Buoyancy in Tropical Cyclones and Other Rapidly Rotating Atmospheric Vortices

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Introduction

- Main motivation: Understanding the dynamics of tropical cyclones.

- For intensification of a tropical cyclone toroidal circulation needed.

- Buoyancy from deep convective clouds possible mechanism for driving the toroidal circulation.

- Dissagreements on the role of the buoyancy in tropical cyclones.

Conventional buoyancy

$$\vec{b} = \vec{g}\frac{\rho - \rho_{ref}}{\rho} \tag{1}$$

$\vec{g} = -g\widehat{k}$ acceleration of gravity

ρ density of the parcel

$\rho_{ref}(z)$ density of the environment at the same height as the parcel

Assumption: the pressure of the air parcel is equal to that of the environment at the same height.

Violated in rapidly rotating vortices!

What determens the vertical momentum of a parcel?

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$$p = p_{ref} + p^{'}$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p_{ref}}{\partial z} - \frac{1}{\rho}\frac{\partial p^{'}}{\partial z} - g = g\frac{\rho_{ref}}{\rho} - \frac{1}{\rho}\frac{\partial p^{'}}{\partial z} - g$$

$$\frac{Dw}{Dt} = b - \frac{1}{\rho}\frac{\partial p^{'}}{\partial z} \qquad (2)$$

Buoyancy in rapidly rotating vortices

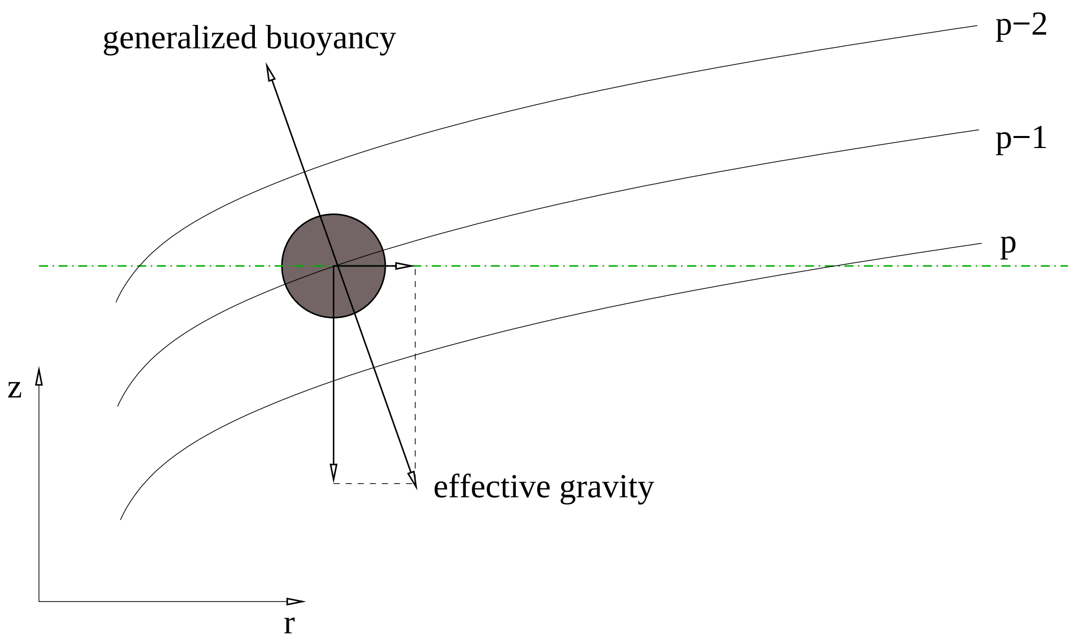


Fig.1. Schematic of radial-height cross-section in a rapidly rotating vortex.

Effective gravity:

$$\vec{g}_e = (\frac{v^2}{r} + fv)\hat{r} - g\hat{k}$$

Generalized buoyancy:

$$\vec{b} = \vec{g}_e \frac{\rho - \rho_{ref}}{\rho}. \qquad (3)$$

ρ density of the parcel

ρ_{ref} density of the environment at the same isobaric surface as the parcel

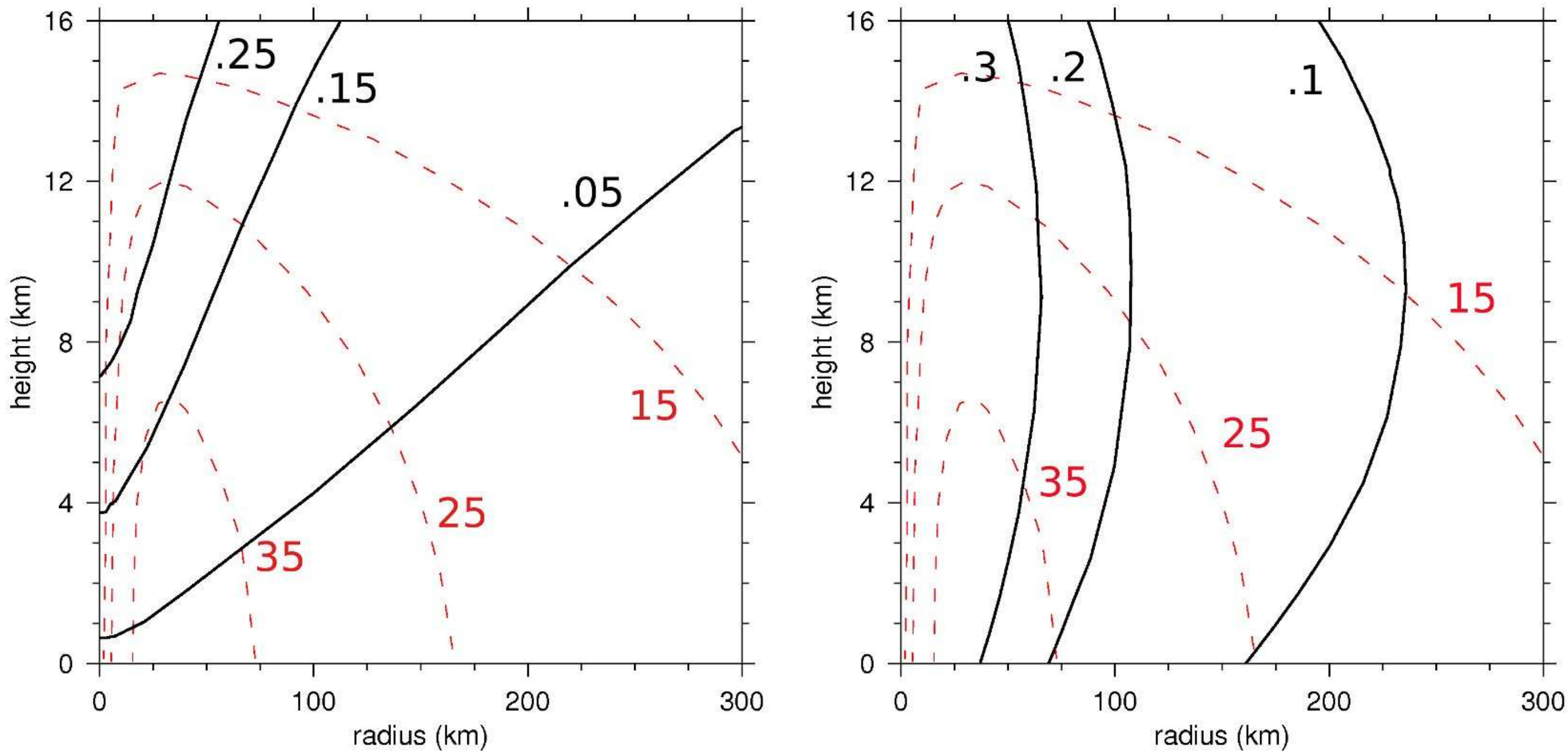
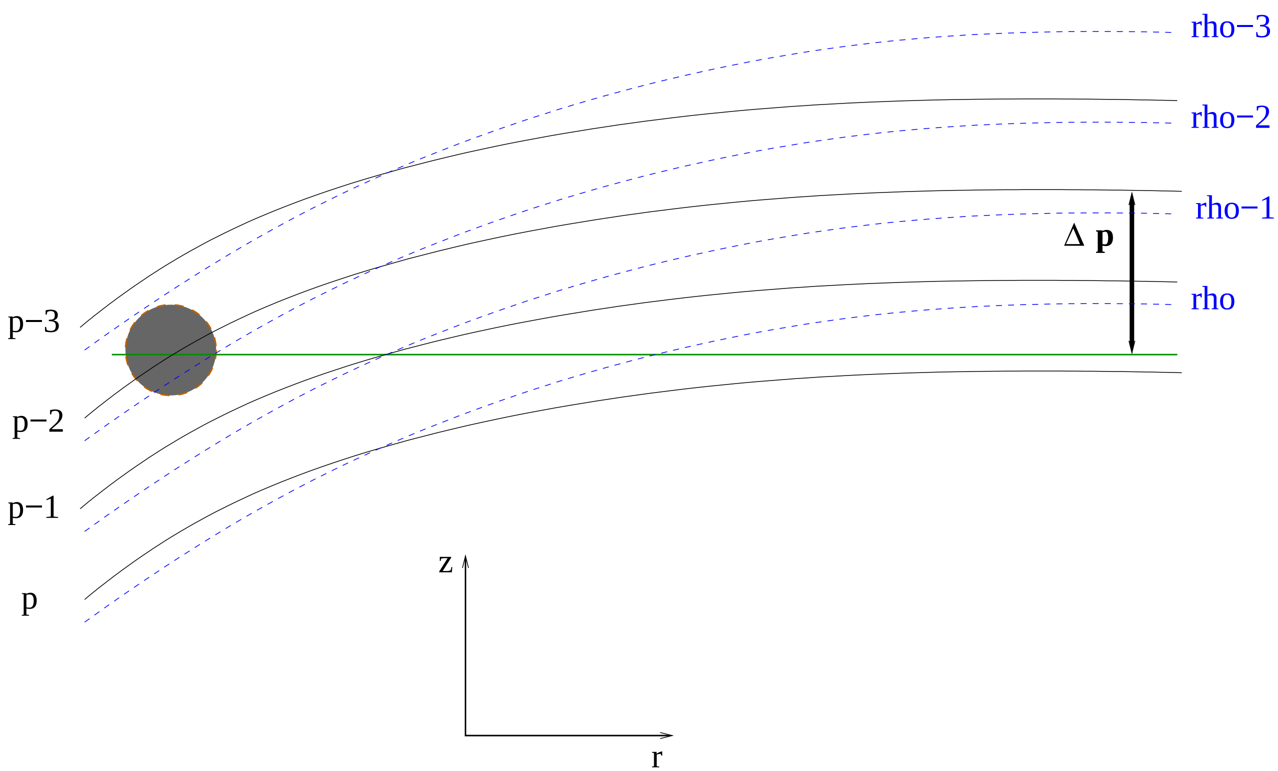


Fig.2. Radial-height cross section in a balanced TC strength vortex. Buoyancy $[m/s^2]$ is calculated by comparing parcel and environmental temperature: (left) at constant pressure, (right) at constant height.



$$\frac{Dw}{Dt} = b - \frac{1}{\rho}\frac{\partial p'}{\partial z}$$

Fig.1a. Schematic of radial-height cross-section in a rapidly rotating vortex.

System buoyancy and local buoyancy

ρ_o, p_0, and v_0 axisymmetric density, pressure and tangential wind.

System buoyancy:

$$\vec{b}_B = \vec{g}_e \frac{\rho_0 - \rho_{ref}}{\rho_0}.$$

Local buoyancy: The unbalanced buoyancy due to the individual convective clouds.

Gradient wind balance:

$$\frac{\partial p_0}{\partial r} = \rho_0(\frac{v_0^2}{r} + f v_o).$$

Hydrostatic balance:

$$\frac{\partial p_0}{\partial z} = -\rho_0 g.$$

Thermal wind balance:

$$\frac{g}{\rho_0}\frac{\partial \rho_0}{\partial r} + (\frac{v_0^2}{r} + f v_0)\frac{1}{\rho_0}\frac{\partial \rho_0}{\partial z} = -(\frac{2v_0}{r} + f)\frac{\partial v_0}{\partial z}. \qquad (4)$$

$$\frac{dz}{dr} = \frac{1}{g}(\frac{v_0^2}{r} + fv_0). \tag{5}$$

For any given point on a characteristic surface (r, z) tells the radial distance and the height.

$$dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz = 0 \Longrightarrow \frac{dz}{dr} = -\frac{\partial p/\partial r}{\partial p/\partial z} = -\frac{\rho_0(\frac{v_0^2}{r}+fv_o)}{-\rho_0 g}$$

$$\frac{d\ln\rho_0}{dr} = -\frac{1}{g}(\frac{2v_0}{r} + f)\frac{\partial v_0}{\partial z}. \tag{6}$$

Given $\rho_0(z, R)$ and $v_0(r, z, t = 0)$, (6) and (7) can be solved for $\rho_0(r)$ and $z(r)$ on a given isobaric surface. Then integrating the hydrostatic equation we can find a value of another isobaric surface.

Toroidal vorticity tendency

Absolute vorticity:

$$\vec{\zeta}_a = \nabla \times \vec{v} + f\hat{z}$$

Vorticity equation:

$$\frac{\partial \vec{\zeta}_a}{\partial t} = \nabla \cdot (\vec{\zeta}_a \vec{v} - \vec{v}\vec{\zeta}_a) + \frac{1}{\rho^2}\nabla\rho \times \nabla p \qquad (7)$$

Baroclinicity vector:

$$\vec{B} = \frac{1}{\rho^2}\nabla\rho \times \nabla p$$

In z-coordinates:

$$B_\lambda = \frac{1}{\rho^2}(\frac{\partial\rho}{\partial z}\frac{\partial p}{\partial r} - \frac{\partial\rho}{\partial r}\frac{\partial p}{\partial z}) = \frac{1}{\rho^2}\frac{\partial\rho}{\partial z}\frac{\partial p}{\partial r} + g\frac{\partial}{\partial r}\ln\rho.$$

In p-coordinates:

$$B_\lambda = g\frac{\partial}{\partial r}\ln\rho.$$

In pressure coordinates the baroclinic forcing on the toroidal vorticity has only one term that is proportional to the radial gradient of the density along an isobaric surface.

The radial gradient of the buoyancy on isobaric surfaces is a more appropriate measure of the horizontal vorticity tendency!

$$(r, \lambda, z), \;\; \vec{v} = (u, v, w)$$

If initially:

$$\vec{v} = \widehat{\lambda} v(r, z)$$

$$\vec{\zeta}_a = \widehat{r}(-\frac{\partial v}{\partial z}) + \widehat{z}(\frac{\partial v}{\partial r} + \frac{v}{r} + f)$$

then:

$$\frac{\partial \eta}{\partial t} = \frac{\partial}{\partial z}(\frac{v^2}{2r} + fv) + B_\lambda$$

Conclusions

- Buoyancy force depends on the choice of reference density. It is not unique. Therefore, the perturbation pressure gradient force is not unique either. Only their sum is unique.

- For rapidly rotating vortices the generalized buoyancy definition is more appropriate. It has a radial component which is due to the Coriolis force and the centrifugal force.

- If the chosen ρ_{ref} is the one that is in thermal wind balance with the tangential flow, then the system buoyancy is zero by definition and the buoyancy forces are a result of asymmetric buoyancy anomalies.

- In pressure coordinates, the azimuthal vorticity tendency equals the radial density gradient along the isobaric surface only. In z and σ coordinates there is an additional term which corresponds to the vertical change of density in the presence of centrifugal and Coriolis forces.