# Lecture 12 outline:

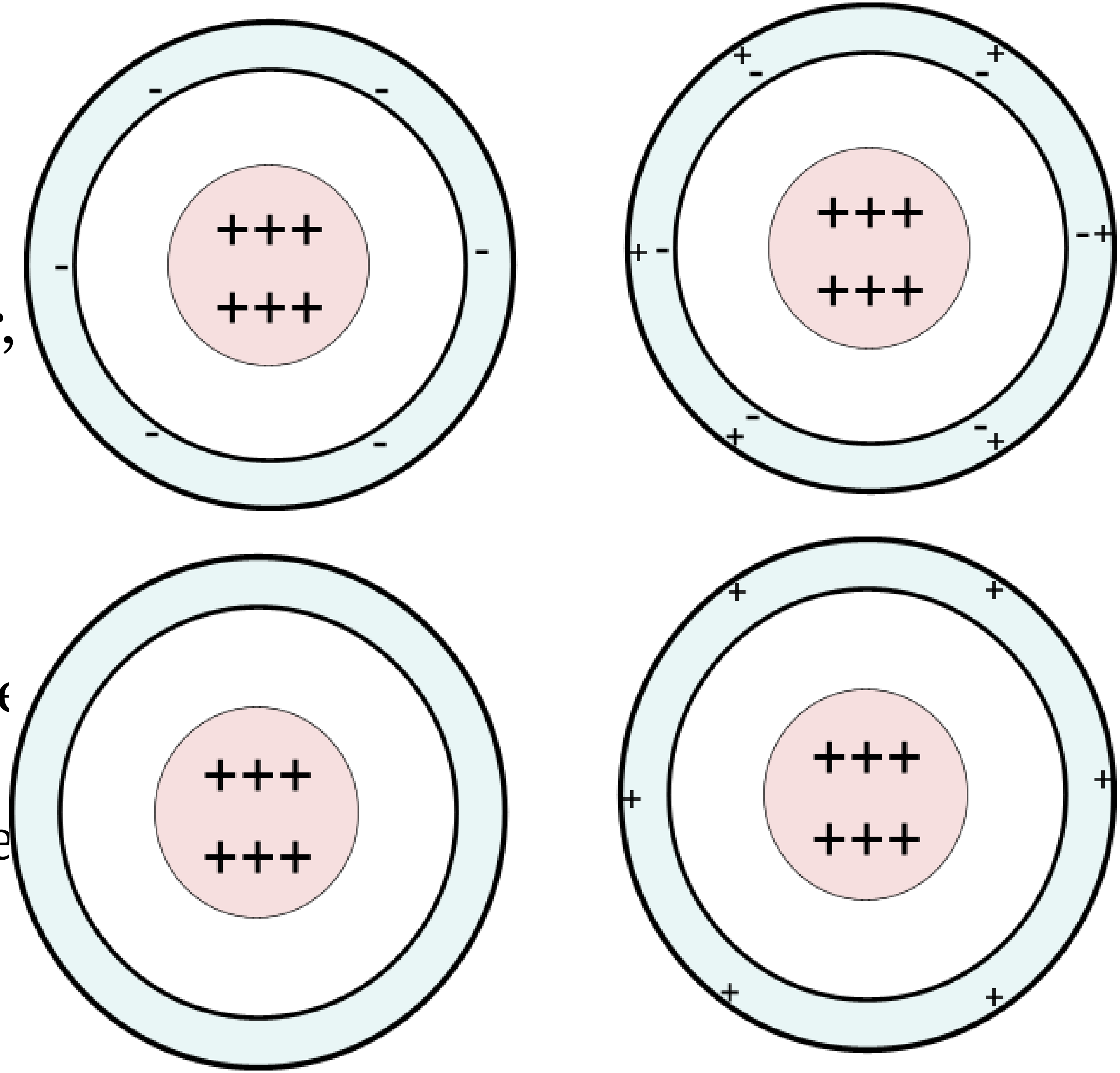
- Behavior of conductors
  - Field lines and intuition
  - Concentric conductors
  - Charge distribution on “lumpy” conductors (for Isaac Edelman)

Field lines must start on + charges

and end on - charges or infinity

Consider concentric and offset conductors

Six charges are placed on a spherical conductor, which is then surrounded by an initially uncharged thick spherical “shell”. How will the charges rearrange themselves once the two objects are put together?



Consider setup from previous slide. There are 6 charges "Q" on the inner conducting sphere, and the outer sphere is initially uncharged. What is the field at a distance r from the center of the inner sphere (where r is outside of the outer sphere)?

A) $$E = \frac{1}{4\pi\epsilon_0}\frac{6Q}{r^2}$$

B) $$E = \frac{1}{4\pi\epsilon_0}\frac{3Q}{r^2}$$

C) $$E = 0$$

D) $$E = -\frac{1}{4\pi\epsilon_0}\frac{6Q}{r^2}$$

Conducting sphere of radius $r_0$ has surface charge density $\sigma_0$.

It is then surrounded by a neutral spherical conductor. What are the charge densities at

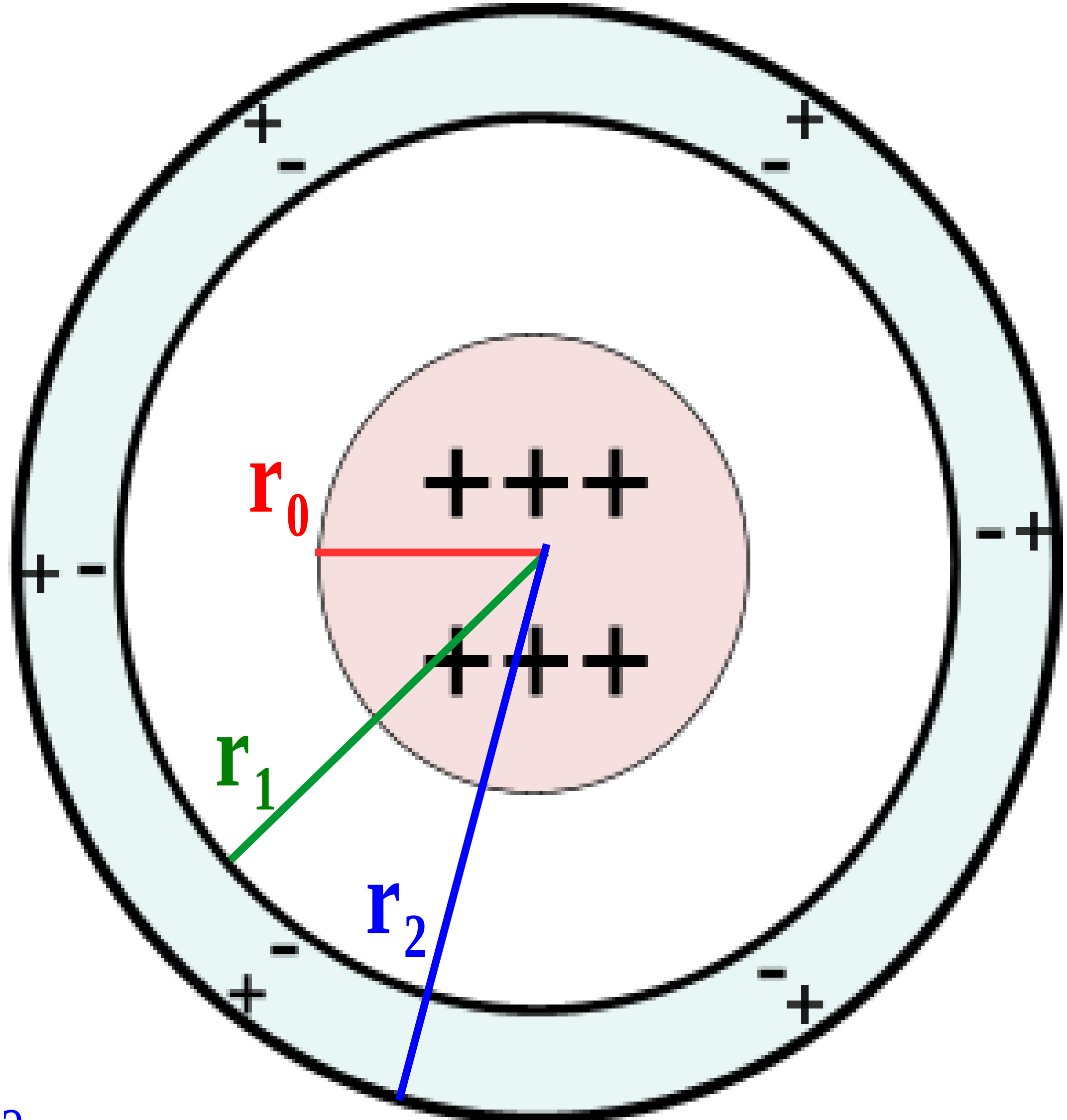
$r_1$ and $r_2$

(A) $\sigma_0$ and $\sigma_0$

(B) $\sigma_0 r_0/r_1$ and $\sigma_0 r_0/r_2$

(C) $\sigma_0 r_1^2/r_0^2$ and $\sigma_0 r_1^2/r_0^2$

(D) $\sigma_0 r_0^2/r_1^2$ and $\sigma_0 r_0^2/r_2^2$



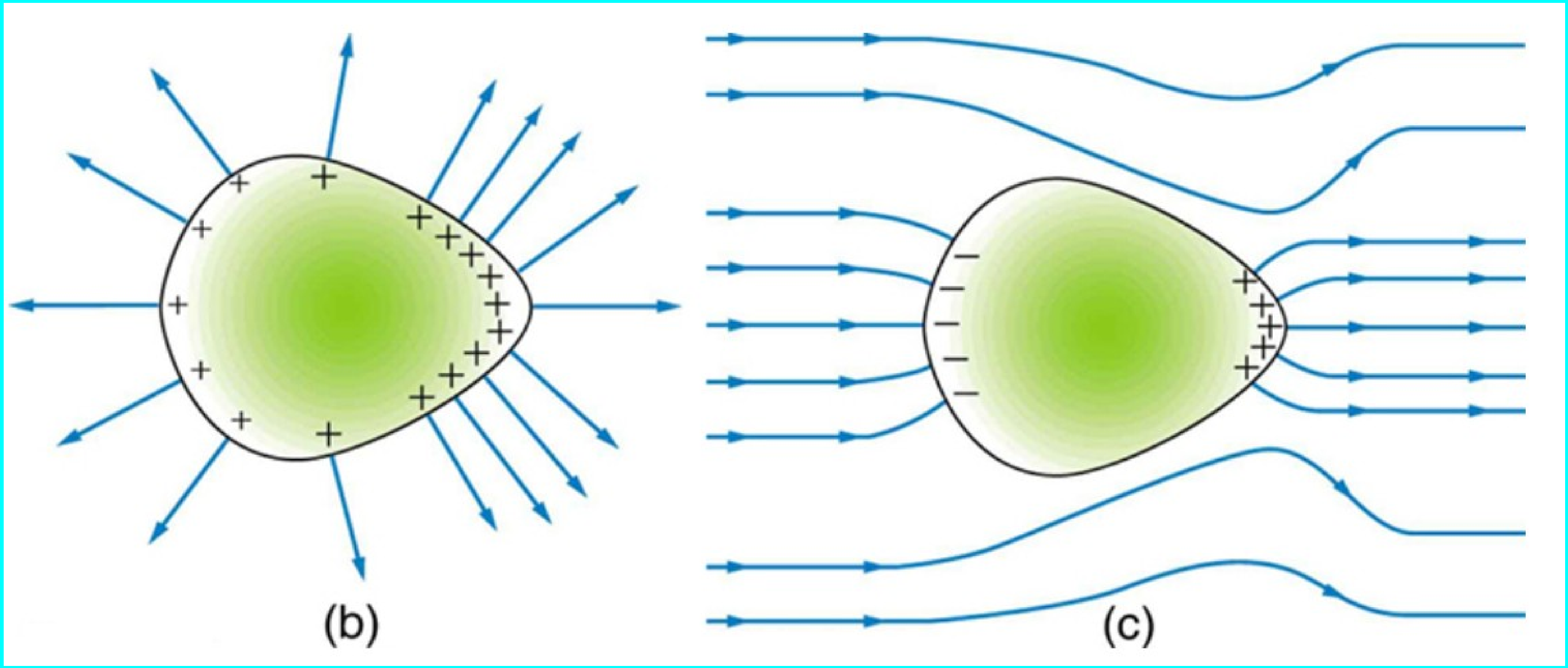
# WHY DOES $\sigma$ VARY WITH CURVATURE?

The picture below is an exaggeration.

Charge only varies as the 4th root of curvature

(and even that is an approximation for ellipsoids

$$\sigma \sim K^{1/4}$$



# Relation between charge density and curvature of surface of charged conductor

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# Of lightning rods, charged conductors, curvature, and things[a)]

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# The lightning-rod fallacy

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It is accepted generally that the electric field strength on the surface of an isolated charged conductor is greatest where the surface curvature is greatest. We show here that there is, in fact, no relationship between these two maxima and that, in general, they are located at different points on the surface. Two classes of analytic examples are offered: one using conformal mapping techniques and the other involving small perturbations of a conducting spherical surface.

It is amusing how pervasive a misconception can be, even in as cut and dried a subject as electrostatics. In this paper we confront the "common knowledge" that the electric field at the surface of an isolated conductor is greatest where the curvature is greatest. It is in fact *true* that when the curvature is singular the **E** field is also singular. The coronal discharge near sharp points is exploited in lightning rods and familiar in electrostatic demonstrations. Proofs that $|\mathbf{E}|$ becomes infinite at sharp outer edges and conical apices can be found in standard textbooks.[1] locations of the maxima can be salvaged by requiring that the conductor be convex.

The nonexistence of a relationship between maximum $|\mathbf{E}|$ and the maximum curvature is rooted in the fact that they depend in entirely different ways on the shape of the surface. Curvature depends only locally on the shape of the surface. At any point of the surface the curvature is determined by the first two derivatives, at that point, of the function specifying the surface. The curvature at that point is independent of what the surface does at points a finite dis-

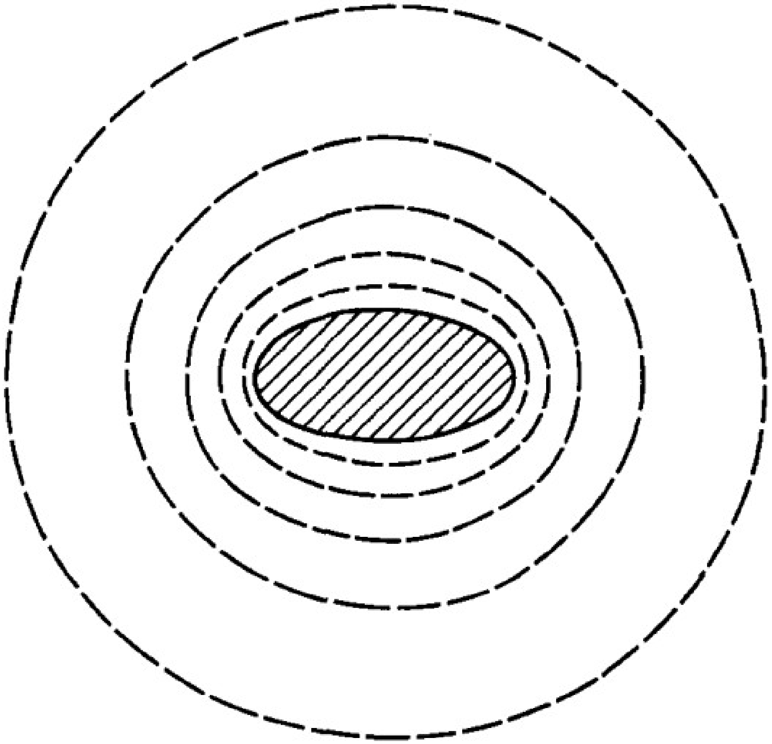


Fig. 1. Proof that |**E**| is always greatest where the curvature is greatest. The equipotentials (dashed lines) are most closely spaced, and hence the **E** strength is greatest, where the curvature is greatest.

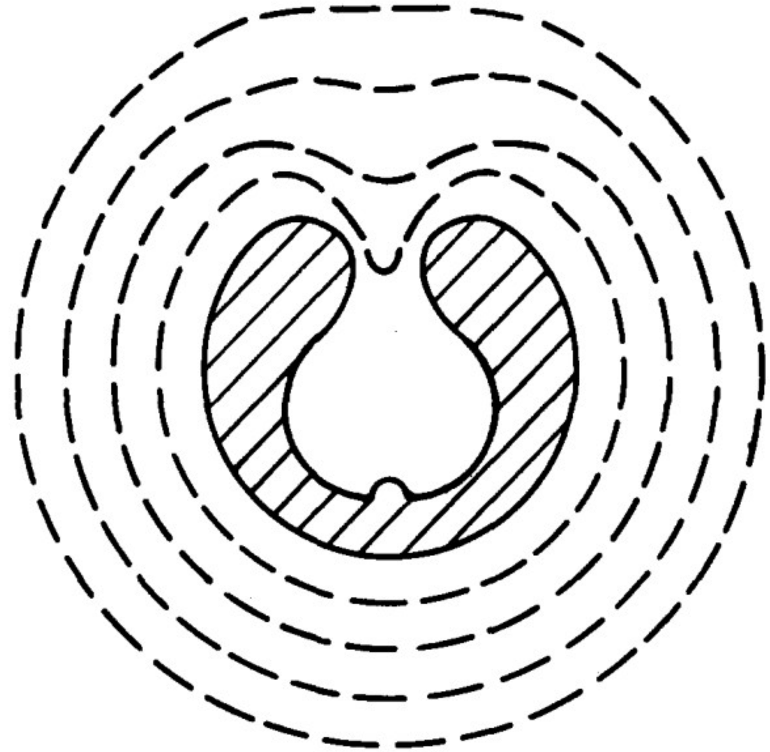
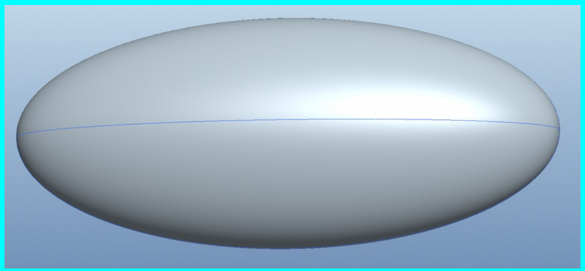


Fig. 2. Proof that |**E**| is not always greatest where the curvature is greatest. (Figure 2, like Fig. 1, represents a cross section of a solid conductor formed by rotating the figure about the vertical symmetry axis.) The curvature is greatest at the bottom of the hollow, on the small hemispherical pimple, but the **E** field there can be made arbitrarily small by narrowing the gap at the top.



# Any equipotential could be a metal surface.

$$\sigma = \epsilon_0 \Delta E_n$$