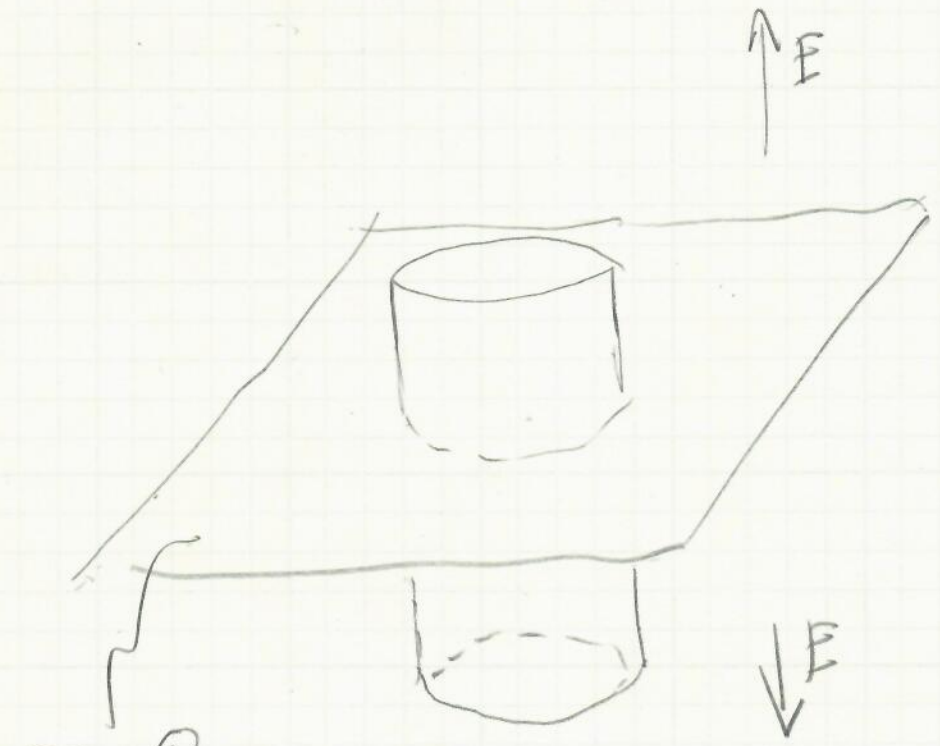
Lecture 10

Student Questions

Field of a plane

Field of a ring

Field of a disk



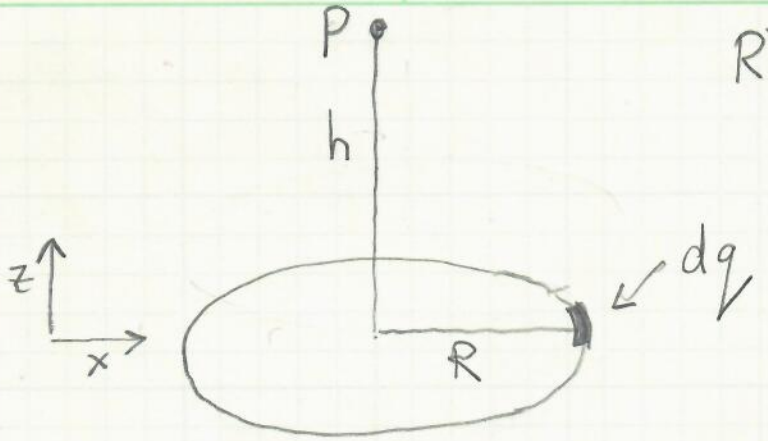
$\sigma = \frac{Q}{A}$

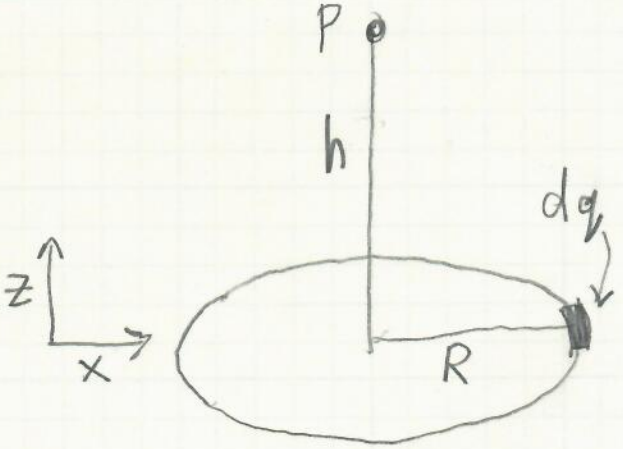
clicker $Q = 10\,nC$

3×3 m square

Ring of Charge

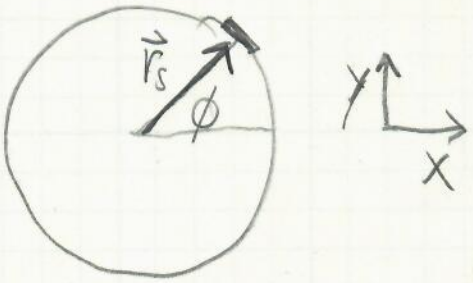
$\lambda = \frac{Q}{2\pi R}$ $\quad \vec{E}_P = ?$



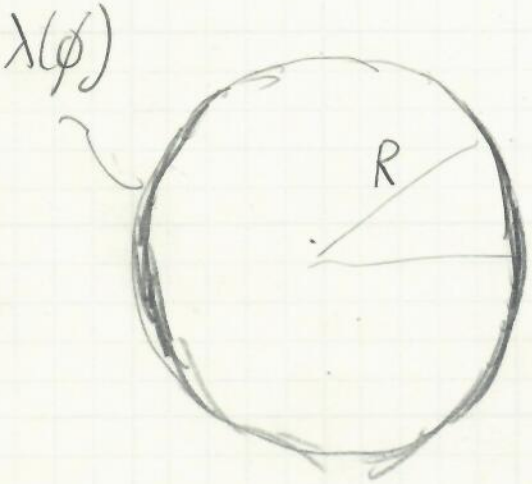


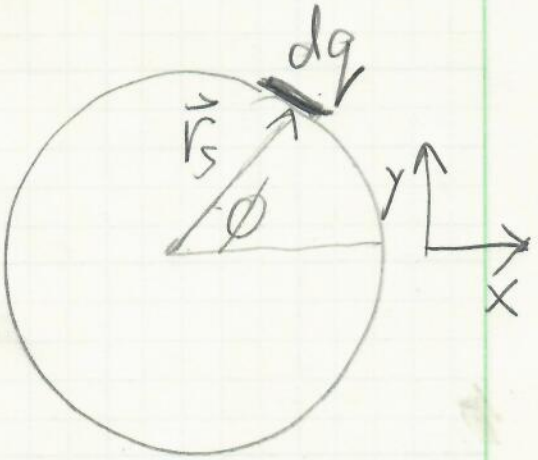
Non-uniform ring

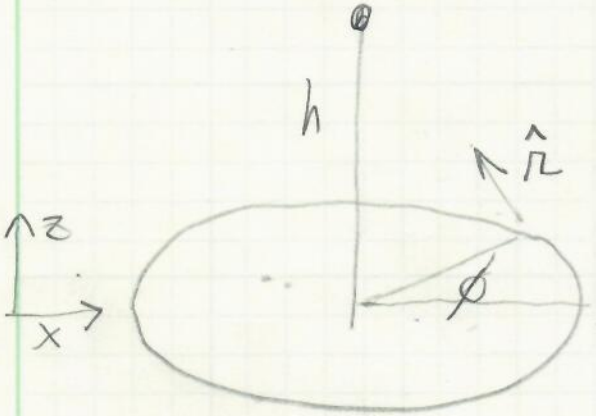
$\lambda = \lambda_0 \cos^2\phi$



What if $\lambda = \lambda_0 \cos^2\phi$







$\vec{r}_T = h\hat{z}$

$\vec{r}_s = R\cos\phi\,\hat{x} + R\sin\phi\,\hat{y}$

$\vec{r} = h\hat{z} - R\cos\phi\,\hat{x} - R\sin\phi\,\hat{y}$

$r^2 = h^2 + R^2\cos^2\phi + R^2\sin^2\phi$

$= h^2 + R^2$

$\hat{r} = \dfrac{h\hat{z} - R\cos\phi\,\hat{x} - R\sin\phi\,\hat{y}}{\sqrt{h^2+R^2}}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq\,\hat{r}}{r^2}$$

$dq = \lambda\, d\ell = \lambda_0\cos^2\phi\, R\, d\phi$

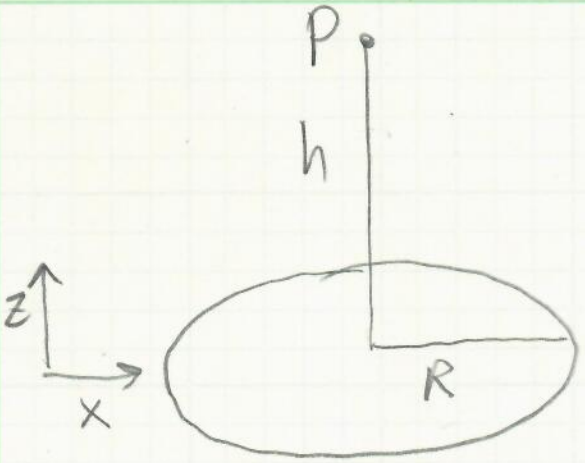
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^{2\pi} \underbrace{\lambda_0\cos^2\phi\, R\, d\phi}_{dq}\ \underbrace{\frac{1}{h^2+R^2}}_{1/r^2}\ \underbrace{\left(\frac{h\hat{z} - R\cos\phi\,\hat{x} - R\sin\phi\,\hat{y}}{\sqrt{h^2+R^2}}\right)}_{\hat{r}}$$

$\overline{\cos^2\phi} = \frac{1}{2}$

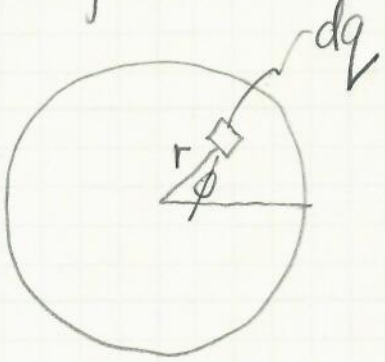
$\hat{x} + \hat{y} \to 0$

$$\vec{E} = \frac{\lambda_0 R}{4\pi\epsilon_0}\,\frac{1}{2}\phi\Big|_0^{2\pi}\,\frac{h}{(h^2+R^2)^{3/2}}$$

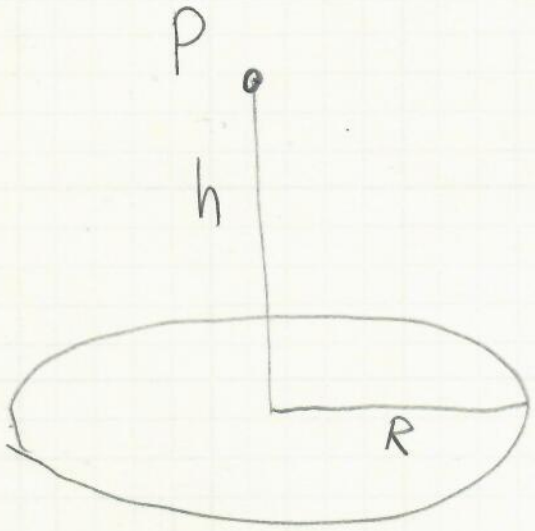
$$= \frac{\lambda_0 h R}{4\epsilon_0}\,\frac{1}{(h^2+R^2)^{3/2}}$$

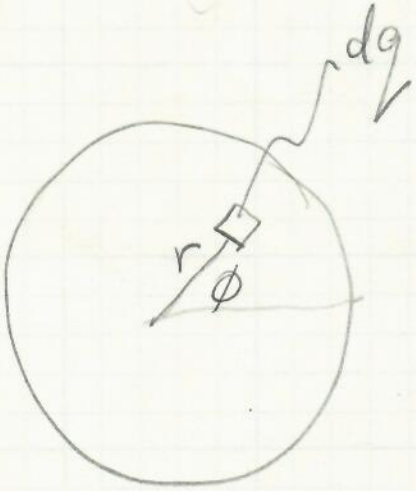


Field of disk



Field of disk





$$\vec{r}_T = h\hat{z}$$

$$\vec{r}_s = r\cos\phi\hat{x} + r\sin\theta\hat{y}$$

$$\vec{n} = h\hat{z} - r\cos\phi\hat{x} - r\sin\theta\hat{y}$$

$$n^2 = h^2 + r^2$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{n^2}\hat{n} \qquad \text{But } dq = \sigma\, da$$

$$da = r\, d\phi\, dr$$

$$\vec{E} = \frac{\sigma}{4\pi\epsilon_0}\iint (r\,d\phi\,dr)\left(\frac{1}{h^2+r^2}\right)\left(\frac{h\hat{z} - r\cos\phi\hat{x} - r\sin\phi\hat{y}}{h^2+r^2}\right)$$