QUARTERLY JOURNAL OF THE ROYAL METEOROLOGICAL SOCIETY
Q. J. R. Meteorol. Soc. **133**: 2133–2136 (2007)
Published online in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/qj.190

Notes and Correspondence

The low-level katabatic jet height versus Monin–Obukhov height

B. Grisogono,[a]* L. Kraljević[b] and A. Jeričević[b]
[a] *AMGI Department of Geophysics, Faculty of Science, University of Zagreb, Croatia*
[b] *MHSC Weather Service, Zagreb, Croatia*

ABSTRACT: In this short note we discuss a long-standing problem in modelling the atmospheric boundary layer (ABL) over complex terrain: namely, an excessive use of the Monin–Obukhov length scale L_{MO}. This issue becomes increasingly relevant with the ever-increasing resolution of numerical weather-prediction and climate models, which typically use L_{MO} in one way or another for parametrizing the surface layer, or at least for formulating the lower boundary conditions. Hence, inevitably, the models under-represent a significant part of the mesoscale flow variability.

We focus here on the stable ABL over land: in particular, sloped cooled flows. However, a qualitatively similar reasoning applies to the corresponding unstable ABL. We show that for sufficiently stratified flows over moderately sloped surfaces, Monin–Obukhov scaling is inadequate for describing the basic ABL dynamics, which is often governed by katabatic and drainage flows. Copyright © 2007 Royal Meteorological Society

KEY WORDS stable ABL; drainage flows; katabatic wind; Prandtl model; scaling

Received 30 June 2007; Revised 5 November 2007; Accepted 7 November 2007

1. Introduction

In complex boundary-layer flows, where the interaction of the air flow with inclined surfaces can be complex, subtle, and dependent on many parameters, one needs to refine the ordinary methods for flux estimations that are used over nearly-horizontal surfaces. This relates, for example, to atmospheric-boundary-layer (ABL) schemes in numerical weather prediction (NWP), and climatological and air-pollution models, all of which need near-surface flux parameters. Monin–Obukhov theory has been most often used for scaling near-surface fluxes (e.g. Stull, 1988; Zilitinkevich *et al*., 2002), even though it has been shown that this theory and its scaling is sometimes inadequate (Munro and Davies, 1978; Mahrt, 1998, 2007a, 2007b; van der Avoird and Duynkerke, 1999; Munro, 2004). In this study we mostly neglect convective conditions and focus on a stable ABL. The use of the Monin–Obukhov length scale L_{MO} is often questionable for katabatic flows (e.g. Grisogono and Oerlemans, 2001a, 2001b) and other stable ABL flows (e.g. Mahrt, 1998, 2007a; Zilitinkevich and Calanca, 2000; Jeričević and Grisogono, 2006; Zilitinkevich and Esau, 2007).

Apparently there is a need for an extension of Monin–Obukhov similarity theory to handle sloping terrain (e.g. Mahrt, 1981, 1998; Grisogono and Oerlemans, 2001b; Zilitinkevich *et al*., 2006; Baklanov and Grisogono, 2007). Although friction acts at the inclined surface, turbulence production is not governed by the surface but by the low-level jet. We tackle this issue by comparing L_{MO} with the height of the low-level katabatic jet, z_j. When $L_{MO} > z_j$, then L_{MO} ought to be used with caution, because it does not capture a short-enough scale to be relevant for the effects of turbulent eddies on the fluxes.

Important related questions include the existence of a critical Richardson number, Ri, in the ABL and the possible increase of the eddy Prandtl number, Pr, with increasing Ri (Kondo *et al*., 1978; Mahrt, 1998, 2007b; Monti *et al*., 2002; Zilitinkevich and Esau, 2007). The critical Ri, employed in linear theory for infinitesimal perturbations, seems to be precluded in the ABL, where pre-existing finite-amplitude disturbances are almost always present. These disturbances include various buoyancy waves, two-dimensional modes (meandering or pancake motions), and other more complicated imprints of unknown dynamics. Our simple approach uses the Prandtl model (Prandtl, 1942; Defant, 1949) for estimating z_j. This model has been extended for the vertically-varying eddy diffusivity and Coriolis effects (Grisogono and Oerlemans, 2001a, 2001b; Stiperski *et al*., 2007; Kavčič and Grisogono, 2007); moreover, Stiperski *et al*. (2007) also show that the Prandtl model may work even for finite-amplitude disturbances (as long as the one-dimensionality assumption holds). This will give some confidence to our

* Correspondence to: B. Grisogono, AMGI Department of Geophysics, Faculty of Science, Horvatovac bb, 10000 Zagreb, Croatia.
E-mail: bgrisog@gfz.hr

reasoning about an asymptotic relation between Pr and Ri.

2. The length-scale comparison

Here we briefly recall the definitions for L_{MO} (e.g. Stull, 1988) and the classical z_{j} (e.g. Egger, 1990; Grisogono and Oerlemans, 2001b). The former is defined as:

$$L_{\mathrm{MO}} = -\frac{\overline{\theta}}{gk} \cdot \frac{u_*^3}{\overline{w'\theta'}}, \tag{1}$$

implying that the turbulent flow is horizontally homogeneous (e.g. Mahrt, 1981, 1998; Stull, 1998), and thus that there is no dynamically-relevant slope significantly affecting the flow. Here, u_* is the friction velocity, $\overline{w'\theta'}$ is the near-surface heat flux (already divided by the density and the specific heat at constant pressure), g is the acceleration due to gravity, $\overline{\theta}$ is a relevant potential temperature, and k is the von Karman constant.

In stark contrast to Equation (1), for an ABL that is slightly tilted–say by 5° or so, which is hardly visible to the eye–the wind receives a direct contribution from buoyancy forces. Probably the simplest meaningful model for the latter flow regime is that of Prandtl (1942) (e.g. Defant, 1949; Egger, 1990). Of course, this generally applies to both statically unstable (i.e. anabatic) and stable (i.e. katabatic) ABL flows. We focus here on sloped cooled (katabatic) flows. It is straightforward to show that:

$$z_{\mathrm{j}} = \frac{\pi}{4}\left(\frac{4K^2 Pr}{N^2 \sin^2\alpha}\right)^{\frac{1}{4}} \tag{2}$$

(e.g. Egger, 1990; Grisogono and Oerlemans, 2001b). Here K is the eddy heat conductivity (giving eddy diffusivity for momentum if multiplied by Pr), N is the buoyancy frequency, and α is the constant slope angle.

Using K-theory to express the near-surface momentum and heat fluxes,

$$u_*^2 = KPr\frac{\partial U}{\partial z}$$

and

$$-\overline{w'\theta'} = K\frac{\partial \Theta}{\partial z},$$

and using the definition for (gradient) Ri,

$$Ri = \frac{N^2}{\left(\frac{\partial U}{\partial z}\right)^2},$$

where U is the mean wind speed, we find the squared ratio:

$$Br = \left(\frac{L_{\mathrm{MO}}}{z_{\mathrm{j}}}\right)^2 = \frac{8}{(k\pi)^2}|\sin\alpha|\left(\frac{Pr^5}{Ri^3}\right)^{\frac{1}{2}}. \tag{3}$$

It is obvious from Equation (3) that as long as α is very small (e.g. $\alpha < 5°$) and $Pr \approx Ri \sim 1$, then $L_{\mathrm{MO}} < z_{\mathrm{j}}$, i.e. $Br < 1$, and the classical Monin–Obukhov theory may apply in the context considered here; furthermore, one may proceed as before in modelling the stable ABL. However, Equation (3) also shows that for moderate and steeper slopes (say $\alpha \gtrapprox 5°$), sufficiently stratified flows with $Pr > 1$ are inevitably susceptible to more momentum than heat mixing, and then $z_{\mathrm{j}} < L_{\mathrm{MO}}$, i.e. $Br > 1$. In such flows, L_{MO} is too large to represent the near-surface fluxes dictated now by the low-level katabatic jet. Therefore, L_{MO} becomes the less relevant scale for turbulent processes in the stable ABL (e.g. Mahrt, 1998; van der Avoird and Duynkerke, 1999; Grisogono and Oerlemans, 2001b). As an instructive example, Figure 1 summarizes our findings; the particular choice of the values plotted does not change our main result or proof of concept. For instance, if an NWP model with a horizontal resolution of 8 km resolves terrain with a mountain height of 1 km, the corresponding slope is over 5°, and thus is prone to more or less persistent sloped flows (e.g. Egger, 1990; Parmhed *et al.*, 2004). For these flows, $Br > 1$, and the related near-surface flow does not satisfy the assumptions related to L_{MO}. Hence (e.g. Mahrt, 1998; Grisogono and Oerlemans, 2001b, 2002), such a sloped strongly-stable ABL, driven by cooling from below, does not possess the classical surface layer described by L_{MO}.

3. Discussion

In an observational case provided by Greuell *et al.* (1997), and reconsidered in further detail by Grisogono and Oerlemans (2001b), z_{j} was about 5–7 m and L_{MO} about 18 m. The related near-surface flow was dominated by the katabatic wind, which could not have been described properly by Monin–Obukhov theory: when z_{j} is so low, there is simply no room for a classical surface layer where the fluxes would be nearly constant. Using Equation (3) and Figure 1, one can still attempt to extend the Monin–Obukhov theory so as to include shallow (simple) katabatic flows in NWP and climate models, i.e. to avoid $Br > 1$. A first candidate for such an extension or modification would be:

$$L_{\mathrm{MOD}}^{-1} = aL_{\mathrm{MO}}^{-1} + bz_{\mathrm{j}}^{-1},$$

where a and b are unknown coefficients that should be found from observational data. Meanwhile, the values of a and b for certain limiting cases could be assessed analytically. The surface-layer similarity in such cases should be modelled via this z/L_{MOD}.

An alternative way of modelling turbulent fluxes in a stable ABL, which we only sketch here in passing, could be based on the WKB theory (Grisogono and Oerlemans, 2001a, 2001b, 2002; Parmhed *et al.*, 2004; Kavčič and Grisogono, 2007). It seems that L_{MO} is only a good scale for eddy diffusivity, as such, which should vary on a scale larger than z_{j}, while the fluxes should be determined by

DOI: 10.1002/qj

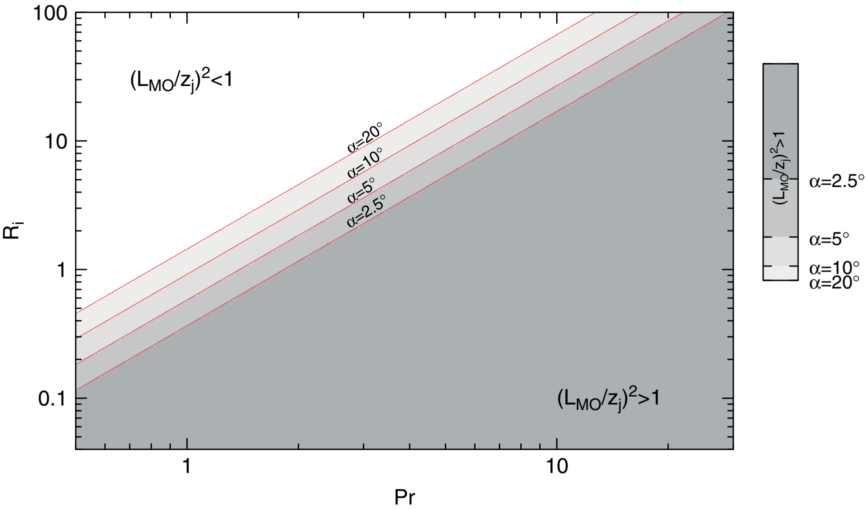

Figure 1. Log-log display of the squared ratio of Monin–Obukhov length vs. low-level katabatic jet height, $Br = (L_{MO}/z_j)^2$, for four different values of terrain slopes (the lines). Whenever this ratio is larger than one, $Br > 1$, pertaining to the lower right part of the plot, L_{MO} is not the relevant scale for the near-surface turbulent fluxes. Higher the slope or/and stronger stratification, earlier the onset of $Br > 1$ and hence the validity of the proposed scaling with z_j. This figure is available in colour online at www.interscience.wiley.com/qj

z_j, or alternatively by L_{MOD}. For example, the height of the maximum eddy diffusivity in the case mentioned above was 20 m, which is very close to $L_{MO} = 18$ m. On the other side, there is also evidence that classical local scaling may work even for sloped flows (e.g. Heinemann, 2004); according to our study, such flows exhibit $Br < 1$.

Suppose that flows under strong stability over a given slope somehow reach a constant ratio in Equation (3) asymptotically; then moderate changes in either Ri or Pr no longer affect this ratio. Thus, given this condition, the supposed constancy of Br implies the constancy of the square-root factor in Equation (3). So in that case $Pr \sim Ri^{\frac{3}{5}}$ asymptotically; this is consistent with ideas from other studies that $Pr \sim Ri^q$, with $0.2 \lessapprox q \lessapprox 0.8$ (e.g. Kondo *et al.*, 1978; Monti *et al.*, 2002; Mahrt, 2007b). This is in a broad agreement with the findings of Richardson (1920), Mauritsen *et al.* (2007) and Zilitinkevich and Esau (2007) that turbulence may exist at any Ri, given sufficiently high Pr. The dimensionless parameter Br in Equation (3) could be generalized to other types of low-level jets. In this way, one might avoid a somewhat similar scaling of L_{MO} with, for example, the top of a very stable ABL, which is often poorly and ambiguously defined.

5. Conclusions

We have compared the Monin–Obukhov length with the height of the low-level katabatic jet estimated from the Prandtl model for simple sloped flows. For a given slope, we have shown when Monin–Obukhov scaling becomes inadequate to describe the lower part of the ABL dynamics, which is governed by katabatic wind. Specifically, Equation (3) and Figure 1 indicate the region of the (Pr, Ri) subspace in which the classical L_{MO} may not describe turbulent processes related to low-level jets. In short, in this note we propose another vertical scaling for the lower part of a very stable ABL. It is plausible that a similar reasoning can be deployed for other types of low-level jets, or anabatic ABL flows. This information is useful for NWP and climate modelling, as well as for air-pollution and dispersion calculations based on these models, because it enables *a priori* estimation of where and when near-surface turbulent fluxes based on L_{MO} will be wrong. Furthermore, we have suggested a first step to remedy this failure.

Acknowledgements

We thank Danijel Belušić, Larry Mahrt, Sergej Zilitinkevich, Leif Enger and Thorsten Mauritsen for constructive discussions. This study was supported by project BORA 119-1193086-1311 and partly by two other projects, numbers 004-1193086-3036 and 119-1193086-1323, all provided by the Croatian Ministry of Science, Education and Sports. This work has also been partly supported by EMEP4HR project number 175183/S30, provided by the Research Council of Norway.

References

Baklanov A, Grisogono B. 2007. Atmospheric boundary layers: Nature, theory and applications to environmental modelling and security. *Boundary-Layer Meteorol.* **125**: 157–160.

Defant F. 1949. Zur theorie der Hangwinde, nebst bemerkungen zur Theorie der Berg- und Talwinde. *Arch. Meteorol. Geophys. Bioklim.* **A1**: 421–450.

Egger J. 1990. Thermally forced flows: theory. Pp. 43–57 in *Atmospheric Processes Over Complex Terrain*, Blumen W (ed). American Meteorological Society.

Grisogono B, Oerlemans J. 2001a. Katabatic flow: analytic solution for gradually varying eddy diffusivities. *J. Atmos. Sci.* **58**: 3349–3354.

Grisogono B, Oerlemans J. 2001b. A theory for the estimation of surface fluxes in simple katabatic flows. *Q. J. R. Meteorol. Soc.* **127**: 2725–2739.

Grisogono B, Oerlemans J. 2002. Justifying the WKB approximation in pure katabatic flows. *Tellus* **54A**: 453–462.

DOI: 10.1002/qj

Greuell W, Knap W, Smeets P. 1997. Elevational changes in meteorological variables along a midlatitude glacier during summer. *J. Geophys. Res.* **102**(D22): 25 941–25 954.

Heinemann G. 2004. Local similarity properties of the continously turbulent stable boundary layer over Greenland. *Boundary-Layer Meteorol.* **112**: 283–305.

Jeričević A, Grisogono B. 2006. The critical bulk Richardson number in urban areas: verification and application in a numerical weather prediction model. *Tellus* **58A**: 19–27.

Kavčič I, Grisogono B. 2007. Katabatic flow with Coriolis effect and gradually varying eddy diffusivity. *Boundary-Layer Meteorol.* **125**: 377–387.

Kondo J, Kanechika O, Yasuda N. 1978. Heat and momentum transfers under strong stability in the atmospheric surface layer. *J. Atmos. Sci.* **35**: 1012–1021.

Mahrt L. 1981. Modelling the depth of the stable boundary layer. *Boundary-Layer Meteorol.* **21**: 3–19.

Mahrt L. 1998. Stratified atmospheric boundary layers and breakdown of models. *Theor. Comp. Fluid Dyn.* **11**: 263–279.

Mahrt L. 2007a. Bulk formulation of the surface fluxes exteded to weak-wind stable conditions. *Q. J. R. Meteorol. Soc.* (to appear).

Mahrt L. 2007b. The influence of small-scale nonstationarity on the turbulent flux for stable stratification. *Boundary-Layer Meteorol.* **125**: 245–264.

Mauritsen T, Svensson G, Zilitinkevich S, Esau I, Enger L, Grisogono B. 2007. A total turbulent energy closure model for neutral and stably stratified atmospheric boundary layers. *J. Atmos. Sci.* **11**: 4117–4126.

Monti P, Fernando HJS, Princevac M, Chan WC, Kowalewski TA, Pardyjak ER. 2002. Observations of flow and turbulence in the nocturnal boundary layer over a slope. *J. Atmos. Sci.* **59**: 2513–2534.

Munro DS. 2004. Revisiting bulk heat transfer on Peyto Glacier, Alberta, Canada, in light of the CIG parameterization. *J. Glaciol.* **50**: 590–600.

Munro DS, Davies JA. 1978. On fitting the log-linear model to wind speed and temperature profiles over a melting glacier. *Boundary-Layer Meteorol.* **15**: 423–437.

Parmhed O, Oerlemans J, Grisogono B. 2004. Describing surface-fluxes in katabatic flow on Breidamerkurjökull, Iceland. *Q. J. R. Meteorol. Soc.* **130**: 1137–1151.

Prandtl L. 1942. *Führer durch die Strömungslehre*, pp. 373–375. Vieweg und Sohn: Braunschweig.

Richardson LF. 1920. The supply of energy from and to atmospheric eddies. *Proc. R. Soc. London* **A97**: 354–373.

Stiperski I, Kavčič I, Grisogono B, Durran DR. 2007. Including Coriolis effects in the Prandtl model for katabatic flow. *Q. J. R. Meteorol. Soc.* **133**: 101–106.

Stull RB. 1988. *An Introduction to Boundary Layer Meteorology*. Kluwer: Dordrecht, The Netherlands.

Van der Avoird E, Duynkerke PG. 1999. Turbulence in a katabatic flow. Does it resemble turbulence in stable boundary layers over flat surfaces? *Boundary-Layer Meteorol.* **92**: 39–66.

Zilitinkevich S, Calanca P. 2000. An extended theory for the stably stratified atmospheric boundary layer. *Q. J. R. Meteorol. Soc.* **126**: 1913–1923.

Zilitinkevich SS, Esau I. 2007. Similarity theory and calculation of turbulent fluxes at the surface for the stably stratified atmospheric boundary layer. *Boundary-Layer Meteorol.* **125**: 193–205.

Zilitinkevich SS, Baklanov A, Rost J, Smedman A-S, Lykosov V, Calanca P. 2002. Diagnostic and prognostic equations for the depth of the stably stratified Ekman boundary layer. *Q. J. R. Meteorol. Soc.* **128**: 25–46.

Zilitinkevich SS, Savijärvi H, Baklanov A, Grisogono B, Myrberg K. 2006. Forthcoming meetings on planetary boundary-layer theory, modelling and applications. *Boundary-Layer Meteorol.* **119**: 591–593.

DOI: 10.1002/qj