$$V = -\int_a^b \vec{E}\cdot d\vec{l}$$

Let $E = k\frac{q}{r^2}\hat{r}$

$$V = -\int_\infty^r \frac{kq}{r'^2}\hat{r}\cdot dr\,\hat{r}$$

If ∞ is reference potential gets very simple

$$= \frac{kq}{r}\Big|_\infty^r = \frac{kq}{r} - 0$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{|\vec{r}-\vec{r}_i'|} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i}$$

$$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{\mathscr{r}}$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, d\tau}{\mathscr{r}}$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\, da}{\mathscr{r}}$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dl}{\mathscr{r}}$$

Click

Potential

If $\nabla \times \vec{F} = 0$ then $\vec{F} = \nabla \phi$

Because $\int (\nabla \times \vec{F}) \cdot da = \oint \vec{F} \cdot dl$ Formula 1.57

Saying $\nabla \times F = 0$ everywhere is equivalent to

$\oint \vec{F} \cdot d\vec{l} = 0$ on any closed loop

Saying $\oint \vec{F} \cdot d\vec{l} = 0$ is equivalent to

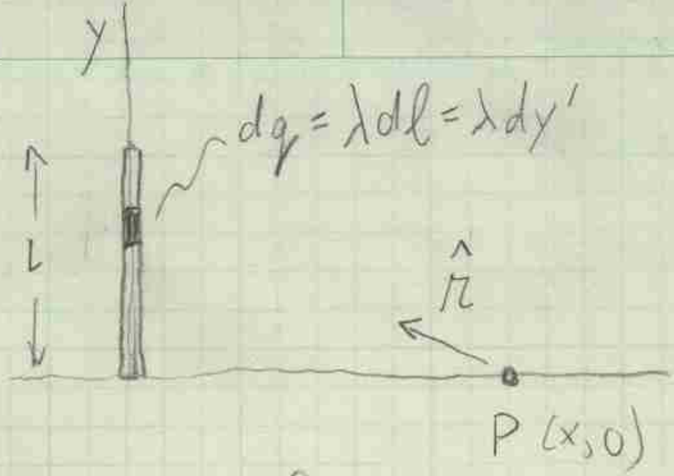
$\int_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{l}$ is path independent

Show pic
Show voltmeter

$$-\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l} = \Delta V$$

$$+\int_{\vec{a}}^{\vec{b}} = -\int_{\infty}^{\vec{a}} + \int_{\infty}^{\vec{b}}$$

$$\int_{a}^{\infty} + \int_{\infty}^{b} = \int_{a}^{b}$$



$$\vec{r} = \hat{x}x - y'\hat{y}$$

$$|r| = x^2 + y'^2$$

$$\hat{r} = \frac{x\hat{x} - y'\hat{y}}{\sqrt{x^2+y'^2}}$$

$$E = \frac{1}{4\pi\epsilon_0}\int \frac{dq\,\hat{r}}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda dy'}{x^2+y'^2}\,\frac{x\hat{x} - y'\hat{y}}{\sqrt{x^2+y'^2}}$$

$$E_y = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{-\lambda y'\,dy'}{(x^2+y'^2)^{3/2}} \qquad u = x^2+y'^2 \qquad du = 2y'dy'$$

$$= \frac{-\lambda}{4\pi\epsilon_0}\int_{x^2}^{x^2+L^2} \frac{1}{2}\,u^{-3/2}\,du$$

$$E_y = \frac{\lambda}{4\pi\epsilon_0}\,\frac{1}{\sqrt{u}}\Bigg|_{x^2}^{x^2+L^2}$$

$$E_y = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{x^2+L^2}} - \frac{1}{\sqrt{x^2}}\right)$$

Let $L >> x \qquad \frac{x}{L} << 1$

$$= \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{L\sqrt{\left(\frac{x}{L}\right)^2+1}} - \frac{1}{x}\right)$$

$$E_y = \frac{\lambda}{4\pi\epsilon_0}\left(\underbrace{\frac{1}{L}\left(1-\frac{1}{2}\frac{x^2}{L^2}\right)}_{0} - \frac{1}{x}\right)$$

$$= -\frac{\lambda}{4\pi\epsilon_0 x}$$

$$(1+\delta)^{-1/2}$$

$$= 1 - \frac{1}{2}\delta$$