# The Problem of Rainfall (over Tropical Oceans)*

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*Sponsored by US National Science Foundation

“Dynamics is good, dynamics is impressive; but it is thermodynamics that does the work.”

— With apologies to Mark Twain

Specific Dry Entropy:

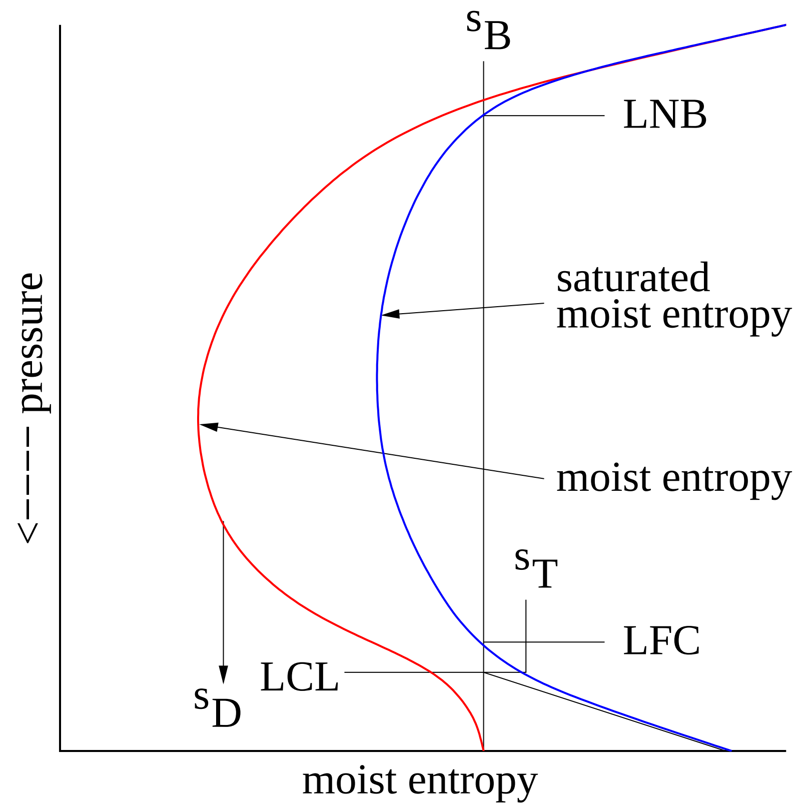
$$s_d \approx C_P \ln\left(\frac{T}{T_R}\right) - R\ln\left(\frac{p}{p_R}\right) \approx C_P \ln\left(\frac{\theta}{T_R}\right)$$

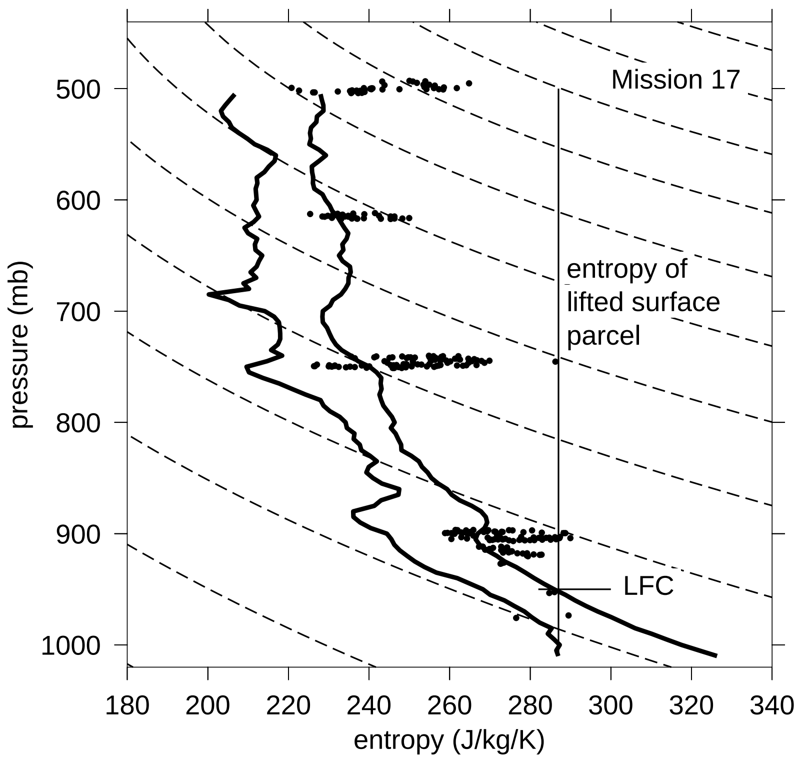
Specific Moist Entropy:

$$s \approx C_P \ln\left(\frac{T}{T_R}\right) - R\ln\left(\frac{p}{p_R}\right) + \frac{Lr_V}{T_R} \approx C_P \ln\left(\frac{\theta_e}{T_R}\right)$$

Saturated Specific Moist Entropy:

$$s_s \approx C_P \ln\left(\frac{T}{T_R}\right) - R\ln\left(\frac{p}{p_R}\right) + \frac{Lr_S}{T_R} \approx C_P \ln\left(\frac{\theta_{es}}{T_R}\right)$$




Since $s_s = s_s[\theta, p, r_S(\theta, p)]$, we can invert: $\theta = \theta(s_s, p)$.

Deep convective inhibition index: $\text{DCIN} = s_T - s_B$.

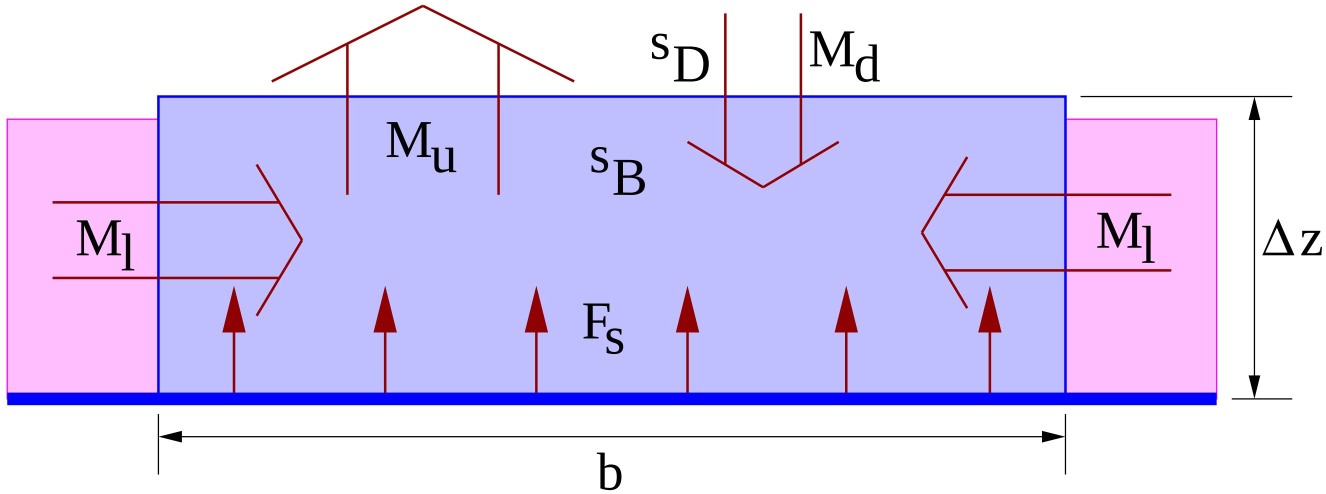
Downdraft entropy deficit: $s_B - s_D$.

Two Ways To Reduce DCIN:

1. Decrease $s_T$ – Most effective mechanism is via dry adiabatic lifting associated with fast-moving, wave-like disturbances – however, effect is necessarily transient, resulting in little rain. Ekman pumping can also act to decrease $s_T$ via lifting.

2. Increase $s_B$ – Moistening via surface evaporation – slow-moving disturbances can act over a long period, producing a lot of rain.

Planetary Boundary Layer (PBL) Control Volume for Boundary Layer Quasi-Equilibrium (Raymond 1995; Emanuel 1995):




- Mass balance: $(M_u - M_d)b^2 - (M_l)4b\Delta z = 0$

- Entropy balance: $(M_u s_B - M_d s_D - F_s)b^2 - (M_l s_B)4b\Delta z = 0$

- Updraft-downdraft condition: $M_d = aM_u$

Boundary Layer Quasi-Equilibrium Results:

- Downdraft mass flux:

$$M_d = \frac{F_s}{s_B - s_D}$$

- Updraft mass flux:

$$M_u = \frac{F_s}{a(s_B - s_D)}$$

Surface Entropy Flux:

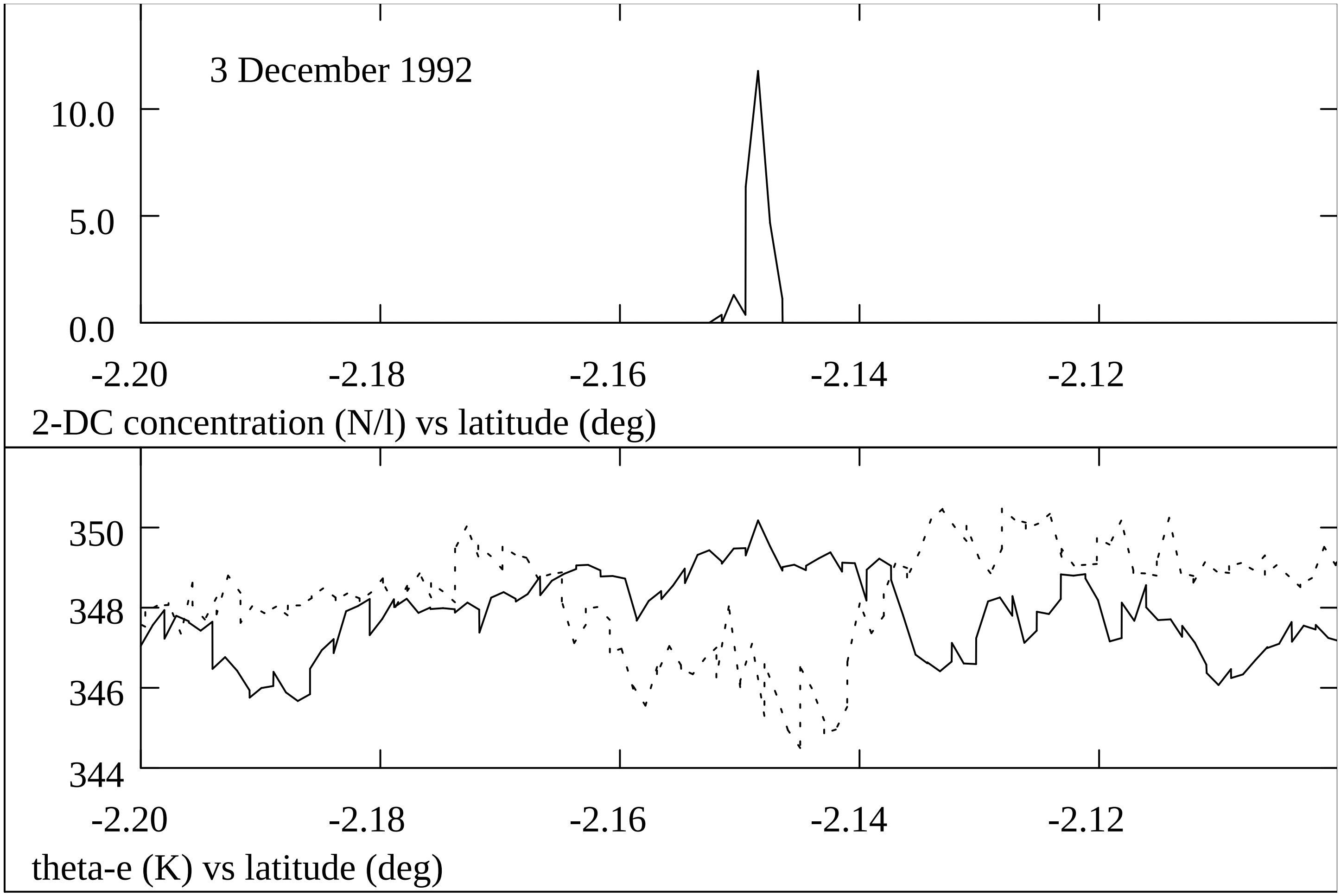
$$F_s = \rho_B C_D U_{eff}(s_{sss} - s_B)$$

$\rho_B$: boundary layer density; $C_D = O(10^{-3})$: drag coefficient;

$U_{eff} = (U_B^2 + W^2)^{1/2}$: effective wind in boundary layer ($W \approx$ 3 m s$^{-1}$); $U_B$: boundary layer wind speed;

$s_{sss}$: saturated moist entropy at temperature and pressure of sea surface; $s_B$: boundary layer entropy.

## Aircraft Measurements from TOGA COARE:



Precipitation Rate and Boundary Layer Quasi-equilibrium:

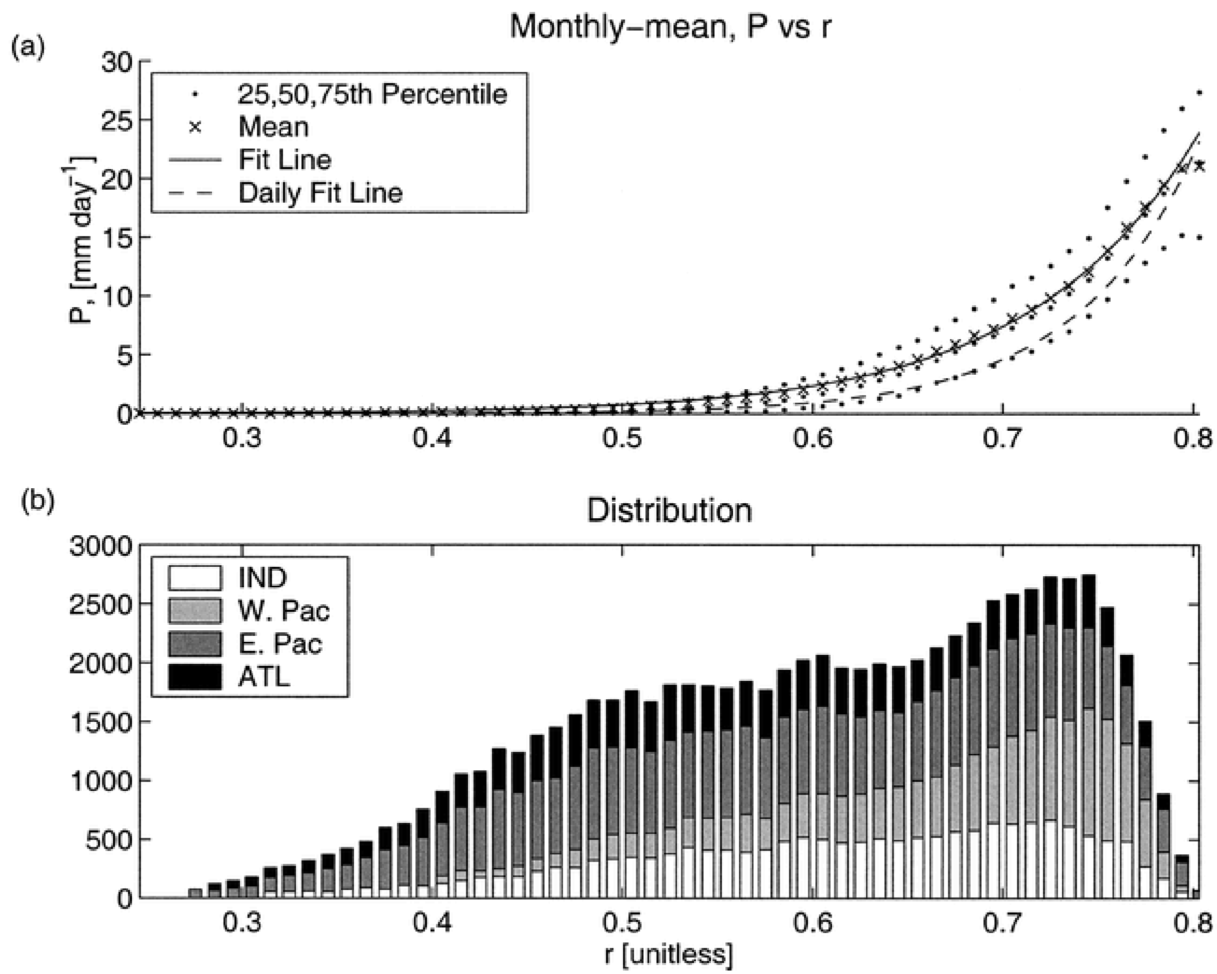
Assume that precipitation rate is proportional to the transport of moisture out of the PBL by updrafts

$$P = \epsilon M_u r_B = \frac{\epsilon F_s r_B}{a(s_B - s_D)} = \left(\frac{T_R F_s}{L}\right)\left(\frac{\epsilon}{a\sigma}\right)$$

- $0 \le \epsilon < O(1)$: dimensionless precipitation efficiency;
- $a = M_d/M_u$: downdraft to updraft mass flux fraction;
- $\sigma = (r_B - r_D)/r_B = T_R(s_B - s_D)/(r_B L) < 1$: dimensionless entropy difference between boundary layer and downdrafts.

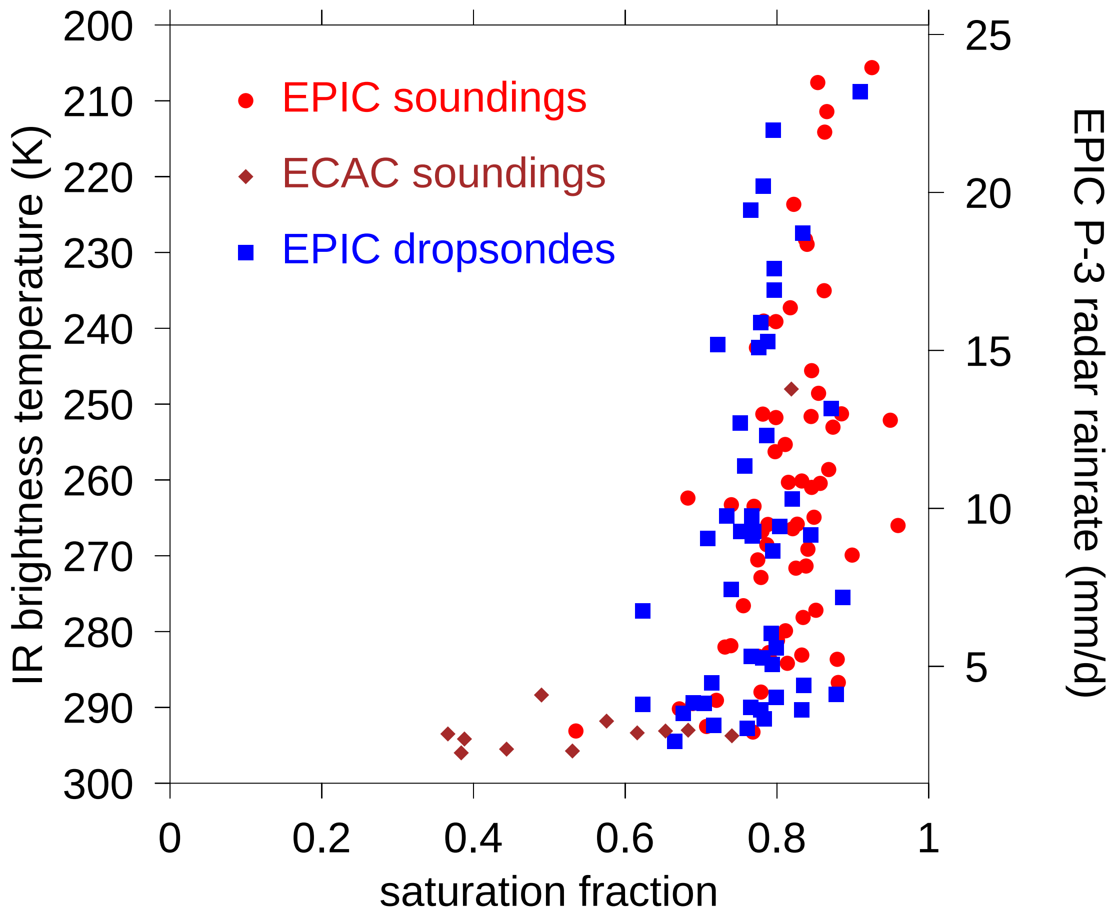
## What Do Observations Say?

Bretherton, Peters, and Back (2004):




$$S = \int r_V dp \Big/ \int r_S dp \approx \int (s - s_d) dp \Big/ \int (s_s - s_d) dp$$

Raymond, Sessions, and Fuchs (2007):



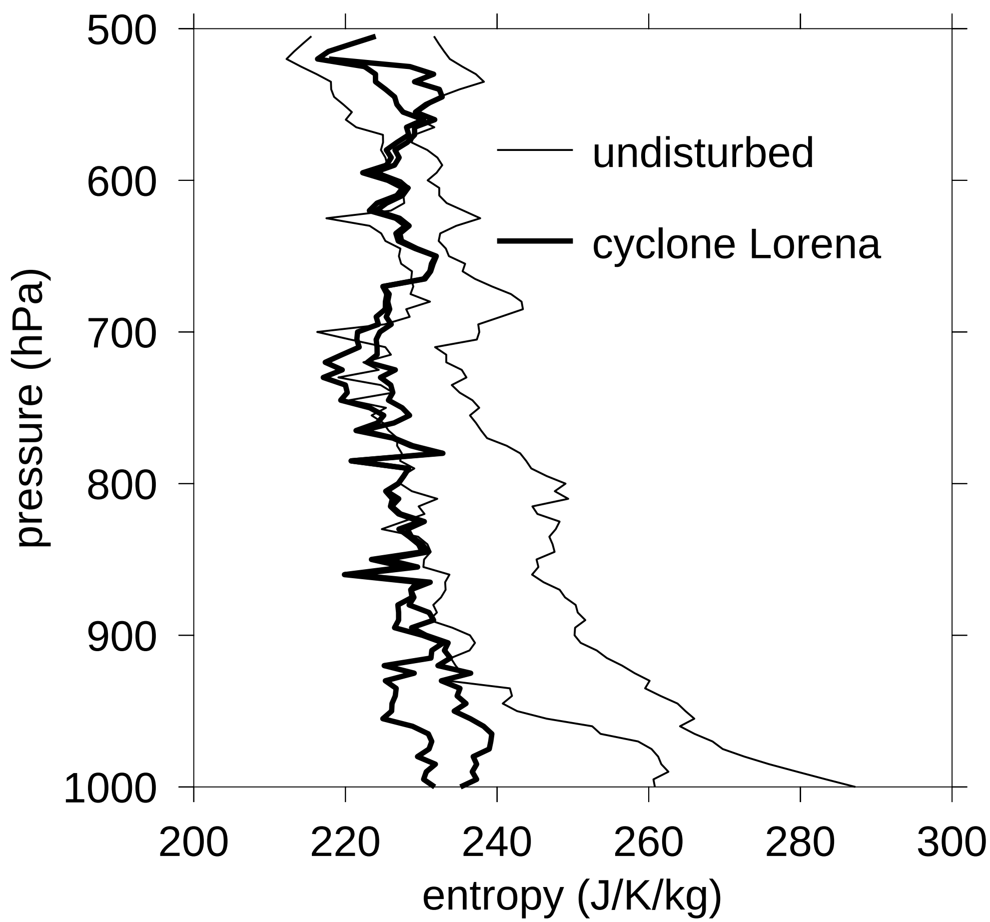
Saturation Fraction Dependence:

Dry environments evaporate cloud and precipitation, resulting in weaker cloud development and less precipitation.

$$P = \left(\frac{T_R F_s}{L}\right)\left(\frac{\epsilon}{a\sigma}\right)$$

This is consistent with a precipitation efficiency $\epsilon(S)$ which increases strongly with saturation fraction $S$.

Entropy Soundings in Disturbed and Undisturbed Environments (Raymond, Sessions, and Fuchs 2007):



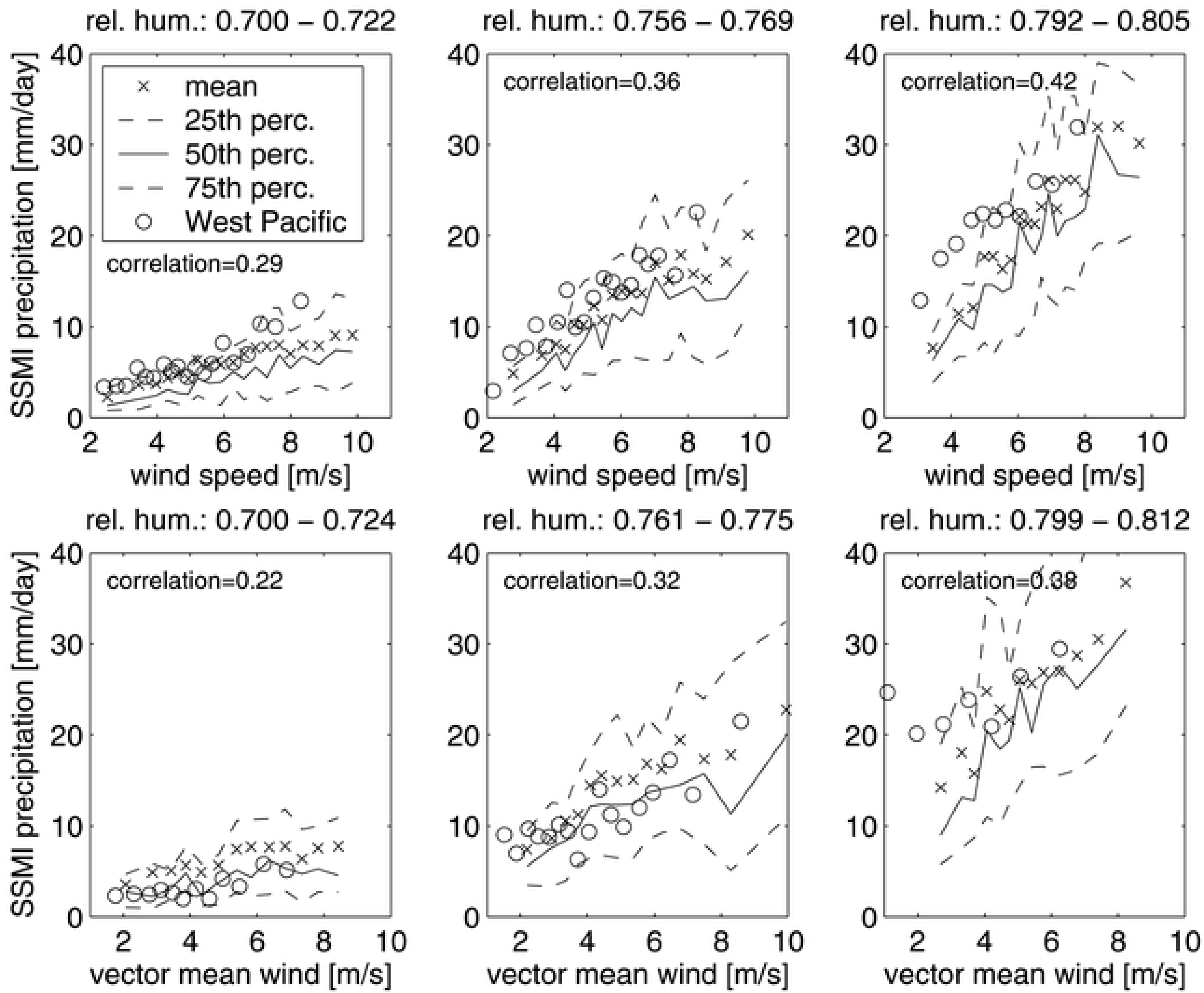
Disturbed Environments:

Disturbed environments with heavier mean precipitation rates tend to be moister and more stable than undisturbed environments.

$$P = \left(\frac{T_R F_s}{L}\right)\left(\frac{\epsilon}{a\sigma}\right)$$

Increased humidity and stability reduce the saturation deficit of downdrafts $\sigma$ and may also reduce the downdraft fraction $a$, both of which increase the precipitation rate.

Back and Bretherton (2005):



Wind Dependence:

At constant saturation fraction, precipitation increases with wind speed.

$$P = \left(\frac{T_R F_s}{L}\right)\left(\frac{\epsilon}{a\sigma}\right)$$

Since surface fluxes increase with wind speed, the multiplicative dependence of $P$ on $F_s$ explains this result.

Conclusions:

- Boundary Layer Quasi-Equilibrium produces plausible hypotheses explaining the observed dependence of precipitation rate over tropical oceans on environmental profiles and wind speed.

- These hypotheses are testable using cloud-resolving numerical models and the right kind of cloud physical observations.