Lecture 8

Differential Gauss
Thunderstorms
Field of Continuous Distributions
Potential of Continuous Distrib
Field of a half line

Ch 2 covers fully half of Phys 2

Coulomb's Law

Gauss Law

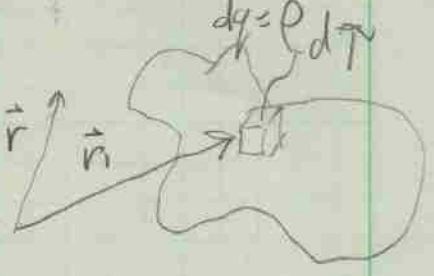
Electric Potential

Electrostatic Energy

Questions/Ch 2

Coulomb $\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i \hat{\pi}_i}{\pi_i^2}$

$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq \hat{\pi}}{\pi^2}$



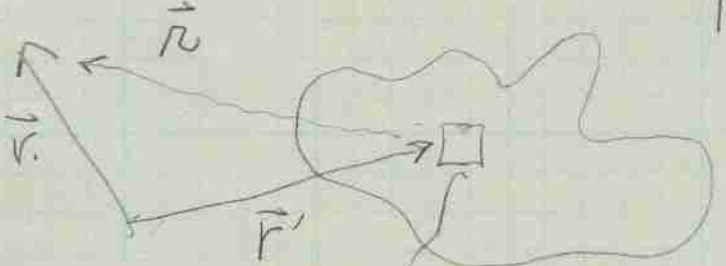
$dq = \rho\, d\tau$

spherical $= \rho(r', \theta', \phi')\, r'^2 \sin\theta'\, dr'\, d\theta'\, d\phi'$

cylindrical $= \rho(s', \phi', z')\, s'\, ds'\, d\phi'\, dz'$

cartesian $= \rho(x', y', z')\, dx'\, dy'\, dz'$

$$\vec{E}(r, \theta, \phi) = \frac{1}{4\pi\epsilon_0} \int_0^\infty dr' \int_0^{2\pi} d\phi' \int_0^{\pi} d\theta'\, \rho(r', \theta', \phi') r'^2 \sin\theta' \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^{3/2}}$$



$dq = \sigma\, da$

Given $\oint \vec{E} \cdot d\vec{a} = \int \nabla \cdot \vec{E}\, d\tau$

Since $\rho = \frac{Q}{\tau} \rightarrow d\rho = \frac{dQ}{\tau}$

$Q = \int \rho\, d\tau$ ← We've already used this

$Q = \int_0^R r^2 dr \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \rho\, d\theta$

What happens to Gauss's Law

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} \rightarrow \frac{1}{\epsilon_0}\int \rho\, d\tau$$

$$\oint \nabla \cdot E\, d\tau =$$

Let volume be small

$$\nabla \cdot E\, \Delta\tau = \frac{\rho}{\epsilon_0}\Delta\tau$$

$$\nabla \cdot \vec{E} = \rho/\epsilon_0$$

Given $\vec{E} = \rho_0 r^2 \hat{r}$

$$\frac{\rho}{\epsilon_0} = \nabla \cdot \vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\rho_0 r^4$$

$$= \frac{1}{r^2} 4\rho_0 r^3$$

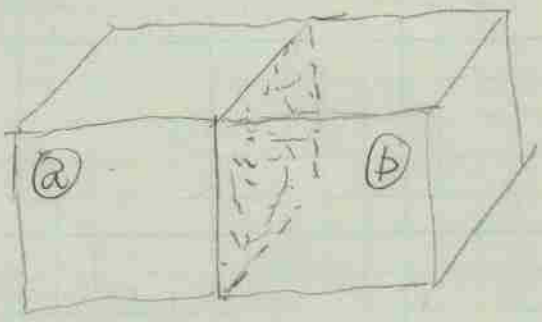
$$\rho = 4\rho_0 r \epsilon_0$$

LEMMA

"PROOF" of divergence theorem

$$\operatorname{div}\vec{E} \equiv \frac{1}{\tau}\lim_{\tau\to 0}\oint \vec{E}\cdot d\vec{a}$$

$$\int \operatorname{div}\vec{E}\, d\tau = ?$$



$$\int_{a+b} \operatorname{div}\vec{E}\, d\tau = \int_a \operatorname{div}\vec{E}\, d\tau + \int_b \operatorname{div}\vec{E}\, d\tau$$

$$\approx \{\operatorname{div}\vec{E}\}\tau_a + \{\operatorname{div}\vec{E}\}\tau_b$$

$$\left\{\frac{1}{\tau_a}\oint_a \vec{E}\cdot d\vec{a}\right\}\tau_a + \left\{\frac{1}{\tau_b}\oint_b \vec{E}\cdot d\vec{a}\right\}\tau_b$$

$$= \oint_a \vec{E}\cdot d\vec{a} + \int_b \vec{E}\cdot d\vec{a}$$

The stippled surface cancels
only the outer surface matters

The definition $\frac{1}{\tau}\lim_{\tau\to 0}\oint \vec{E}\cdot d\vec{a}$ is equivalent to

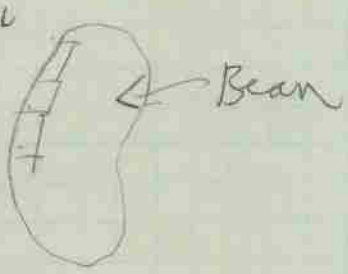
$$\operatorname{div}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\vec{F} = q\vec{E}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_i q_i \frac{\hat{r}_i}{r_i^2}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int dq \frac{\hat{r}_i}{r_i^2}$$

$dq = \rho\, d\tau$



$dq = \sigma\, da$

$dq = \lambda\, dl$

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \qquad \vec{E} = -\nabla V$$

$$V(x,y,z) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{\sqrt{(x-x_i')^2+(y-y_i')^2+(z-z_i')^2}} \qquad E_x = -\frac{\partial V}{\partial x}$$

$$\frac{\partial V}{\partial x} = \frac{1}{4\pi\epsilon_0}\sum_i q_i \left(-\frac{1}{2}\right)\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-3/2}(2)x$$

$$= \frac{1}{4\pi\epsilon_0}\sum_i q_i \frac{x-x'}{|r_i|^3} = \frac{1}{4\pi\epsilon_0}\sum_i q_i \frac{1}{r_i^2}\left(\frac{\vec{r}_x}{r_i}\right) \to \hat{r}_i$$

$$V = -\int_a^b \vec{E} \cdot d\vec{l}$$

Let $E = \frac{kq}{r^2}\hat{r}$

If ∞ is reference potential gets very simple

$$V = -\int_\infty^r \frac{kq}{r^{12}}\hat{r} \cdot dr\,\hat{r}$$

$$= \left.\frac{kq}{r}\right|_\infty^r = \frac{kq}{r} - 0$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{|\vec{r}-\vec{r}_i'|} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, dV}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\, da}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dl}{r}$$

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