"DEL DELIVERS"

HW Due Fri office Hrs 4-6 M/W 575-838-7113

Admission

Reading was 2.2 - Wed 2.1

Did lecture 5

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

scalar in Vector out $\vec{E} = -\nabla V$

$$= -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z}$$

Vector in, scalar out

$\nabla \cdot \vec{E} = \rho/\epsilon_0 \qquad \rho = Q/\tau$

$$\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) \cdot \left(E_x\hat{x} + E_y\hat{y} + E_z\hat{z}\right)$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \rho/\epsilon_0$$

$\frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \vec{E} \qquad \vec{E} = \frac{\vec{F}}{q} = \frac{[N]}{[C]}$

$$\frac{q}{r^2 E} = 4\pi\epsilon_0$$

$$\frac{[C]}{[m^2]\frac{[N]}{[C]}} = \epsilon_0 = \frac{C^2}{N\cdot m^2} = \frac{C^2}{J\cdot m}\ ; \ \frac{1}{\epsilon_0} = \frac{N\cdot m^2}{C^2}$$

$$\frac{[N]}{[C][m]} = \rho\frac{[N]m^2}{[C^2]}$$

$$1 = \rho\frac{[m^3]}{[C]}$$

$$\nabla \cdot \underline{E} \equiv \mathrm{div}\,\vec{E} \equiv \lim_{\tau \to 0} \frac{1}{\tau} \oint E \cdot da$$

Vector in Vector Out

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow \nabla \times \vec{E} = 0 \text{ Electrostatics}$$

$$\mathrm{curl}\,E = \lim_{a \to 0} \frac{1}{a} \oint \vec{E} \cdot d\vec{l}$$

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = \hat{x}\,\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \hat{y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \qquad \vec{E} = -\nabla V$$

$$\nabla \cdot \nabla V = \rho/\epsilon_0 \quad \text{Poisson's Eqn}$$

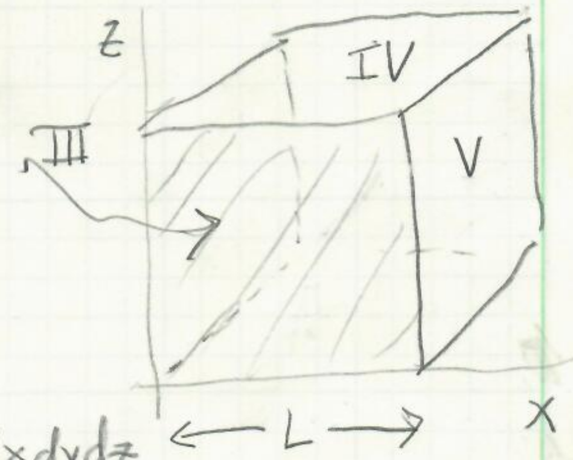
$$\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) \cdot \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) V$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \rho/\epsilon_0 \quad \text{"del-squared"}$$

$$\nabla^2 V = \rho/\epsilon_0 \text{ or } = 0 \text{ no free charge}$$

Let $\vec{E} = y^2\hat{x} + (3xy+z)\hat{y} + xyz\hat{z}$

- Test the divergence theorem on a cube of side L



$$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= 0 + 3x + xy$$

Is $\oint E\cdot d\vec{a} = \int \nabla\cdot E\, d\tau$?

$$\int \nabla\cdot E\, d\tau = \int_0^L\int_0^L\int_0^L 3x + xy \; dx\,dy\,dz$$

$$= L\int_0^L\int_0^L 3x + xy \; dx\,dy$$

$$= L\int_0^L 3xy + \frac{xy^2}{2}\Bigg|_{y=0}^{y=L} = L\int_0^L\left(3xL + \frac{xL^2}{2}\right)dx$$

$$= \frac{3}{2}L^4 + \frac{L^5}{4}$$

IV = Top   V = Right   VI = Back

I = Bottom   II = Left   III = Front

$\hat{n}_I = -\hat{z}$ $\hat{n}_{II} = -\hat{x}$ $\hat{n}_{III} = -\hat{y}$

$\hat{n}_{IV} = \hat{z}$ $\hat{n}_V = \hat{x}$ $\hat{n}_{VI} = \hat{y}$

$d\vec{a}_I = -dx\,dy\,\hat{z}$ $d\vec{a}_{II} = -dy\,dz\,\hat{x}$ $d\vec{a}_{III} = -dx\,dz\,\hat{y}$

I: $\int \vec{E}\cdot d\vec{a} = -\int_0^L dx \int dy\; xy(0) = 0$

IV: $\int \vec{E}\cdot d\vec{a} = \int_0^L dx\int_0^L dy\; xyL = \int_0^L dx\; x\frac{L^3}{2} = \frac{L^5}{4}$

II: $\int \vec{E}\cdot da = -\int_0^L dy\int_0^L dz\; y^2$

IV: $= +\int_0^L dy\int_0^L dz\; y^2$

Σ to 0

III : $\int \vec{E} \cdot d\vec{a} = -\int dx \int dz \, (3xy + z)$ where $y = 0$

$$\int \vec{E} \cdot d\vec{a} = -\int dx \int z\,dz = -L \int z\,dz = -L^3/2$$

VI : $= \int dx \int dz \, (3xL + z)$

$$= \frac{3}{2} L^4 + L^3/2$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{3}{2} L^4 + \frac{L^5}{4}$$

$$\int \nabla \cdot E \, d\tau = \text{same}$$

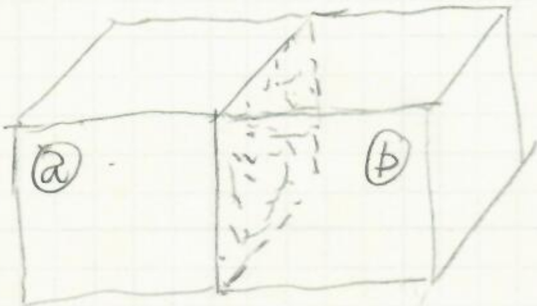
What's ρ ?

$\nabla \cdot E = \rho / \epsilon_0$

$$\text{div}\,\vec{E} \equiv \frac{1}{\tau} \lim_{\tau \to 0} \oint \vec{E} \cdot d\vec{a}$$

"PROOF" of divergence theorem

$$\int \text{div}\,\vec{E}\, d\tau = ?$$



$$\int_{a+b} \text{div}\,\vec{E}\, d\tau = \int_{a} \text{div}\,\vec{E}\, d\tau + \int_{b} \text{div}\,\vec{E}\, d\tau$$

$$\approx \{\text{div}\,\vec{E}\}\tau_a + \{\text{div}\,\vec{E}\}\tau_b$$

$$\left\{\frac{1}{\tau_a} \oint_{a} \vec{E} \cdot d\vec{a}\right\}\tau_a + \left\{\frac{1}{\tau_b} \oint_{b} \vec{E} \cdot d\vec{a}\right\}\tau_b$$

$$= \oint_{a} \vec{E} \cdot d\vec{a} + \int_{b} \vec{E} \cdot d\vec{a}$$

The stippled surface cancels
only the outer surface matters