

# **Atmospheric Predictability experiments with a large numerical model (E. N. Lorenz, 1982)**

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# OUTLINE OF TALK

- ✓ Introduction and Brief History
- ✓ Predictability Experiments ( Lorenz, 1982)
- ✓ Predictability Studies after 1982
- ✓ Conclusions

# What is Predictability?

- The predictability of a system refers to the degree of accuracy with which it is possible to predict the future state of the system.
- Predicting the future state of the system consisting of atmosphere, ocean, land, etc. is the main goal of the field of weather and climate.

# Historical Studies with Model

First Study was done by Thompson (1957).

- Simple barotropic model
- Initial errors tend to grow with time and that the atmospheric flow is not predictable beyond a week.
- Instability of the atmospheric flow is the main reason for limits on the predictability.
- Error between two randomly chosen maps as a convenient upper limit of the error beyond which the flow is completely unpredictable.

# The Butterfly Effect

- Two or more slightly different states , each evolving according to the same physical laws, may in due time develop into appreciably different states.
- Lorenz (1963): “... one flap of a sea gull’s wings would be enough to alter the course of the weather forever.”
- Lorenz (1972): “Predictability: Does the flap of a butterfly’s wings in Brazil set off a tornado in Texas?”

# Dynamical or Statistical Predictability

The state of the atmosphere and its surroundings (weather and climate system) is continually evolving in accordance with a set of physical laws. There are two methods of prediction.

- a) In dynamical prediction, the process of predicting future states of the atmosphere consists of extrapolating forward from the present state according to these laws.
- b) Alternately, in statistical prediction, the rules for extrapolation are established empirically, based on a sequence of past states.

# Is Perfect Forecast Possible?

There is a lack of perfection in predicting weather and climate even with the use of complex models and high speed computers. The reasons are:

- a) Imperfect knowledge of the state of the atmosphere from which one extrapolates.
- b) Inadequacy of the methods by which one extrapolates because of incomplete knowledge of the physical laws and imperfect numerical prediction schemes.



# Abstract

- The instability of the atmosphere places an upper bound on the predictability of instantaneous weather patterns.
- The skill with which current operational forecasting procedures are observed to perform determines a lower bound.
- Estimates of the both bounds are obtained by comparing the ECMWF operational forecast.
- Predictions at least 10-days ahead as skillful as predictions now made 7-days ahead appear to be possible.



# Lower and Upper Bounds

- Lower bound refers to the *minimum accuracy* with which forecasts can be made.
- The performance of the current operational NWP models gives an estimate of the lower bound of predictability (i.e., there is a possibility of doing better than that)
- The upper bound on predictability refers to the *maximum error* for forecasts at a given range.
- Classical predictability studies give an estimate of the upper bound on predictability (i.e., we cannot do better than that)

# Model and data used

- ECMWF 15-level global primitive equation model with moisture and orography.
- Forecast from one up to ten days in advance are prepared.
- 100 days Study Period: 1 December 1980 to 10 March 1981
- Analysis is referred as a zero-day prognosis. For each day of the above period we have  $k=0,1,2,\dots,10$ -day prognoses.
- Analyzed and Predicted 500-mb height fields are transformed into global spherical harmonic sequences, triangularly truncated at wave number 40.

# Anaylsis

Each height field  $z(\lambda, \varphi)$  where  $\lambda$  is longitude and  $\varphi$  is latitude, is therefore represented by a set of  $41 \times 42 = 1722$  spherical-harmonic coefficients  $A_{mn}$  or  $B_{mn}$ , according to the formula.

$$z(\lambda, \varphi) = \sum_{m=0}^{40} \sum_{n=m}^{40} (A_{mn} \cos m\lambda + B_{mn} \sin m\lambda) \times P_n^m(\sin \varphi)$$

$P_n^m$  is the associated Legendre function of degree  $n$  and order  $m$ , suitably normalized. Normalize Spherical Harmonics so that:

For  $m = 0$  average Square of  $P_n^0$  is 1.

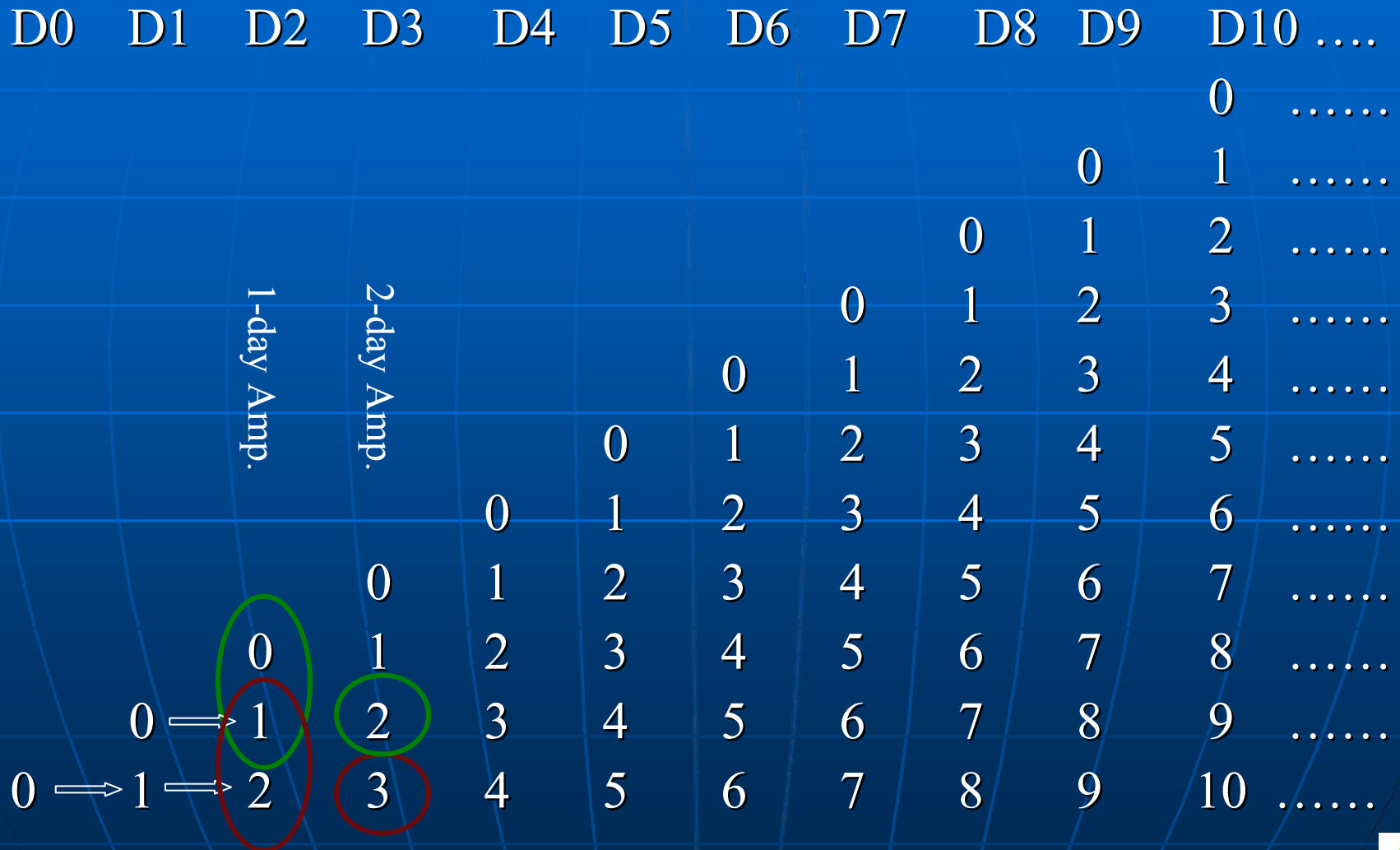
For  $m > 0$  average Square of  $P_n^m \cos m\lambda$  and  $P_n^m \sin m\lambda$

$$\frac{1}{S} \int_S z^2(\lambda, \varphi) dS = \sum_{m=0}^{40} \sum_{n=m}^{40} (A_{mn}^2 + B_{mn}^2)$$

# First Experiment

- The analysis for a given day, regardless of its accuracy, and the one-day prognosis for the same day represent two states which do not differ too greatly.
- One day forecast made from these two states are simply they 1-day and 2-day prognosis for the following day.
- By comparing average difference between 1-day and 2-day prognosis for the same day with the average difference between analysis and 1-day prognoses, we can obtain an estimate of the average one-day amplification of moderately small errors.

# Error Amplification (Small Errors $k - j = 1$ )



# Error Amplification (Large Errors $k - j > 1$ )

D0	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	....
								$k-j=1$		0	.....
								$k-j=2$	0	1	.....
						$k-j=3$	0	1	2	.....	
					$k-j=4$	0	1	2	3	.....	
				$k-j=5$	0	1	2	3	4	.....	
			$k-j=6$	0	1	2	3	4	5	.....	
		$k-j=7$	0	1	2	3	4	5	6	.....	
	$k-j=8$	0	1	2	3	4	5	6	7	.....	
		$k-j=9$	0	1	2	3	4	5	6	.....	
0	1	2	3	4	5	6	7	8	9	10	.....



# Global R.M.S Differences

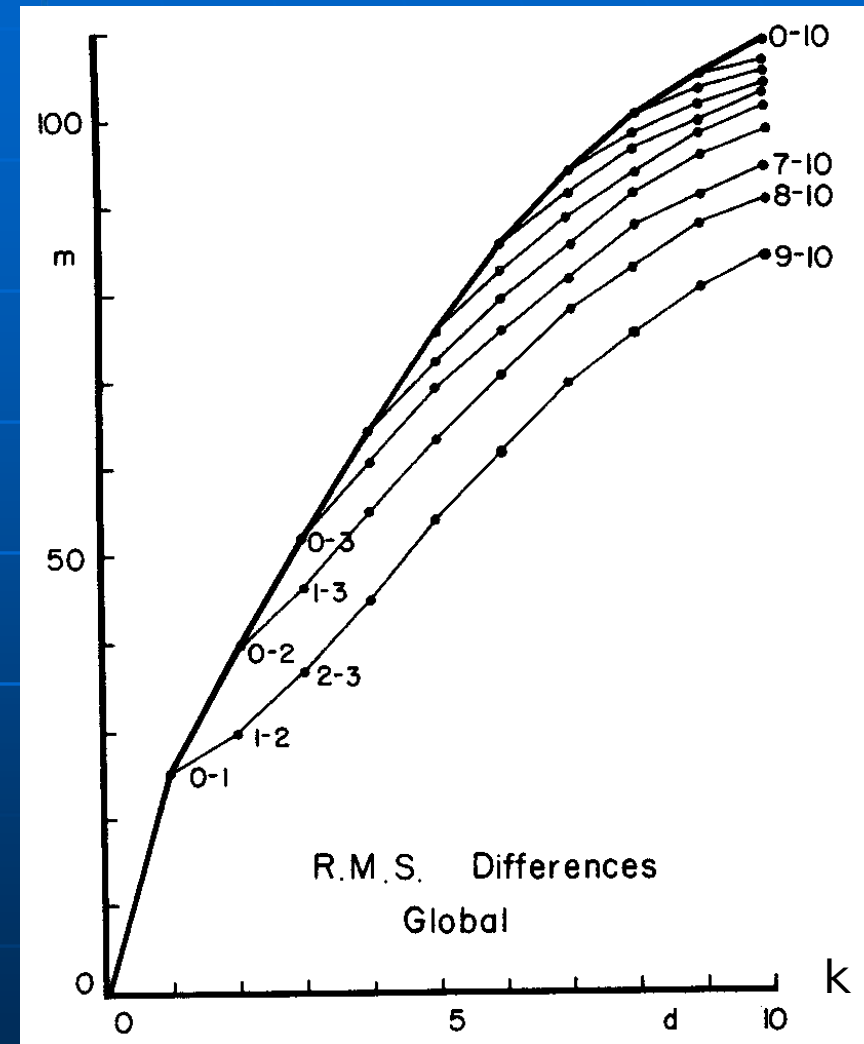
If  $E_{ij}$  is the root-mean square difference between  $j$ -day and  $k$ -day prognoses for the same day, averaged over the globe and over all  $N$  ( $=100$ ) days of the sample, then

$$E_{jk}^2 = \frac{1}{N} \frac{1}{S} \sum_{i=1}^N \int_S [z_{ij}(\lambda, \varphi) - z_{ik}(\lambda, \varphi)]^2 dS$$

Using previous equation for  $z(\lambda, \varphi)$

$$E_{jk}^2 = \frac{1}{N} \sum_{i=1}^N \sum_{m=0}^{40} \sum_{n=m}^{40} [(A_{mn,ij} - A_{mn,ik})^2 + (B_{mn,ij} - B_{mn,ik})^2]$$

Heavy curve connects values of  $E_{0k}$ . Thin curve connects values of  $E_{jk}$  for constant  $k-j$ .



Lorenz curves



# Lorenz's Empirical Formula for Error Growth

Introduce an ensemble of small initial errors and allow it evolve. If  $E$  is the mean error, the exponential growth is given by the equation:

$$dE/dt = aE$$

Doubling time of the errors  $t_d = (\ln 2)/a$

Lorenz introduced a simple assumption that nonlinear error growth is quadratic in  $E$ . The modified error equation is:

$$dE/dt = aE - bE^2$$

The constant  $a$  measures the growth rate of small error. If  $E$  is normalized so that the value which it approaches as  $t \rightarrow \infty$  is unity,  $b=a$ . The solution of the equation is:

$$E/(1 - E) = \exp[a(t - t_0)]$$

Where  $t_0$  is time at which  $E=1/2$ , equivalently

$$E = \frac{1}{2} + \frac{1}{2} \tanh\left[\frac{1}{2} a(t - t_0)\right]$$

# Furthur Experiment: Modified Model

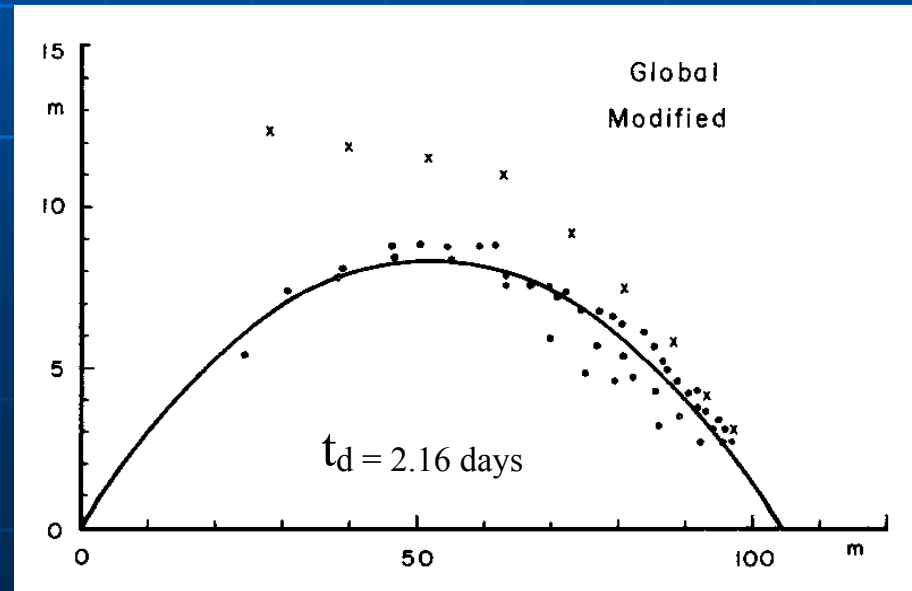
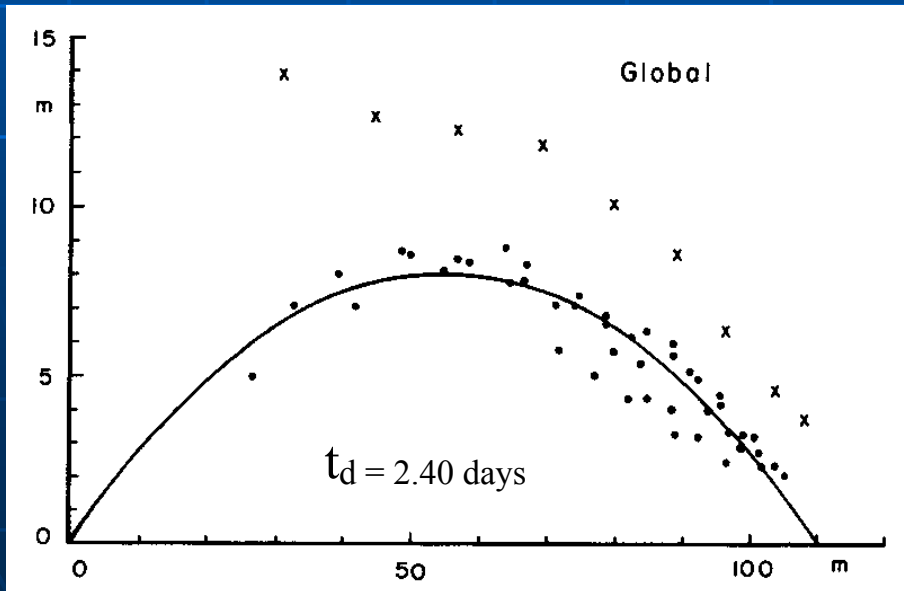
Lorenz replaced the prediction  $X$  for a predictand  $Y$ , where  $X$  stands for any spherical harmonic coefficient  $A_{mn}$  or  $B_{mn}$  in a prognosis, and  $Y$  stands for the same coefficients in an analysis, by the linear function of  $X$ .

$$X' = A + BX$$

Where  $A$  and  $B$  are to be chosen so that  $X'$  possesses the same temporal mean and standard deviation as  $Y$ .

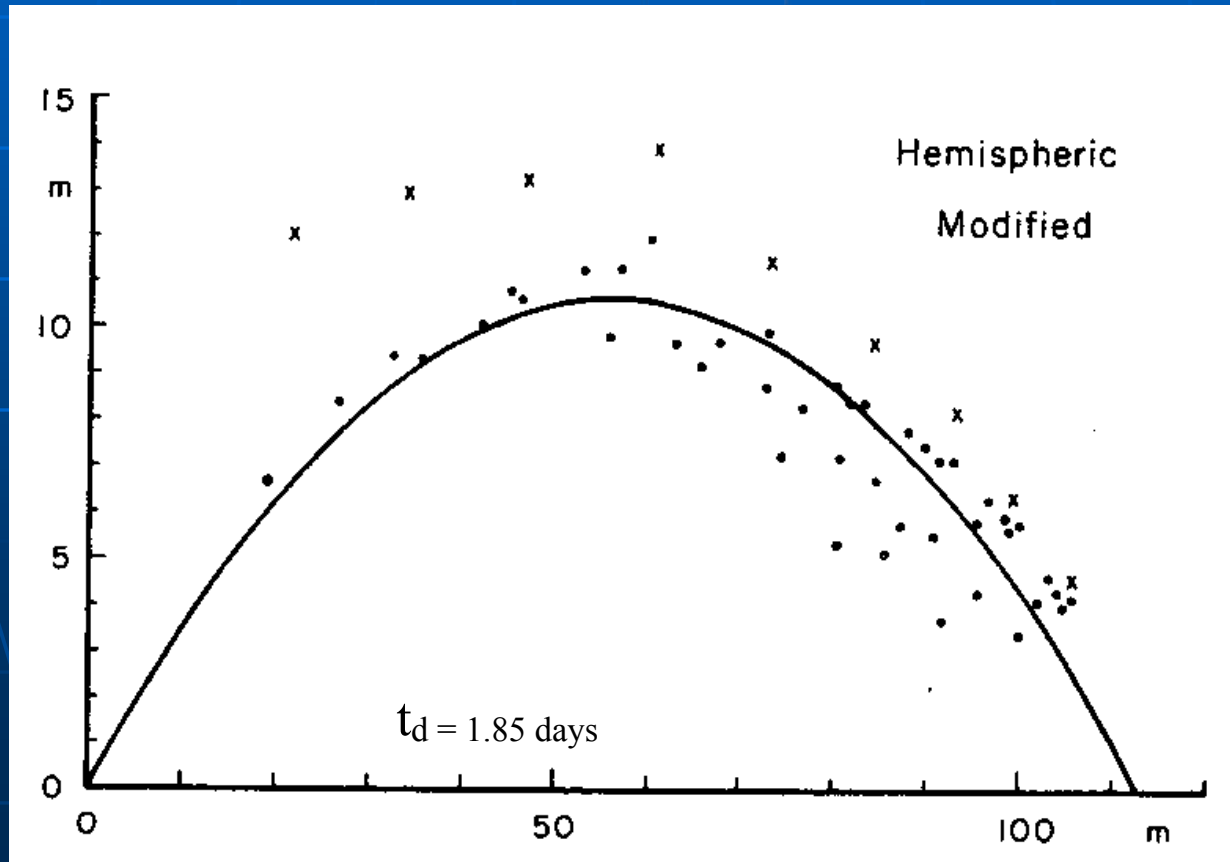
# Results

Increases in global root-mean-square 500-mb height differences,  $E_{j+1,k+1} - E_{jk}$ , plotted against average height differences  $(E_{j+1,k+1} - E_{jk})/2$ , in meters, for each one-day segment of each thin curve in previous figure (large dots), and increases  $E_{0,k+1} - E_{0k}$  plotted against average differences  $(E_{0,k+1} - E_{0k})/2$ , for each one-day segment of heavy curve in previous Fig. 1 (crosses). Parabola of “best fit” to large dots is shown.

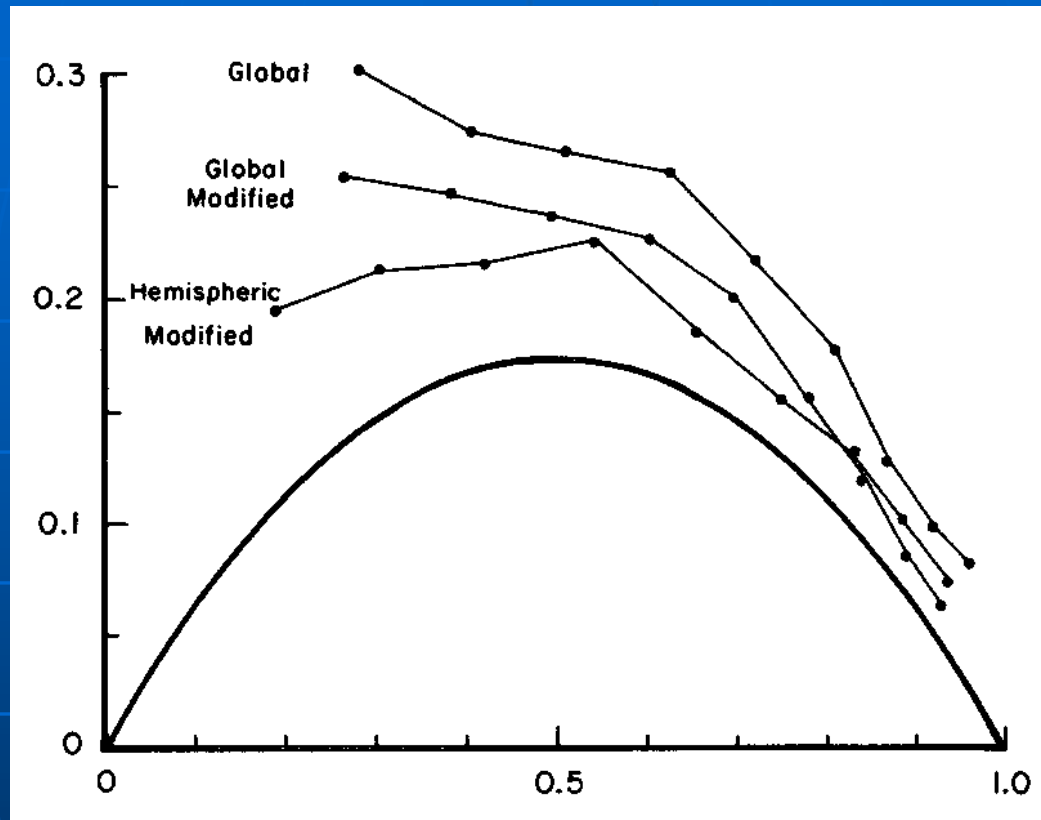


# Modified Model: Northern Hemisphere

Good Prognosis requires good analysis. Model behaves like a better model in regions where data are more plentiful.



# Comparison



Superposition of points marked by crosses for the last three figures, after horizontal and vertical scales have been altered so that parabolas coincide.

# Conclusion

- Without further improvement in one-day forecasting, we may eventually make ten-day forecasts as good as present 7-day forecast and 13.5 day forecast as good as present 10-day forecasts.
- “Additional improvements at extended range may be realized if the one-day forecast is capable of being improved significantly.”

# Present Day Models

- The present day forecasting methods use primitive equations. The numerical models at leading forecasting centers have a horizontal resolution of about 40-50km. Certain processes such as boundary layer transports, clouds and convection are parametrized. Several millions of equations are solved using supercomputers.
- The initial states for the forecast integrations are prepared from observations and data assimilation. The observations come from single-site instruments, balloons, aircrafts, satellites and radars. The observed data are used for preparing the initial states by data assimilation methods which ensure that the component fields are in physical balance.



# Ensemble Forecast

- Because of the uncertainty in the initial states, several integrations are performed using slightly different initial conditions. These integrations produce an ensemble of forecasts giving a range of possible future states.
- ECMWF is using ensemble of about 50.

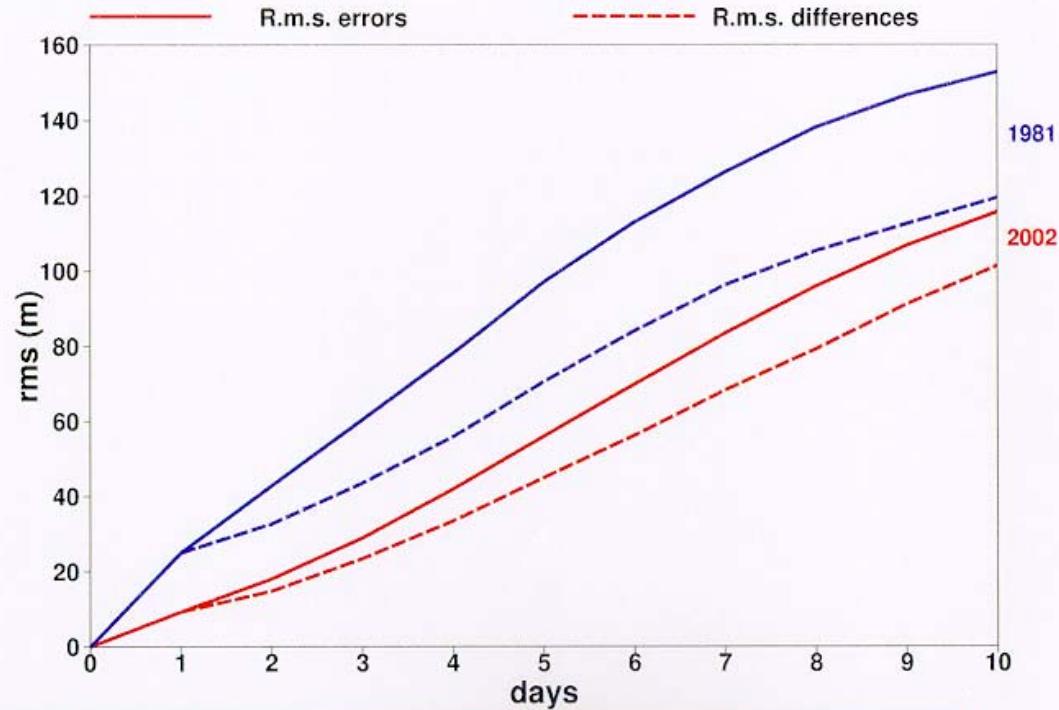
# Evolution of 1-Day Forecast Error, Lorenz Error Growth, and Forecast Skill for ECMWF Model (500 hPa NH Winter)

	1982	1987	1992	1997	2002
<b>“Initial error” (1-day forecast error) [m]</b>	<b>20</b>	<b>15</b>	<b>14</b>	<b>14</b>	<b>8</b>
<b>Doubling time [days]</b>	<b>1.9</b>	<b>1.6</b>	<b>1.5</b>	<b>1.5</b>	<b>1.2</b>
<b>Forecast skill [day 5 ACC ]</b>	<b>0.65</b>	<b>0.72</b>	<b>0.75</b>	<b>0.78</b>	<b>0.84</b>

Thanks to ECMWF

# 1981 - 2002

R.m.s. errors and differences between successive forecasts  
Northern hemisphere 500hPa height Winter



Current Limits of Predictability

A.Hollingsworth,

Savannah

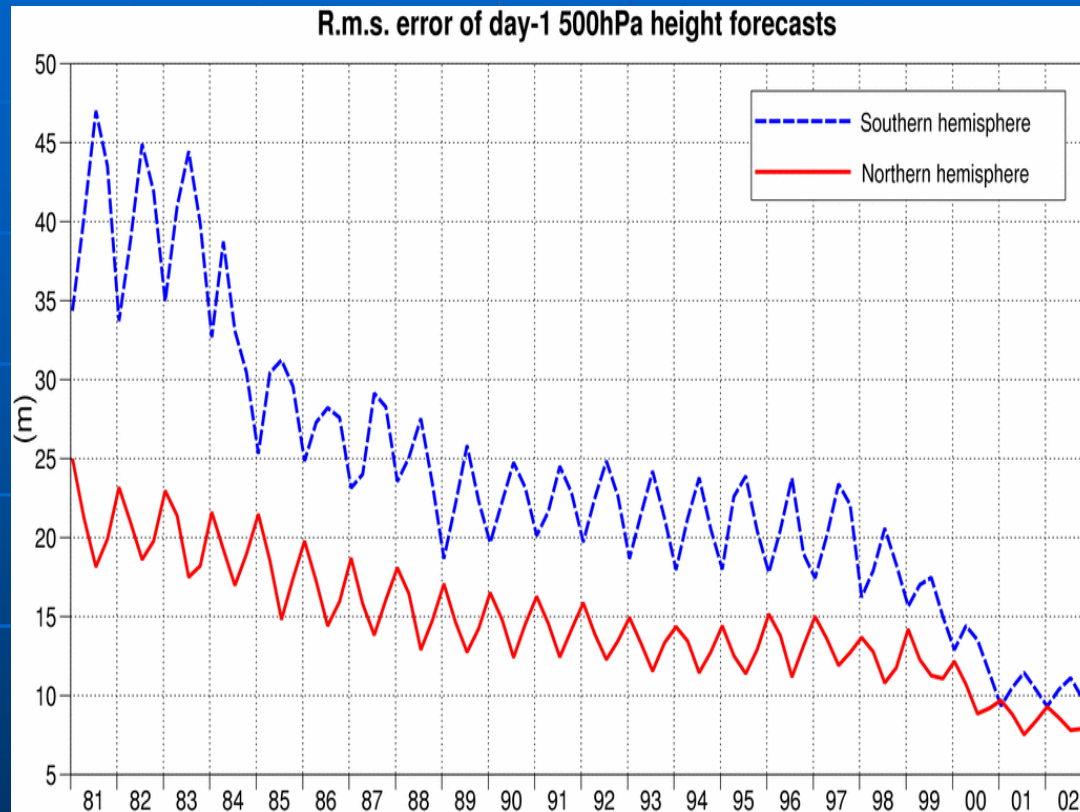
Feb 2003

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Thanks to ECMWF

# ECMWF 500 hPa Error Evolution 1981 – 2002



N.Hem & S.Hem D+1 Forecast error

A. Hollingsworth ECMWF - Sloan Conference on Weather Predictability 2003

# Conclusions

- The largest obstacles in realizing the potential predictability of weather and climate are inaccurate models and insufficient observations, rather than an intrinsic limit of predictability.
- Scientists worldwide have made tremendous progress in improving the skill of weather forecasts by advances in data assimilation, improved parameterizations, improvements in numerical techniques and increases in model resolution and computing power.
- The next big challenge is to build a hypothetical “perfect” model which can replicate the statistical properties of past observed climate (means, variances, covariances and patterns of covariability), and use this model to estimate the limits of weather and climate predictability.

# Weather Prediction Model of ~2020

## Coupled Ocean-Land-Atmosphere Model



**Assumption: Computing power enhancement by a factor of  $10^3$ - $10^4$**

Thanks for your attention

