# Multiple Equilibria in a Cloud Resolving Model: Using the Weak Temperature Gradient Approximation to Understand Self-Aggregation [1]

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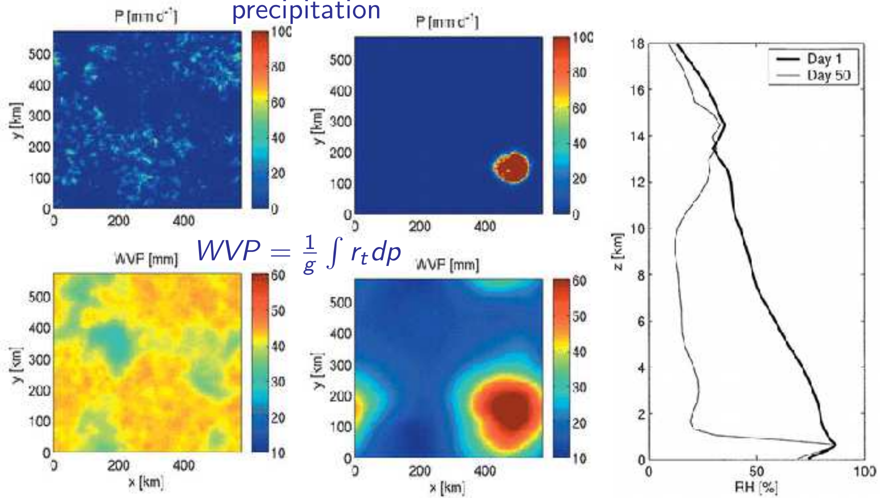
Bretherton et al (2005)

precipitation

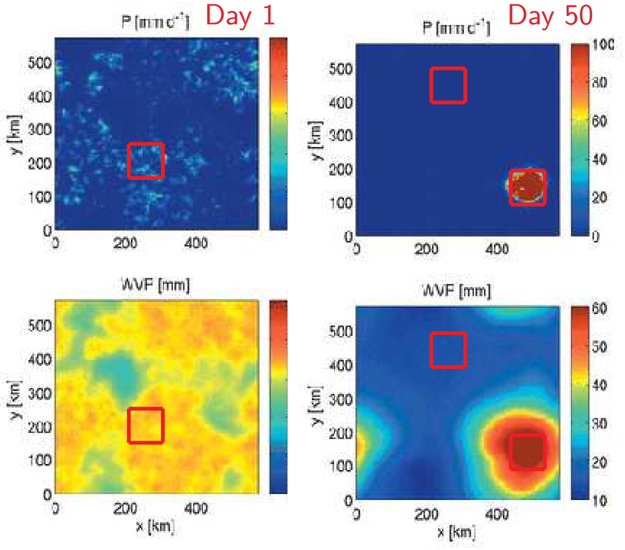



$$WVP = \frac{1}{g}\int r_t dp$$

Day 1 Day 50

# Possible insight from limited domain WTG simulations

Bretherton et al (2005)



# Weak Temperature Gradients in the Tropics

Charney 1963 scaling analysis

Extratropics (small Rossby number)

$$\delta\theta \sim \frac{F_r}{R_o}\theta$$

Tropics (Rossby number not small)

$$\delta\theta \sim F_r\theta$$

- Froude number: $F_r = U^2/gH \sim 10^{-3}$, measure of stratification
- Rossby number: $R_o = U/fL \sim 10^{-5}/f$, characterizes rotation

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In the tropics, gravity waves rapidly redistribute buoyancy anomalies

# Weak Temperature Gradient (WTG) Approximation

Sobel & Bretherton 2000

- Single column model (SCM) representing one grid cell in a global circulation model
- Convection is not explicitly resolved

Temperature (T) and moisture (q) governed by:

$$\frac{\partial T}{\partial t} + \mathbf{u}_h \cdot \nabla T + \omega S = Q^T_{total}$$

$$\frac{\partial q}{\partial t} + \mathbf{u}_h \cdot \nabla q + \omega \frac{\partial q}{\partial p} = Q^q_{total}$$

and $S = \frac{T}{\theta}\frac{\partial \theta}{\partial p}$ is the static stability

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Steady state WTG approximation

and $S = \frac{T}{\theta}\frac{\partial \theta}{\partial p}$ is the static stability

# Weak Temperature Gradient (WTG) Approximation

Sobel & Bretherton 2000

Parameterize the large scale circulations

$$\omega S = Q_{total}^{T}$$

- Conventional parameterizations: specify $\omega$, diagnose $T$
- WTG parameterization: specify $T$, diagnose $\omega$

and $S = \frac{T}{\theta}\frac{\partial\theta}{\partial p}$ is the static stability

# WTG Implementation in a Cloud Resolving Model (CRM)

Raymond & Zeng 2005

$$\frac{\partial\theta}{\partial t} + \nabla\cdot(\rho\mathbf{v}\theta + \mathbf{T}_\theta) \equiv \rho(S_\theta - E_\theta)$$

$$\frac{\partial r_t}{\partial t} + \nabla\cdot(\rho\mathbf{v}r_t + \mathbf{T}_r) \equiv \rho(S_r - E_r)$$

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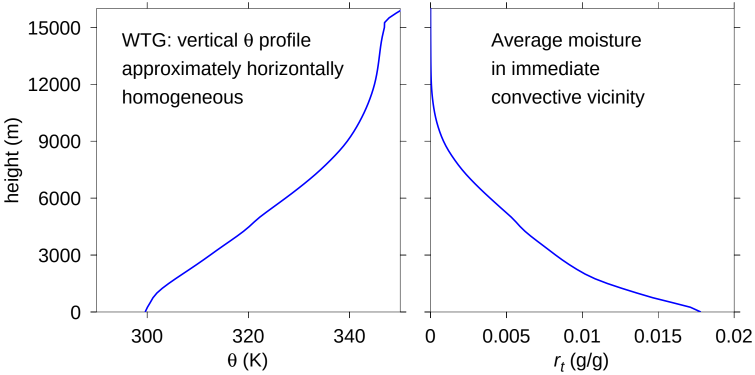
Enforcing WTG generates $w_{wtg}$

$$E_\theta = w_{wtg} \frac{\partial \overline{\theta}}{\partial z} = \sin(\pi z / h) \frac{[\overline{\theta} - \theta_0(z)]}{t_\theta}$$

# Specifying Convective Environment

Reference profiles of potential temperature, $\theta_0$, and moisture, $r_0$, are generated by running model to radiative-convective equilibrium (RCE)




Moisture advected from environment via mass continuity

# WTG Implementation in our CRM

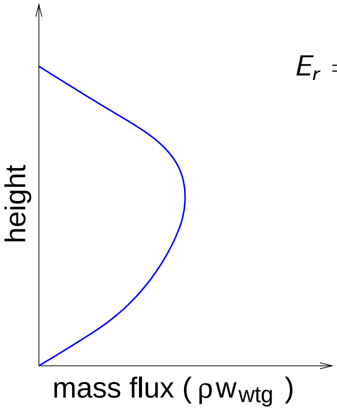
Raymond & Zeng 2005

$$
\begin{aligned}
\frac{\partial\theta}{\partial t} + \nabla\cdot(\rho\mathbf{v}\theta + \mathbf{T}_\theta) &\equiv \rho(S_\theta - E_\theta) \\
\frac{\partial r_t}{\partial t} + \nabla\cdot(\rho\mathbf{v}r_t + \mathbf{T}_r) &\equiv \rho(S_r - E_r)
\end{aligned}
$$

$w_{wtg}$ vertically advects moisture and brings in environmental moisture via mass continuity:

$$
E_r = \frac{(\overline{r}_t - r_x(z))}{\overline{\rho}}\frac{\partial(\overline{\rho}w_{wtg})}{\partial z} + w_{wtg}\frac{\partial\overline{r}_t}{\partial z}
$$

# Environmental moisture in WTG simulations




$$E_r = \frac{(\overline{r}_t - r_x)}{\overline{\rho}} \frac{\partial(\overline{\rho} w_{WTG})}{\partial z} + w_{WTG} \frac{\partial \overline{r}_t}{\partial z}$$

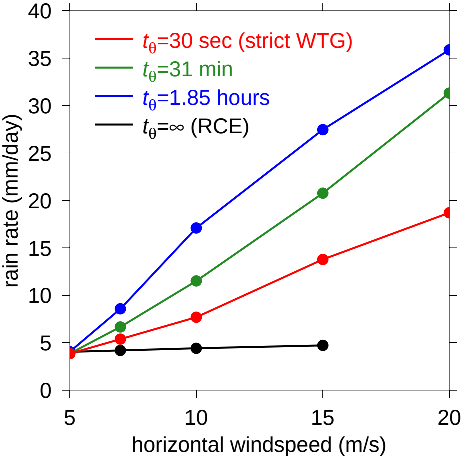
$$r_x = \begin{cases} r_0 & \partial(\overline{\rho} w_{wtg})/\partial z > 0 \\ \overline{r}_t & \partial(\overline{\rho} w_{wtg})/\partial z < 0 \end{cases}$$

# Precipitation in WTG simulations

$$E_\theta = w_{wtg} \frac{\partial \bar{\theta}}{\partial z}$$
$$= \sin(\pi z/h) \frac{[\bar{\theta} - \theta_0(z)]}{t_\theta}$$

$\theta_0(z)$ is RCE profile for $v_y = 5$ m/s on 2D 200 km domain

$t_\theta$ is a measure of the enforcement of WTG



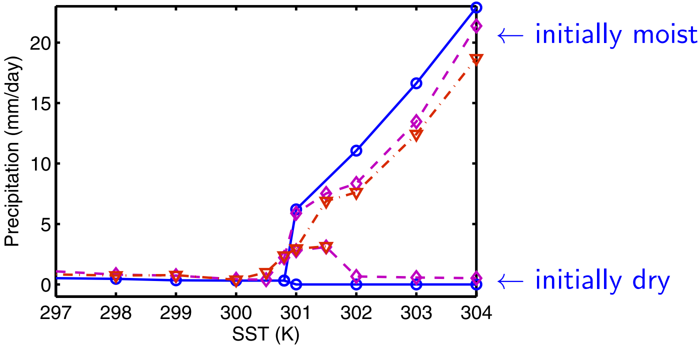
# Multiple Equilibria in Precipitation

Sobel, Bellon & Bacmeister (2007)

- Single column model (SCM) with WTG approximation
- Parallel experiments: identical forcing
  - initially moist troposphere
  - initially dry troposphere
- Observe convective response with different SSTs

# Multiple Equilibria in Precipitation

Sobel, Bellon & Bacmeister (2007)




Reference profile is RCE with SST of 301 K

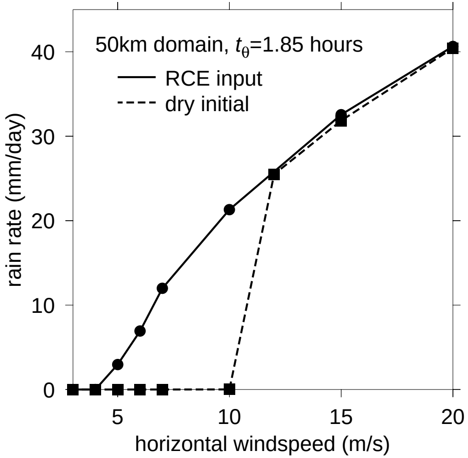
# Multiple Equilibria in a CRM

Sessions, Sugaya, Raymond & Sobel (2010)

- 2-D Cloud Resolving Model (CRM) with WTG approximation
  - periodic boundary conditions
  - horizontal domains vary from 50-200 km (0.5-1 km resolution)
  - vertical dimension of 20 km (250 m resolution)
- Parallel experiments: Initial moisture profile of tropics
  - same as surrounding environment
  - completely dry
- Observe convective response with different surface wind speeds (surface wind speed and SST both control surface fluxes)
- 4 month simulations, averaged over last month of simulation

# Multiple Equilibria in a CRM

Sessions, Sugaya, Raymond & Sobel (2010)




Reference profile is RCE with $v_y = 5$ m/s

# Normalized Gross Moist Stability (NGMS)

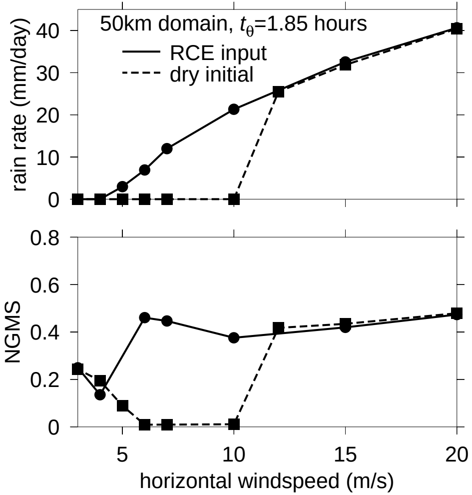
Define NGMS to characterize the convective environment

$$NGMS = \frac{T_R \frac{1}{g} \int \nabla \cdot (s\mathbf{v}) dp}{-L \frac{1}{g} \int \nabla \cdot (r\mathbf{v}) dp} = \frac{moist\ entropy\ export}{moisture\ import}$$

- GMS based on moist static energy budget (Neelin & Held 1987)
- Thorough discussion of NGMS (Raymond et al. 2009)

# NGMS in the Steady State

Sessions, Sugaya, Raymond, Sobel (2010)




In the steady state,

$$Net\ precipitation = \frac{net\ entropy\ forcing}{NGMS}$$

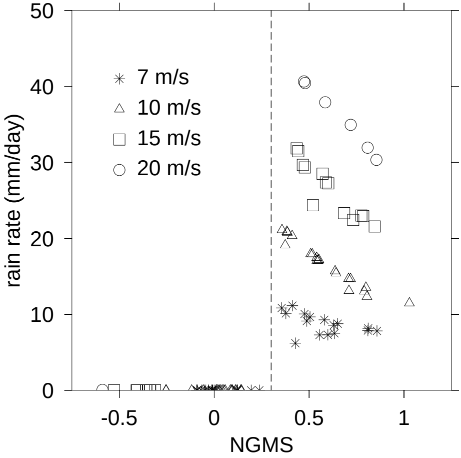
# NGMS as a diagnostic

Sign of NGMS

| | moist entropy import | moist entropy export |
|---|---|---|
| moisture import | $-$ | $+$ |
| moisture export | $+$ | $-$ |

$$NGMS = \frac{T_R \frac{1}{g} \int \nabla \cdot (sv) dp}{-L \frac{1}{g} \int \nabla \cdot (rv) dp} = \frac{moist\ entropy\ export}{moisture\ import}$$

# Steady state precipitation and NGMS




$$NGMS = \frac{moist\ \ entropy\ \ export}{moisture\ \ import}$$

- Dry equilibrium
  NGMS + or -
- $Rain\ rate = \frac{net\ \ entropy\ \ forcing}{NGMS}$
- NGMS=0.3 divides equilibria

# Multiple Equilibria & Self-Aggregation NGMS

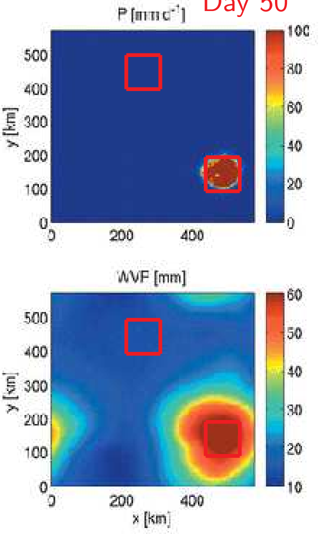
Bretherton et al (2005)

- + GMS in precipitating region
- − GMS in dry region

Sessions et al (2010)

- + NGMS in precipitating equilibria
- small or − NGMS in dry equilibria

Day 50



# Summary

- WTG approximation parameterizes the large scale circulations
- Multiple equilibria in CRM
  - determined by surface fluxes
  - initial moisture determines which state is realized
- NGMS characterizes the steady state atmosphere
  - larger values for precipitating equilibrium
  - smaller or negative values for dry equilibrium

Investigating multiple equilibria in limited domain WTG simulations may be a computationally economic approach to understanding self-aggregation