$$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

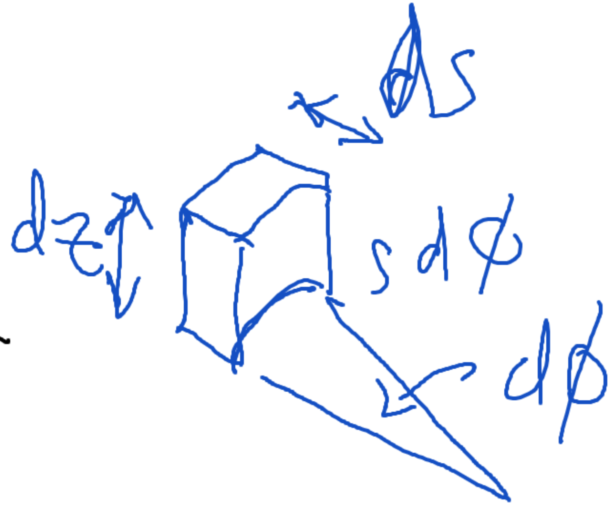
$$\rho = \frac{Q}{\tau} \qquad \rho = \rho_0 s$$

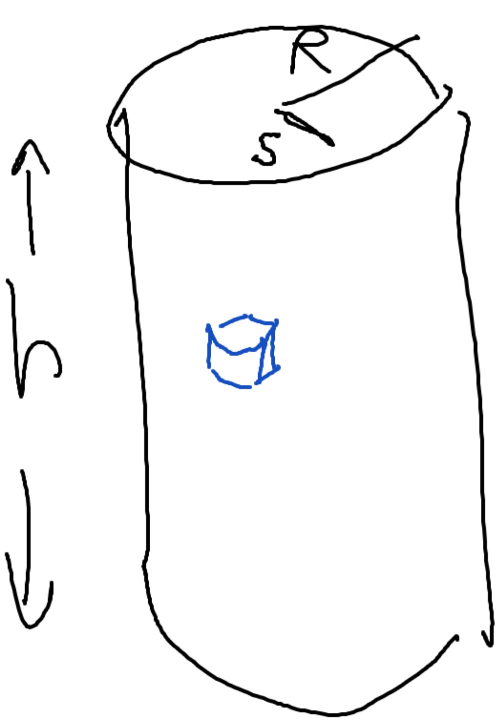
$$dQ = \rho \, d\tau$$

$$\int dQ = \int \rho \, d\tau$$

$$Q = \int \rho_0 s \, d\tau$$

$$Q = \int \rho_0 s \; s \, d\phi \, ds \, dz$$





$$Q = \int \rho \, d\tau \longrightarrow \oint_S \vec{E} \cdot d\vec{a}$$

$$\vec{E} = E_0 \hat{s}$$
$$d\vec{a} = da \, \hat{s}$$

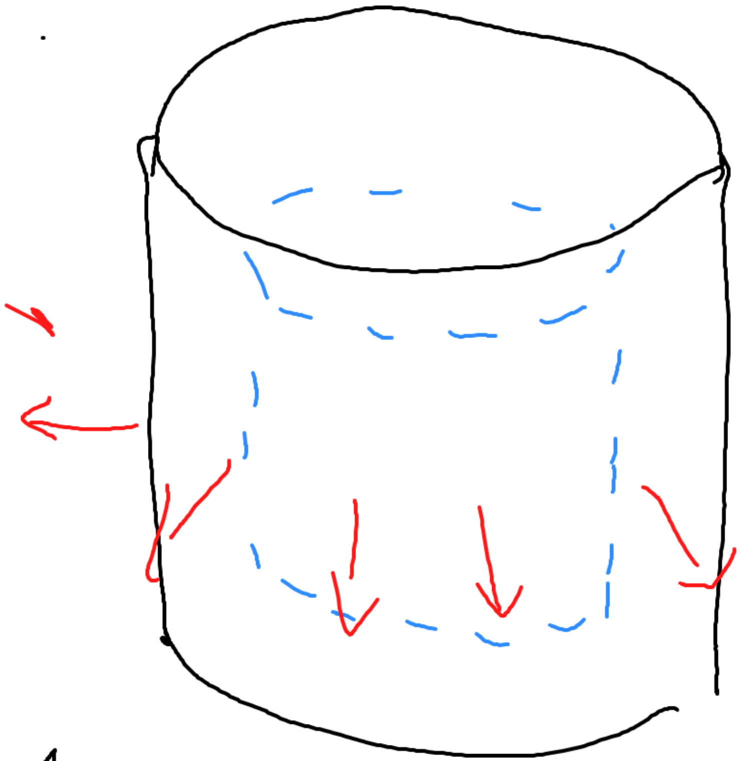
$$Q = \int_0^h dz \int_0^R ds \int_0^{2\pi} d\phi \, \rho_0 s \; s$$

$$Q = h 2\pi \int_0^R \rho_0 s^2 \, ds$$

$$Q = 2\pi h \left. \frac{\rho_0 s^3}{3} \right|_0^R$$

$$Q = \frac{2\pi h \rho_0 R^3}{3}$$

$$\vec{E} \cdot d\vec{a} = E_0 \hat{s} \cdot da \, \hat{s}$$
$$E_0 \, da$$



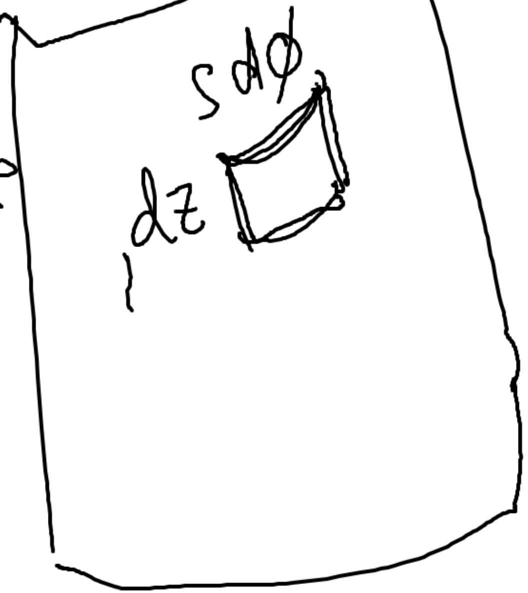
$\int E\,da \quad E\hat{s}$

$\underline{E} = \rho_0 s$

$E = \rho_0 s^2 / 3\epsilon_0\,\hat{s}$

$da = dz\, s\, d\phi$

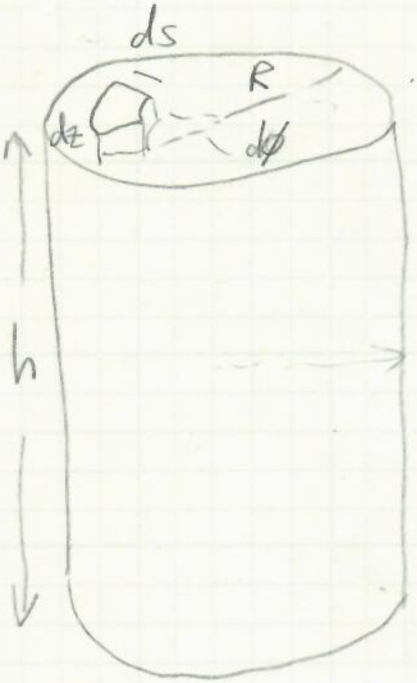
$\int_0^{2\pi} d\phi \int_0^h dz\, sE = \int \vec{E}\cdot d\vec{s}$



$\boxed{2\pi h s} E$

$\int \vec{E}\cdot d\vec{a}$

$2\pi h s E = \dfrac{2\pi \rho_0 h R^3}{3\epsilon_0} \rightarrow \cancel{2\pi h} s E = \dfrac{\cancel{2\pi} \rho_0 h s^{3/2}}{3\epsilon_0}$



$\rho(s) = \rho_0 s$ How to "do" Gauss's Law

$$dQ = \rho\, d\tau$$

$$Q = \int \rho\, d\tau$$

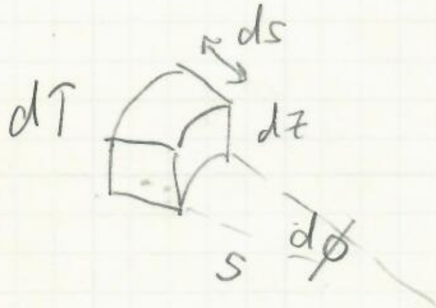
$$d\tau = dz\, ds\, (d\phi\, s)$$

$$Q = \int_0^h dz \int_0^{2\pi} d\phi \int_0^R \rho_0 s\, s\, ds$$

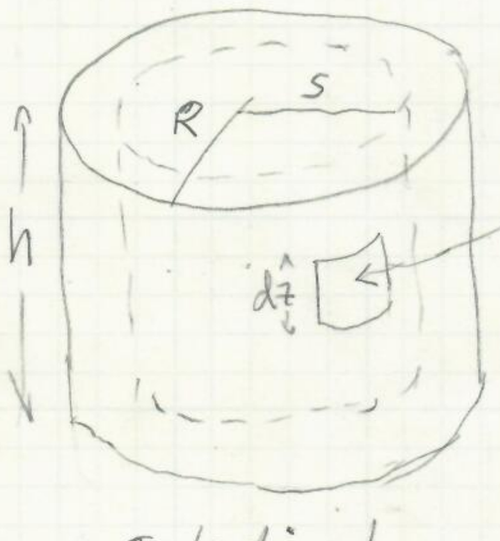
$$Q = z\Big|_0^h \int_0^{2\pi} d\phi \int_0^R \rho_0 s^2\, ds$$

$$= (h)(2\pi)\int_0^R \rho_0 s^2\, ds$$

$$= 2\pi \rho_0 h\, \frac{s^3}{3}\Big|_0^R = \frac{2\pi \rho_0 h R^3}{3}$$



$$\oint E \cdot da = \frac{1}{\epsilon_0} Q_{enc} \qquad 2.13$$



$$d\vec{a} = dz\, s\, d\theta\, \hat{s}$$

$$\vec{E}(s) = E\,\hat{s}$$

$$\vec{E} \cdot d\vec{a} = E\, s\, d\theta\, dz$$

another mistake

$$\int_0^{2\pi} d\theta \int_0^h dz \int_0^s ds'\, s E = \frac{1}{\epsilon_0} Q_{enc} = \frac{2}{3\epsilon_0}\pi \rho_0 h R^3$$

Intentional mistake

$$(2\pi)(h)\, s E = \frac{2}{3\epsilon_0}\pi \rho_0 h s^3$$

$$\boxed{\vec{E} = \frac{1}{3\epsilon_0}\rho_0 s^2\, \hat{s}}$$

What if $\rho(s) = \rho_0 s \cos\phi$?

$dQ = \rho \, d\tau$

$$Q = \int_0^h dz \int_0^{2\pi} \cos\phi \, d\phi \int_0^R \rho_0 s^2 ds$$

$$Q = h \frac{\rho_0 R^3}{3} \int_0^{2\pi} \cos\phi \, d\phi = 0$$

$\therefore$ Since $\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enc} \rightarrow E = 0$

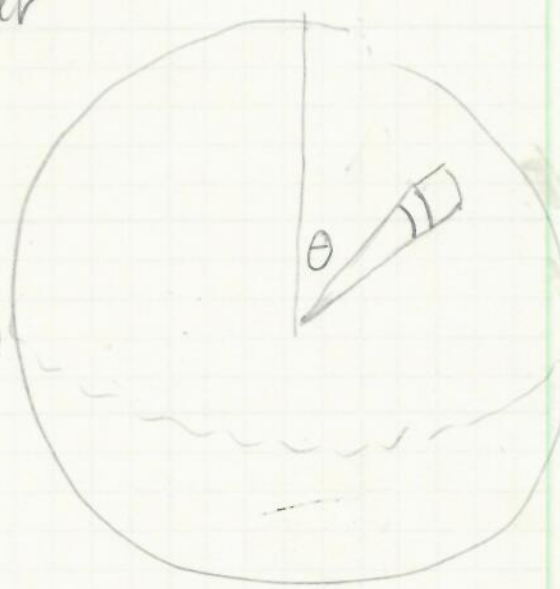
Must pay attention to symmetry when applying Gauss Law

Let's Figure out field from a sphere

$\rho = \rho_0 e^{-r/R}$

$Q_{total} = \int \rho \, d\tau$

$d\tau = r^2 \sin\theta \, d\theta \, d\phi \, dr$



$Q = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{R} dr \, \rho_0 e^{-r/R} r^2 \sin\theta$

$Q = \phi \Big|_0^{2\pi} \int d\theta \int dr \ldots$

$= 2\pi \int_0^{\pi} \sin\theta \, d\theta \int_0^{R} \rho_0 r^2 e^{-r/R} dr$

$Q = (2\pi)(2) \int_0^R \rho_0 r^2 e^{-r/R} dr$

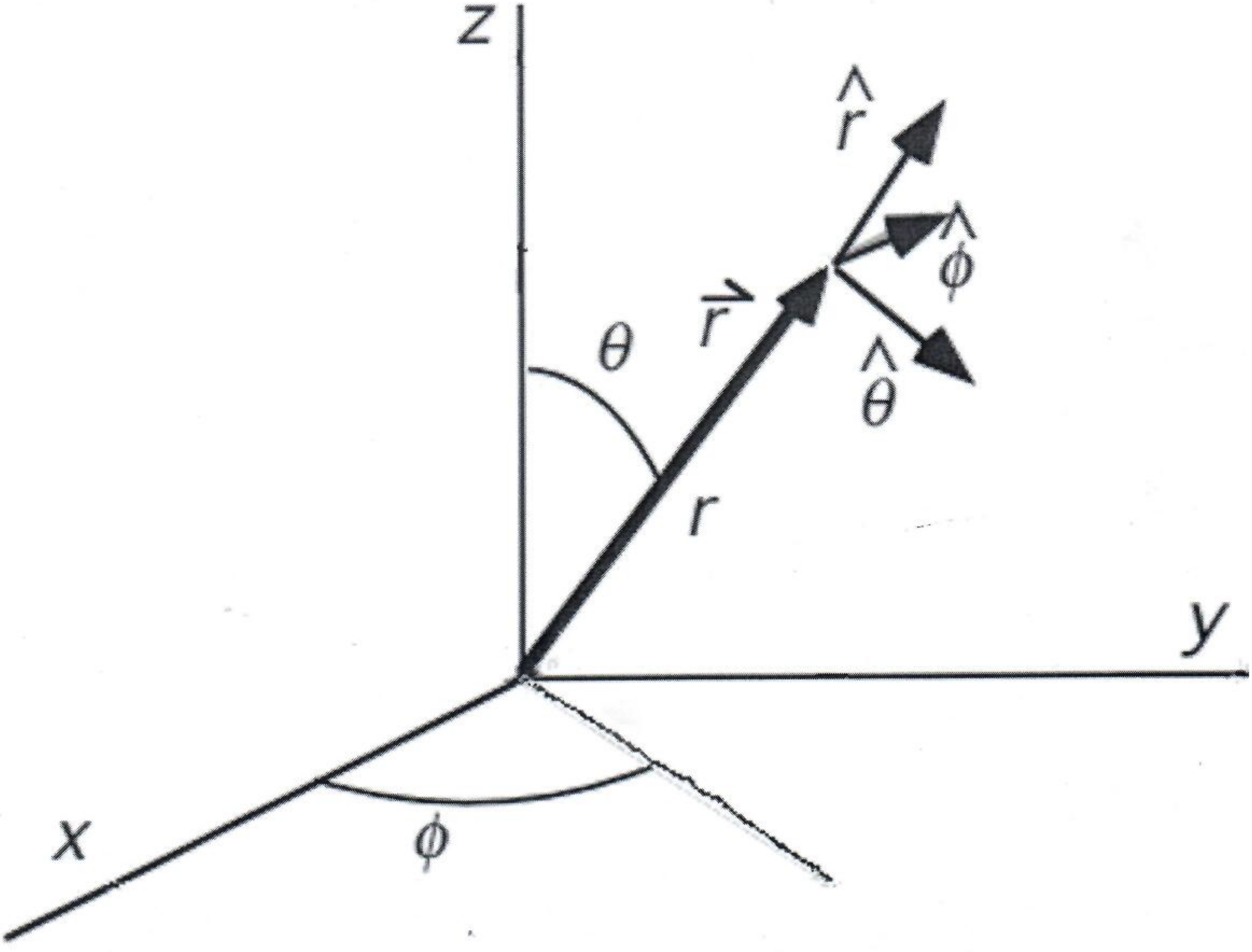
$u = r^2 \quad dv = e^{-r/R} \quad du = 2r\,dr \quad v = -Re^{-r/R}$

$Q = 4\pi\left(uv - \int v\,du\right) = 4\pi\left(-r^2 R e^{-r/R}\right)\Big|_0^R + 4\pi \int 2rRe^{-r/R} dr$

$Q = 4\pi\left(-\frac{R^3}{e} + 0\right) + 8\pi\left(uv - \int v\,du\right)$

$u = r \quad du = dr \quad dv = Re^{-r/R} \quad v = -R^2 e^{-r/R}$

$Q = \frac{-4\pi R^3}{e} + 8\pi\left(-rR^2 e^{-r/R}\right) + 8\pi \int R^2 e^{-r/R} dr$


z
$\hat{r}$
$\hat{\phi}$
$\vec{r}$
θ
$\hat{\theta}$
r
y
x
ϕ