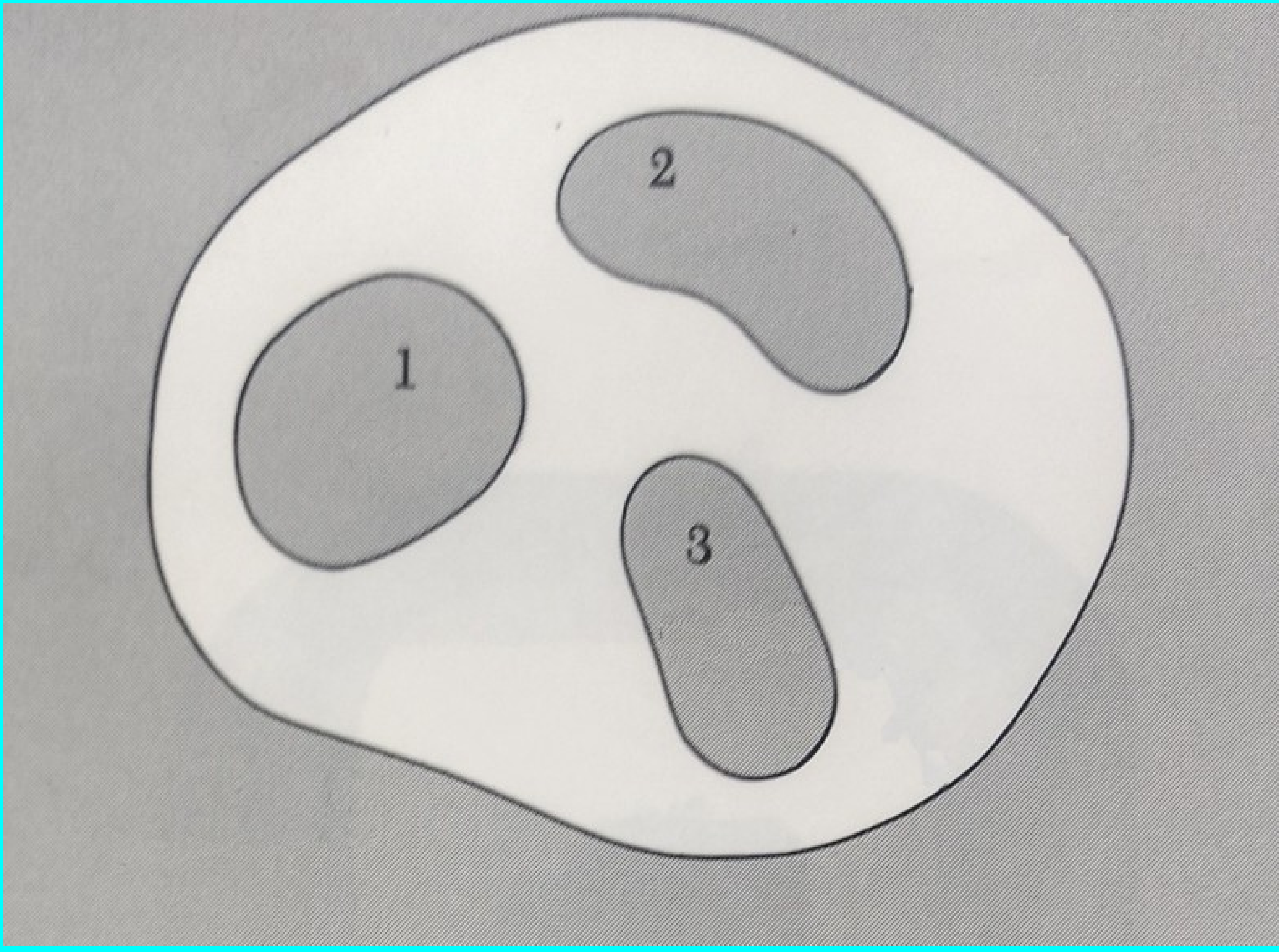
First Uniqueness Theorem

- If you have found a solution to Laplace or Poisson for arbitrary **potentials** on a set of conductors, it is the ONLY solution.



Second Uniqueness Theorem

- If you have found a solution to Laplace or Poisson for arbitrary **charges** on a set of conductors, it is the ONLY solution.



Method of Images

- Given a charge Q (or set Q_i) above a plane conductor the field above the conductor will be the same as if there were an equal and opposite charge below the conductor

Method of Images