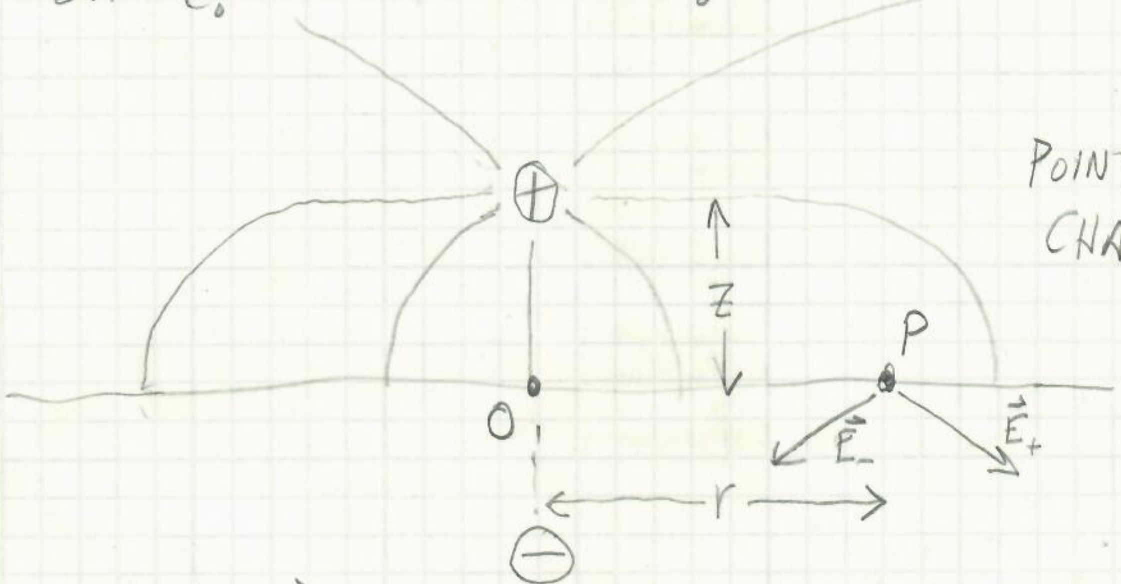
$$-\frac{\partial V}{\partial n} = \frac{\sigma}{\epsilon_0} \qquad \Delta\vec{E}\cdot\hat{n} = \frac{\sigma}{\epsilon_0}$$

POINT IMAGE CHARGE



$$\vec{\mathscr{r}} \equiv \vec{r}_T - \vec{r}_S$$

$$\vec{\mathscr{r}}_+ = r\hat{x} - z\hat{z} \qquad \vec{\mathscr{r}}_- = r\hat{x} + z\hat{z}$$

$$|\mathscr{r}_+| = \sqrt{r^2+z^2} = |\mathscr{r}_-|$$

$$\hat{\mathscr{r}}_+ = \frac{\vec{\mathscr{r}}_+}{\mathscr{r}_+} = \frac{r\hat{x}-z\hat{z}}{\sqrt{r^2+z^2}}$$

$$\vec{E}_P = \vec{E}_+ + \vec{E}_-$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q\,\hat{\mathscr{r}}_+}{\mathscr{r}_+^2} + \frac{-q\,\hat{\mathscr{r}}_-}{\mathscr{r}_-^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r\hat{x}-z\hat{z}+(-r\hat{x}-z\hat{z})}{(r^2+z^2)^{3/2}}\right]$$

$$\vec{E}_P = \frac{q}{4\pi\epsilon_0}\,\frac{-z\hat{z}}{(r^2+z^2)^{3/2}}$$

$$\Delta\vec{E}\cdot\hat{n} = \frac{\sigma}{\epsilon_0} = \vec{E}_P\cdot\hat{z} = \frac{-z}{(r^2+z^2)^{3/2}}\,\frac{q}{4\pi\epsilon_0}$$

$$\therefore\ \sigma = \frac{-qz}{4\pi}\,\frac{1}{(r^2+z^2)^{3/2}}$$

$$Q_{\text{INDUCED}} = \int\sigma\,da = -\int_0^\infty \underbrace{\frac{qz}{4\pi}\,\frac{1}{(r^2+z^2)^{3/2}}}_{\sigma}\ \underbrace{2\pi r\,dr}_{da} = -q\,!!$$