wege entstehen, indem wir sie gehen
ways emerge in that we go them

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Turbulence energetics in stably stratified geophysical flows: Strong and weak mixing regimes

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Overview of today's talk

Introduction

Past

- *Previous/classical understanding of the physical process*
- *Treatment / Application*

Present

- *New Findings*
- *Proposed Solution*

Validation of proposed solution

Implications

- *What will change in the light of new findings*
- *Future direction*

Introduction .. Turbulence

- the *vertical* transport of moisture, heat, momentum, and pollutants is dominated by turbulence
- eddies can vary in size - anywhere from 100-3000 m, however, eddies exist in size as small as a few millimeters

what creates turbulence?

- solar heating generating thermals - nothing more than larger eddies
- wind shear
- deflected flow around obstacles such as trees and buildings, creating turbulent wakes downstream of the obstacle
- in summary, turbulence allows the boundary layer to respond to changes in surface forcings (daytime heating, for example).
- This does not occur in the free troposphere, the free troposphere acts like the earth's surface does not exist.

Big Whorls have little whorls,
Which feed on their velocity;
And little whorls have lesser whorls,
And so on to viscosity

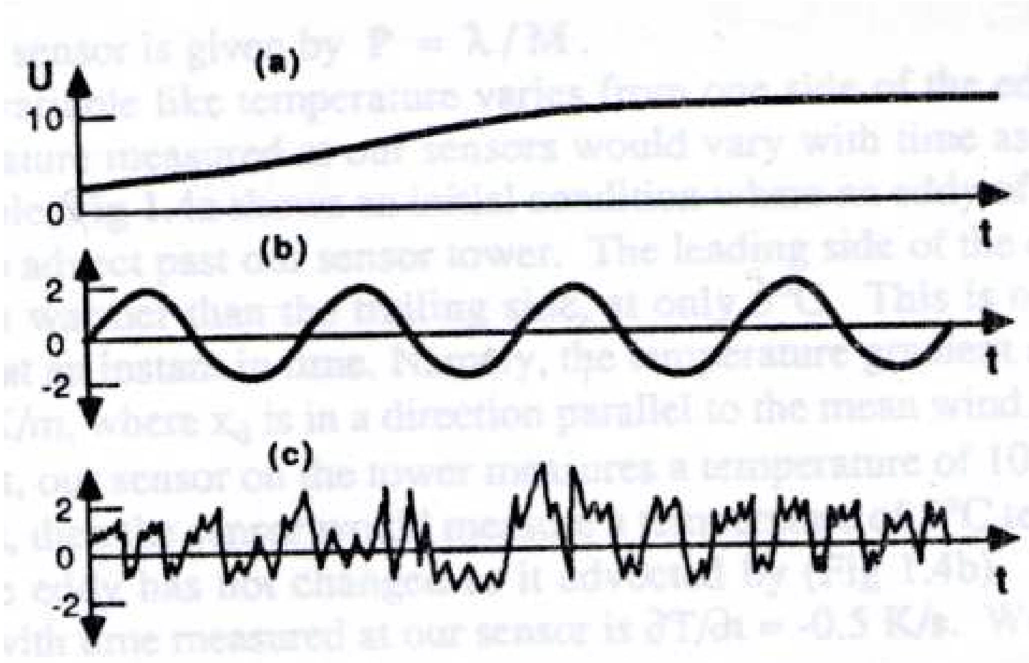
Introduction .. Turbulent Energies

mean velocity $\overline{\mathbf{u}} = (\overline{u}, \overline{v}, \overline{w})$,

$$u = \overline{u} + u'$$

Mean Shear

$$\overline{\mathbf{S}} = \mathbf{i}\ \partial\overline{u}/\partial z + \mathbf{j}\partial\overline{v}/\partial z$$



Source Stull 1988

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Let us consider stable stratification

$\overline{\rho}$ decreases with increasing height: $\partial\overline{\rho}/\partial z < 0$

Let us consider fluid element displaced upward (downward) over a distance δz differs in density from the ambient fluid by: $\rho' = (\partial\overline{\rho}/\partial z)\delta z$

and experiences the downward (upward)

acceleration: $(g/\rho_0)\rho' = (g/\rho_0)(\partial\overline{\rho}/\partial z)\delta z$

where $g = 9.81$ m s^{-2} is the acceleration of gravity
and ρ_0 is a reference density.

In other words, the stable density stratification prevents vertical velocity fluctuations. This effect is the stronger the larger the vertical gradient of the mean buoyancy, b, defined as
$b \equiv -g\rho/\rho_0$, or its square root

$$N \equiv (\partial\overline{b}/\partial z)^{1/2}$$

called the Brunt–Väiäslä frequency. So we can say that Gradient Richardson Number which is a dimensionless ration between N and S.

$$\mathrm{Ri} = N^2/\mathbf{S}^2$$

- It is widely believed to be true in general i.e. at Richardson number (Ri) exceeding a critical value Ri_c local shear cannot maintain turbulence and flow becomes laminar. Recent studies show turbulence can exist at Ri >> 1

  (Richardson, 1920; Prandtl, 1930; Taylor, 1931; Chandrasekhar, 1961; Miles, 1961; Monin and Yaglom, 1971; Turner, 1973)

- The turbulence completely decays at when Ri exceeds a critical value Ri_c
- The same symbol (Ri_c) and name (critical Richardson number) are applied to the hydrodynamic instability threshold, $Ri_{c\text{–}instability}$, varying from 0.25 to 1
  (Taylor, 1931; Miles, 1961; Abarbanel *et al*., 1984, 1986; Miles, 1986).
- As follows from the perturbation analysis, sheared flows are hydro-dynamically unstable only at subcritical Richardson numbers

$$Ri < Ri_{c\text{–}instability}$$

At infinitesimal perturbations are stable hence shear cannot maintain turbulence

it has been recognised that very-short-wave perturbations in sheared flows are dynamically stable even under neutral stratification, so that the stable static stability simply shifts the dynamic instability towards larger wavelengths (Sun, 2006). Hence, perturbation analysis cannot be fully conclusive in answering the question of whether or not the shear can maintain turbulence at large Ri.

Traditional approach to characterize the turbulence energetics is by Turbulent Kinetic Energy (TKE) and is modelled by using the TKE budget equation

However, this reasoning is inapplicable to finite perturbations: they cause internal gravity waves with inherent orbital motions and local shears, including horizontal shears of vertical velocities, which are not affected by static stability and immediately generate turbulence (Phillips, 1972, 1977)

So Ri and $Ri_{c\text{-instability}}$ should not be confused with one an other and analysis presented in this study is limited to energetics of turbulence

essential turbulent mixing at large Ri, modern turbulence closures are equipped with Ri dependencies of the turbulent Prandtl number, PrT ≡ K_M/K_H , preventing appearance of Ri_c, and/or with nonzero background turbulent diffusivities, preventing unrealistic laminarisation

Many experiments show general existence of turbulence up to Ri = 10^2

In the free atmosphere, where Ri typically varies from 1 to 10 and often approaches 10^2, pronounced turbulence has been observed almost continuously at all levels (Lawrence *et al*., 2004), not to mention that the effective eddy viscosity, K_M, and conductivity, K_H , are orders of magnitude larger than the molecular ones (Kim and Mahrt, 1992). The same is true for the deep ocean

- meteorological observations over very cold and smooth surfaces bear witness to a considerable decrease (but never total degeneration) of turbulence in a thin near-surface layer with perceptible wind shears and extremely strong temperature increments (e.g. Smedman *et al*., 1997).

- Degeneration of turbulence was occasionally observed in strongly stratified airflows over smooth land surfaces (Monti *et al*., 2002) and in some laboratory experiments (Strang and Fernando, 2001).

- in real system laminar flow is non-existent and turbulence exists at Ri >> 1 in atmosphere and deep oceans.

- if we say that like any other mechanical system TKE alone can not fully describe the turbulent flows.

- Introduction of budget equations of Turbulent Potential Energy (TPE) and Total Turbulent Energy (TTE) which is conserved in by shear in any stratification could help.

Validation

$$\frac{\mathrm{D}E_{\mathrm{K}}}{\mathrm{D}t} + \frac{\partial \Phi_{\mathrm{K}}}{\partial z} = -\boldsymbol{\tau} \cdot \overline{\mathbf{S}} + \beta\ F_z - \varepsilon_{\mathrm{K}},$$

$$\frac{\mathrm{D}E_{\theta}}{\mathrm{D}t} + \frac{\partial \Phi_{\theta}}{\partial z} = -F_z \frac{\partial \overline{\theta}}{\partial z} - \varepsilon_{\theta}.$$

$$\delta E_{\mathrm{P}} = \frac{g}{\rho_0} \int_z^{z+\delta z} \rho' \mathrm{d}z = \frac{1}{2} \frac{b'^2}{N^2}.$$

$$\frac{\mathrm{D}E_{\mathrm{P}}}{\mathrm{D}t} + \frac{\partial \Phi_{\mathrm{P}}}{\partial z} = -\beta F_z - \varepsilon_{\mathrm{P}},$$

$$\frac{\mathrm{D}E}{\mathrm{D}t} + \frac{\partial \Phi_E}{\partial z} = -\boldsymbol{\tau} \cdot \overline{\mathbf{S}} - \varepsilon_E,$$

The left-hand sides of budget equations are neither productive nor dissipative and describe the energy transports. TTE budget Equation simplifies to $\varepsilon_E = -\boldsymbol{\tau} \cdot \mathbf{S} > 0$, which implies generation of TTE in any stratification and thus argues against any finite value of the energetics critical Richardson number

buoyancy, b ; $b = \beta\theta$
Fz ; mean-flow equations include only the vertical component,
T *;* vertical turbulent flux of momentum: $\boldsymbol{\tau} = \mathbf{i}\tau_{xz} + \mathbf{j}\tau_{yz}$.
T_{xz}, T_{yz} *are tangential components* of the Reynolds stresses

E_k ; is the turbulent kinetic energy
E_θ ; mean squared potential temperature fluctuations

ϕ_K, ϕ_θ ; 3rd order vertical turbulent fluxes;
εK, $\varepsilon\theta$; the molecular dissipation rates
CK, *CP* are dimensionless constants of order unity; and t_T can be expressed through the turbulent length scale *l*

the mean turbulent potential energy (TPE) is defined as $E_{\mathrm{P}} = 1/2(\beta/N)^2\overline{\theta'^2}$. Then, multiplying Equation (3) by $(\beta/N)^2$ and assuming that $N^2 = \beta\partial\overline{\theta}/\partial z$ changes only slowly in space and time gives the following TPE budget equation:

The principal difference between these two concepts is that APE is an integral property of the entire flow-domain (e.g. of the atmosphere as a whole), whereas TPE is determined in each point of turbulent flow

Suppose that the buoyancy flux, βF_z, becomes so large that TKE considerably decreases. According to TTE budget eq, TTE is conserved, so that TPE increases and fluctuations of buoyancy strengthen. In other words, fluid elements acquire stronger accelerations and speed up toward their ‘equilibrium level’, which causes re-establishment of TKE, and decrease of TPE. In its turn, too large TKE causes stronger displacements of fluid elements, hence stronger buoyancy fluctuations and therefore increase of TPE.

TPE fraction, E_P/E, is negligible in neutral stratification and increases with strengthening static stability (increasing Ri). Generally speaking, the dependence of E_P/E on Ri is not universal. However, in the equilibrium turbulence regime, when the left-hand sides of the energy budget equations become zero, Equations yield a simple dependence of E_P/E on the so-called flux Richardson number, $\mathrm{Ri_f} = \beta F_z(\boldsymbol{\tau} \cdot \mathbf{S})^{-1}$

$$\frac{E_{\mathrm{P}}}{E} = \frac{(C_{\mathrm{P}}/C_{\mathrm{K}})\mathrm{Ri_f}}{1 + (C_{\mathrm{P}}/C_{\mathrm{K}} - 1)\mathrm{Ri_f}};$$

$$\tau = -K_M \overline{\mathbf{S}}, \;\; \beta F_z = -K_H N^2;$$

$$\mathrm{Ri_f} = \mathrm{Ri}/\mathrm{Pr_T}.$$

Validation

$$\frac{DE_K}{Dt} + \frac{\partial \Phi_K}{\partial z} = -\tau \cdot \bar{S} + \beta \ F_z - \varepsilon_K,$$

From this eq we can say that: in very strong static stability (at large Ri) the negative buoyancy flux, βF_z, passes a threshold, after which the TKE production, $-\boldsymbol{\tau} \cdot \mathbf{S}$, becomes insufficient to compensate the TKE losses, $-\beta F_z + \varepsilon K$, so that the turbulence can only decay (Prandtl, 1930; Chandrasekhar, 1961; Monin and Yaglom, 1971).

However, the steady-state TKE budget equation, $-\boldsymbol{\tau}\mathbf{S} = -\beta\ F_z + E_K(C_K t_T)^{-1}$, is not closed. The above reasoning says only that the ratio of the TKE consumption to its production, $Ri_f = -\beta\ F_z/(-\boldsymbol{\tau} \cdot \mathbf{S})$ called flux Richardson number, cannot exceed unity. But Rif is an internal turbulent parameter ($\boldsymbol{\tau}$ and F_z depend on each other), which is why the restriction Rif < 1 says nothing about maintenance or degeneration of turbulence at large Ri. To proceed further, the traditional approach assumes that the turbulent Prandtl number, Pr_T, is either constant or limited to a finite maximal value, Pr_{T-max}. If so, it would indeed follow from the TKE budget equation that the equilibrium turbulence exists only at Ri smaller than some critical value $Ri_c < Pr_{T-max}$. The fallacy in this conclusion is that neither theory nor experiments confirm the existence of any upper limit for Pr_T. On the contrary, the presence of turbulence at very large Ri has been disclosed in numerous experiments and numerical simulations, in particular those summarised in Figures 1–4 below.

Validation

LES data in show a well-pronounced monotonic dependence: the ratio E_P/E sharply increases with increasing Ri in the interval 0 < Ri < 1 and then levels off approaching the limiting value:
$E_P/E \approx 0.25$.

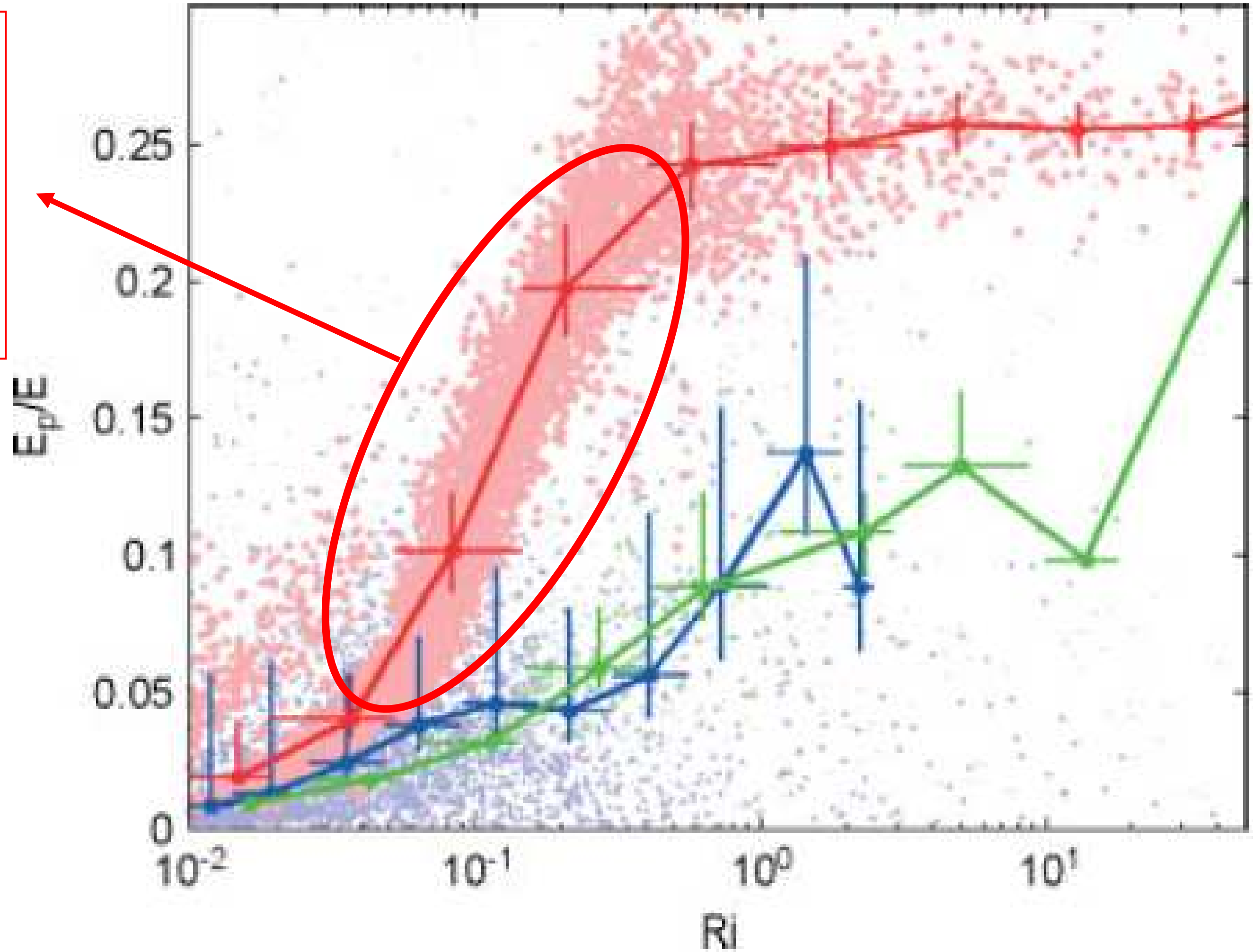

Figure 1. The ratio of the potential to total turbulent energies, E_P/E, versus the gradient Richardson number, Ri. Blue points and curve – meteorological field campaign SHEBA (Uttal *et al*., 2002); green – lab experiments (Ohya, 2001); red/pink – new large-eddy simulations (LES) using NERSC code (Esau, 2004). Vertical error bars show one standard deviation above and below the averaged value within the bin; horizontal bars show the width of the bins.

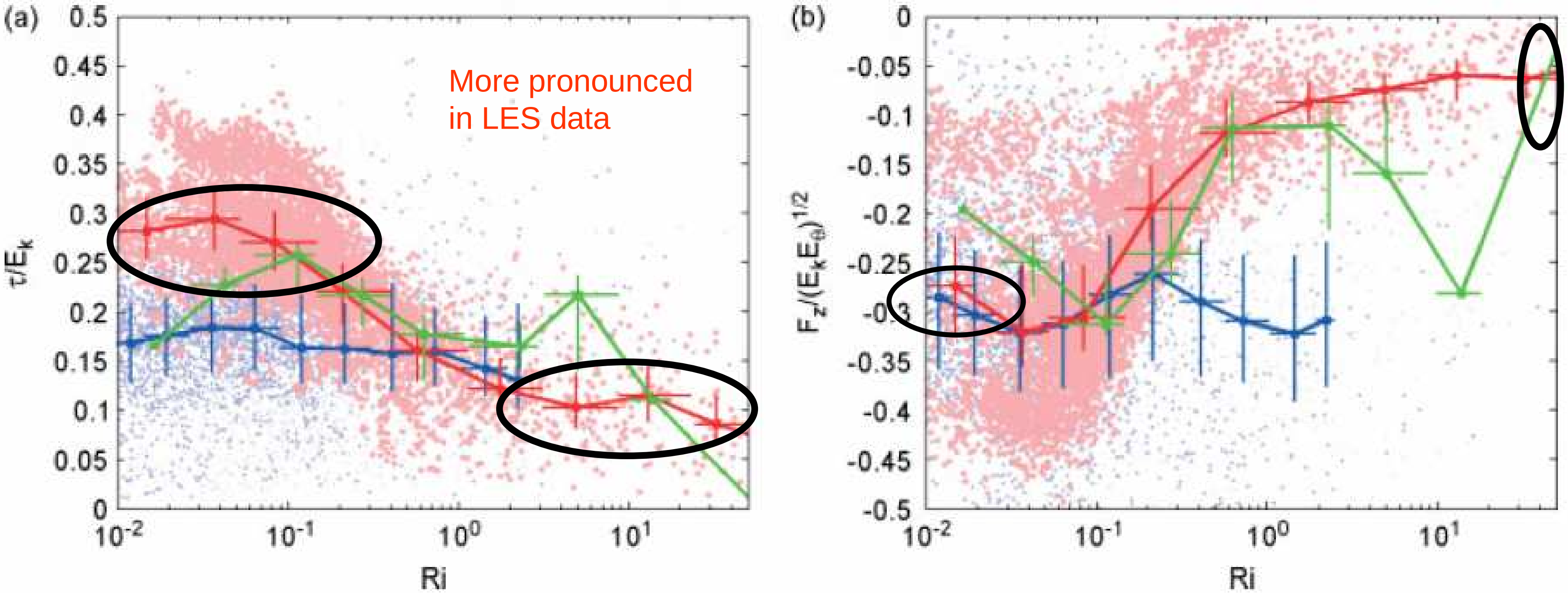

Figure 2. Normalised turbulent fluxes of momentum and heat, (a) τ/E_K and (b) $Fz/(E_K E_\theta)^{1/2}$, versus Ri, using the same data as in Figure 1.

Validation

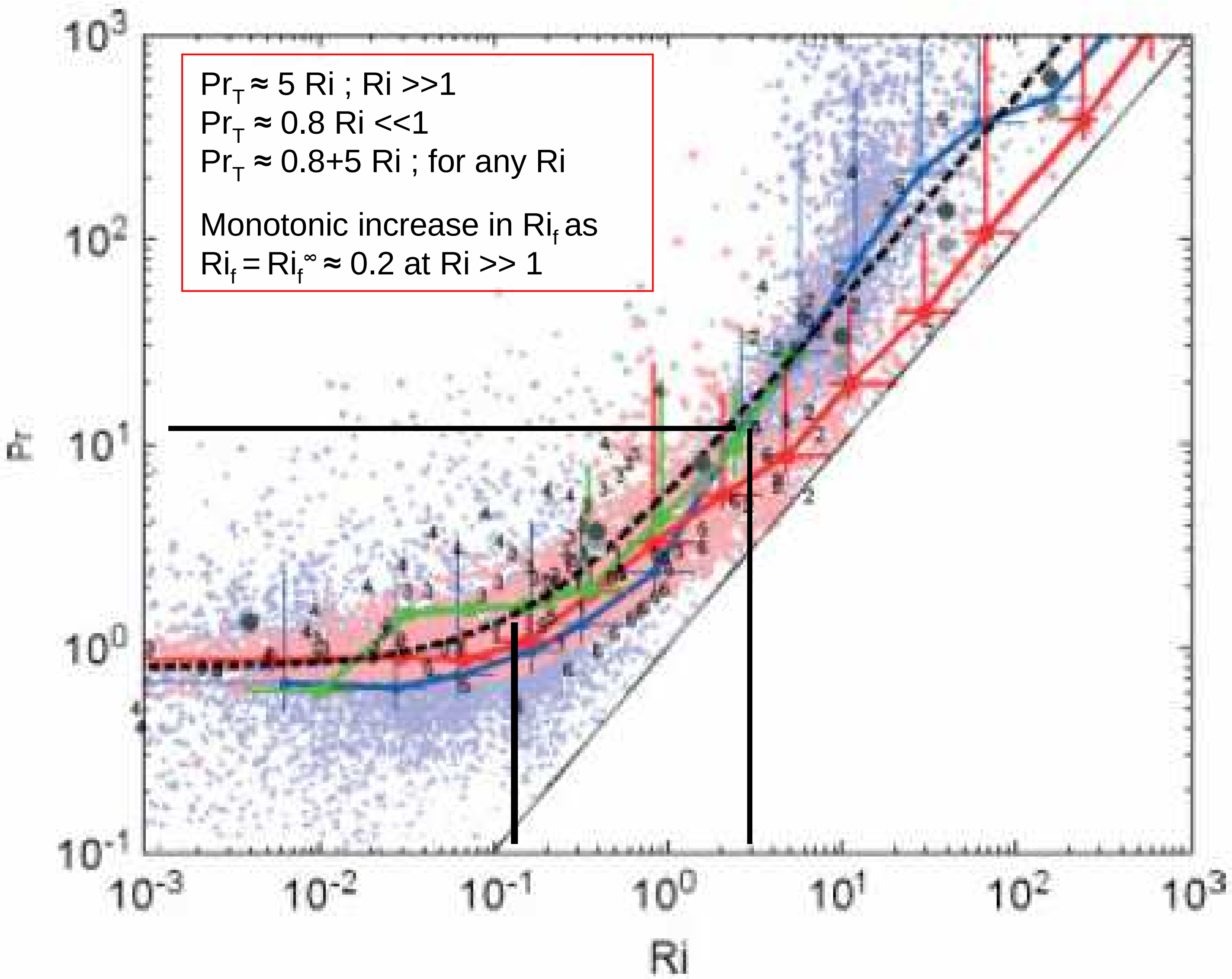

Figure 3. Turbulent Prandtl number $Pr_T = K_M/K_H$ versus Ri.
SHEBA
CASES-99
green – laboratory sheared flow
red – new LES
DNS

Numbers show data from literature

Key findings

- Using empirical very-large-Ri limits disclosed in Figures 1 and 3, namely $E_P/E \approx 0.25$ and $\mathrm{Ri}_f \approx 0.2$, $C_K/C_P \approx 0.6$. Then, using empirical large-Ri limits: $E_P/E \approx 0.25$ and $E_K/E = (E - E_P)/E \approx 0.7$, $\varepsilon_E \approx 0.7 C_K^{1/2} E^{3/2} l^{-1}$
- Then using the very-large- Ri limit: $\tau/E_K \approx 0.1$ after Figure 2, the equilibrium TTE budget equation, $\varepsilon_E = -\boldsymbol{\tau} \cdot \mathbf{S}$, yields the asymptotic formula:

$$E \approx 0.02(C_K S l)^2 > 0 \text{ at } \mathrm{Ri} \gg 1.$$

TTE is positive in any stationary, homogeneous sheared flow and confirms the argumentation against the energetics critical Richardson number

- In particular, it allows refining the definition of the stably stratified atmospheric boundary layer (ABL) as the *strong-mixing* stable layer, in contrast to the also stable but *weak-mixing* free atmosphere.
- two turbulent regimes are characterised by the small and the large Ri, respectively, it is natural to expect that the ABL outer boundary, $z = h$, should fall into the threshold interval: $0.1 < \mathrm{Ri} < 1$.

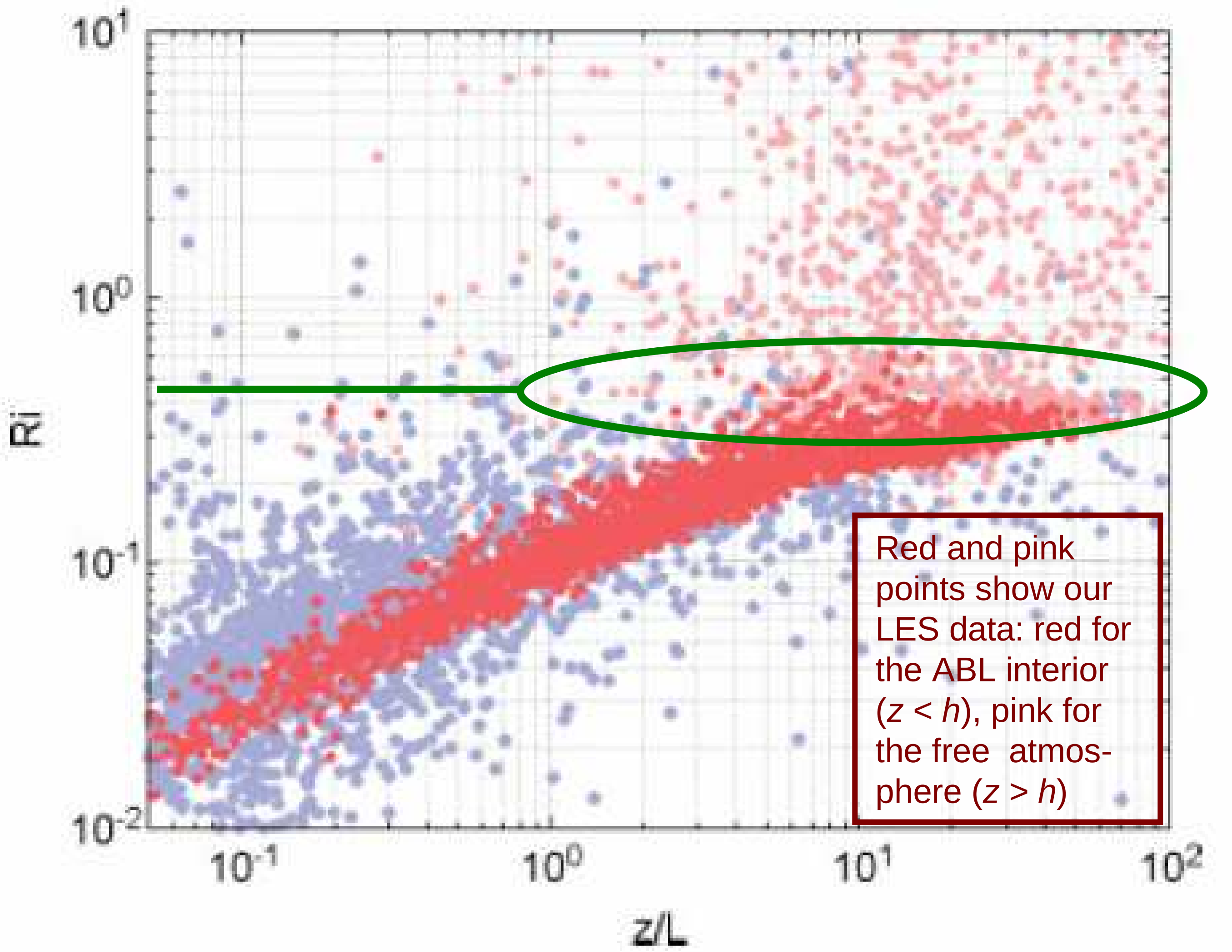

Figure 4. The gradient Richardson number within and above the stable ABL: Ri versus z/L, where $L = \tau^{3/2}(-\beta F z)^{-1}$ is the Monin–Obukhov length scale. Red points (for $z < h$) and pink points (for $z > h$) show LES data (NERSC code: Esau, 2004); blue points show atmospheric data (Uttal *et al.*, 2002).

Summary and Future Direction

- The above analyses disprove the concept of the energetics' critical Richardson number in its classical sense.
- existence of turbulence at very large Ri, up to Ri > 10^2
- What is factually observed is a threshold interval of Richardson numbers, 0.1 < Ri < 1, separating two regimes of essentially different nature but both turbulent.
- (Ri < 0.1) *strong mixing* capable of very efficiently transporting both momentum: $\tau/E_K \approx 0.3$ and heat: $-F_z/(E_K E_\theta)^{1/2} \approx 0.4$;
- (Ri > 1) *weak mixing* quite capable of transporting momentum: $\tau/E_K \to$ constant ≈ 0.1; but rather inefficient in transporting heat: $-F_z/(E_K E_\theta)^{1/2}$ drops to ~0.04 at Ri = 50
- Turbulent flows, as any other mechanical systems, are not fully characterised by their kinetic energy. TTE is more promising.
- This explains persistent occurrence of turbulence in the free atmosphere and deep ocean at Ri>>1, clarifies the principal difference between turbulent boundary layers and free flows, and provides the basis for improving operational turbulence closure models

Thanks for your attention!