Invited/accepted to: "Advances in Environmental Fluid Mechanics"

Editors: Dragutin T. Mihailovic & Carlo Gualtieri

Publisher: World Scientific, London, New York, Singapore (2009)

Some Recent Advances in Modeling Stable Atmospheric Boundary Layers

B. Grisogono

Dept of Geophysics, Faculty of Science & Math., Univ. of Zagreb, 10000 Zagreb, Croatia (bgrisog@gfz.hr)

Keywords: Convective ABL; Mixing length-scale; Monin-Obukhov length; Numerical Modeling; Parameterization; Prandtl number; Richardson numbers; Stratified turbulence; "z-less" turbulence; very stable boundary layer.

Abstract

The atmospheric boundary layer (ABL) is the lowest part of the atmosphere that is continuously under the influence of the underlying surfaces through mechanical (roughness and shear) and thermal effects (cooling and warming), and the overlying, more free layers. Such boundary layers and the related geophysical turbulence exist also in oceans, seas, lakes and rivers. Here we focus on those in the atmosphere; however, similar reasoning as presented here also applies to the other geophysical flows mentioned. Since most of human activities and overall life take place in the ABL, it is easy to grasp the need for an ever better understanding of the ABL: its nature, state and future evolution. In order to provide a reasonable and reliable short- or medium-range weather forecast, a decent climate scenario, or an applied micrometeorological study (for e.g. agriculture, road construction, forestry, traffic), etc., the state of the ABL and its turbulence should be properly characterized and marched forward in time in concert with the other prognostic fields. This is one of many tasks of numerical weather prediction and climate models. Many of these models have problems in handling rapid surface cooling under weak or without synoptic forcing (e.g. calm nighttime mountainous or even hilly conditions).

Overall research during the last ~ 10 years or so, strongly suggests that the evolution of the stable ABL is still poorly understood today. There we make a contribution by assessing some recent advances in the understanding of nature, theory and modeling of the stable ABL (SABL). In particular, we address inclined very (or strongly) stratified SABL in more details. We show that a relatively thin and very SABL, as recently modeled using an improved "z-less" mixing length scale, can be successfully treated nowadays; the result is quietly extended to other types of the SABL. Finally, a new generalized "z-less" mixing length-scale is proposed. At the same time, no major improvements in modeling weak-wind strongly-stable ABL is reported yet.

1. Introduction

Most of human life takes place in the atmospheric boundary layer (ABL). The ABL behaves as an active intra- and inter-layer among various underlying surfaces on one side (e.g. sea, inclining terrain, urban areas), and the rest of the capping atmosphere. There the ABL exchanges, modulates or even alters a multitude of information ranging from radiative and moisture fluxes, to momentum, heat and species fluxes, etc. After a very brief review of a few recent advancements related to the convective ABL (CABL), we focus on the (very) stable ABL (SABL). This is justified because in the very SABL (VSABL) progressively smaller eddies still play significant roles in the nature and ultimate fate of the layer, which is usually not the case for the CABL where the largest eddies determine most of turbulent flow properties. Almost needless to say, small eddies are difficult to measure at statistically meaningful levels, as well as to calculate the related turbulent fluxes due to these relatively small eddies [1, 2, 12], which may not only exist but also significantly contribute to the fluxes. Hence, it is generally less known about the SABL than about the CABL today [3, 11]. Turbulent structures associated with the CABL and SABL are under various influences due to e.g. surface fluxes, near-surface temperature inversions, low-level jets (LLJ), wind meandering, unsteadiness, internal boundary layers, etc. [16, 17, 18, 19, 31]. These features strongly affect and often determine the ABL turbulence; therefore, these structures should be included in new ABL turbulence parameterizations for numerical weather prediction (NWP), i.e. meteorological, air-chemistry and even climate models.

A contemporary research overview of the CABL structures, with the emphasis on shear affecting the CABL evolution, is in [8]. The attention there is paid to the surface layer, mixed layer and entrainment layer – all in the view of both barotropic and baroclinic effects, recalling observations, numerical modeling and analytic works. The CABL evolution is explained in terms of Monin-Obukhov length, *L*, friction and convective velocity scales, u_* and w_*, respectively, and the inversion height; various results are often inferred there from the authors successful large-eddy simulations, LES. The latter technique is generally better suited for the CABL than for the SABL [3]. Although Fedorovich and Conzemius, [8], display several results, we stress here only a few. As the contribution of wind shear to the turbulence production increases, the coherent structure in the CABL alters from quasi-hexagonal cells to horizontal convective rolls oriented parallel to the mean flow vector. The CABL may behave as a single layer only in the shear-free case. Finally, most of modeled CABL structures can be described by Richardson numbers. A few other intriguing features of the CABL, e.g. dispersion of air pollutants, nonlinear interplays between advection and diffusion, etc. are in e.g. [24]. Certain

ABL effects on NWP model initialization and data assimilation are in e.g. [25]. These aspects deserve a few independent review articles or even a new book.

Several specific questions will be discussed related to the SABL's stratified turbulence. We focus on e.g. a proper treatment of L over sloped surfaces, and the corresponding "z-less" length-scale [28, 29] which is active above the surface layer overlying the sloping surface (if the latter sub-layer even exists in a VSABL). These features should help preventing NWP and air-chemistry models' problems like runaway cooling, frictional decoupling [4, 5, 23], or a systematic thermal bias [4, 31]. One of the VSABL types is that due to weak-wind stable conditions [6, 19], another is katabatically driven SABL [5], etc. [12, 13, 16, 17, 18]; furthermore, one often talks about the SABL in plural (different types of SABLs). Among several reasons for the importance of these usually thin and very stratified boundary-layer flows, one of them being a more proper weather and climate simulations over e.g. Antarctica and Greenland, we also explain the vertical diffusion of the slope parallel wind component (the one induced by katabatic flows and Coriolis force). Through this vertical diffusion, in principle, the long-lived VSABL might interact with the polar vortex. A few recommendations for modeling purposes will be provided as well. This study continues on a few other recent works of the author and the collaborators [3, 5, 7, 12]. We will end up with a couple of plausible derivations, brief enough but hopefully inviting for a scientific contemplation and scrutiny while reflecting on how much of misinterpretation and parameterization mishaps we have had during the last few decades. Apparently, nature continues yet to be nice to us, to our clumsy modeling approaches and sophisticated visions that often end up in the middle of the nowhere.

In this Introduction a necessary background and an incomprehensive overview have been just given. The next (largest) section is the core of this contribution going through certain details of the SABL mentioned above. The main results are outlined in the concluding section.

2. Recent improvements in modeling the SABL

2.1. *Mixing length-scale*

Figure 1 illustrates the starting point of this study: an over-diffusive SABL in a typical mesoscale numerical model (solid curves); the profiles are taken from [5], based on their Fig. 1, simulated by using MIUU model [20, 21, 22, 24, 25, 26]. The overall perspective to this problem has been also elaborated elsewhere recently [3, 4, 12]. The simulation details will be given in the next subsection; here we first wish to boldly introduce the problem of the SABL over-diffusion and its remedy in Fig. 1.

The dashed-dotted curves, shown on both panels in Fig. 1 for the downslope velocity U and potential temperature θ, respectively, represent the corresponding simulation with the problem alleviated. The latter simulation (dash-dotted) is a more trustful one because it also corresponds to another model, i.e. a calibrated analytic Prandtl model result [5]. Of course, both models, MIUU and Prandtl, had been previously checked independently against various observations. These totally independent models both qualify as valuable tools for studying role types of SABL flows, their different level of complexity, and the sets of underlying approximations; that give viability and credibility to our approach.

The "z-less" mixing length scale has been most often defined as a local quantity (e.g. a MIUU model default):

$$l_{STAB} = a\frac{(TKE)^{1/2}}{N}.$$

Its modification is [5]:

$$l_{STAB} = \min[a\frac{(TKE)^{1/2}}{N}, b\frac{(TKE)^{1/2}}{\$}], \qquad (1)$$

where *TKE* is turbulent kinetic energy, *N* and *$* is buoyancy and shearing frequency (based on the absolute shear: $\$ = |S|$), respectively, $a \approx 0.5$ and $b = a/2$, valid for the gradient Richardson number $0 < Ri \leq 1$, $Ri = (N/S)^2$; otherwise, for $Ri > 1$, only the 1st term in (1) is kept. If (1) is applied for all $Ri > 0$, then the old formulation above (1), will be valid only for $Ri \geq 4$, provided again $b = a/2$ (namely, this validity goes in (1) as the square of a/b due to $Ri = (N/\$)^2$). Note that in the SABL, *TKE* usually scales as a local u_*^2. (Often-present factor of two around TKE due to related higher-order closures is not tracked down here for simplicity; this should be done properly while coding.) Plausibly define: the weakly stratified SABL exhibiting everywhere $0 < Ri << \infty$ (typically $Ri \leq 1$), and the VSABL characterised by containing (sub)regions with $Ri >> 1$. Dash-dotted curves in Fig. 1 are obtained using (1). While the over-diffusive SABL modeled, Fig. 1 (solid), is much too deep, its properly modeled behavior, i.e. the VSABL (dash-dotted), is in agreement with another model (that of calibrated Prandtl, see below), and it is also numerically and physically stable (e.g. it does not show a sign of frictional decoupling as that in e.g. [23]). It is expected that (1) ought to improve simulations for other types of SABL flows too [5, 16, 17, 18] because the overall turbulence scheme deployed, a higher-order one, so called level 2.5 [20, 21], is slope insensitive. Hence, this scheme as such does not care whether a particular flow is katabatic or not. Since wind shear is generally more changeable than buoyancy frequency in the stable atmosphere, it makes sense to try to use (1).

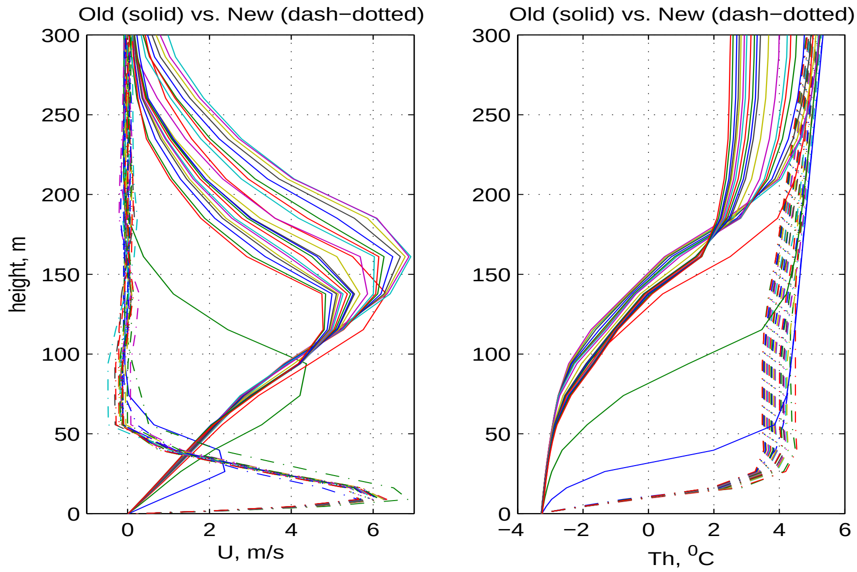

Figure 1. Two numerical simulations of same katabatic flow using two different parameterizations for the "z-less" mixing length-scale in MIUU model [5]. The profiles of the downslope wind component U (left) and potential temperature θ (right), are shown for 24 h of simulations. Over-diffusive SABL (solid) consists of an elevated LLJ and a capping inversion spreading over the lowest ~ 200 m. Using a newly proposed mixing length-scale eqn. (1), the SABL becomes much thinner (dash-dotted) and in agreement with calibrated analytic Prandtl model (see below).

The main advantage of (1) seen in Fig. 1 is the prevention (dash-dotted) of an excessive vertical diffusion of the SABL in time; such over-diffusive behavior is clearly shown in both panels, U and θ, respectively (solid). We shall return to a re-derivation of (1) and its eventual new modification later. Testings for other types of (more realistic, etc.) flows are being performed elsewhere while this paper is being written.

2.2. ***Coriolis effect in the VSABL***

Next, we display a few detailed, additional katabatic flow fields from MIUU model, some of these were shown to correspond very well to the calibrated Prandtl model [5]. Of course, all the fields modeled are coupled among themselves in the dynamically consistent way through the governing equations [20, 21, 22]. Figure 2 displays U and θ, Fig. 2a and 2b, also from Fig. 1, dash-dotted, for a constantly sloped terrain of -2.2°, under a calmly stratified background atmosphere of $\Delta\theta/\Delta z$ = 5K/km with the surface potential temperature deficit of 6.5 K (i.e. same as in [5]). An intriguing feature is in Fig. 2c displaying the slope-parallel wind component, V, i.e. the component continuously diffusing upward [10, 11, 14]. Apparently, this diffusive-like behaviour of a SABL flow component has been theoretically known for a few decades in the school of Lev N. Gutman [11], but it was poorly accessible in the peer-review English literature. It is only this flow component that exhibits the overall vertical diffusion, Fig. 2c, but the essential fields corresponding directly to the katabatic forcing, Fig. 2a and 2b, remain confined in the lowest few tens of meters. While the simulation results shown up to Fig. 2c are very recent [5], the latter figure is one of the new results of this study (shown only in conferences and the proceedings). Incidentally, the poorly modeled fields in Fig. 1 (solid), are qualitatively somewhere between Fig 2a and 2b on one side, and Fig. 2c, on the other side. We shall return to the V-component, Fig. 2c, later.

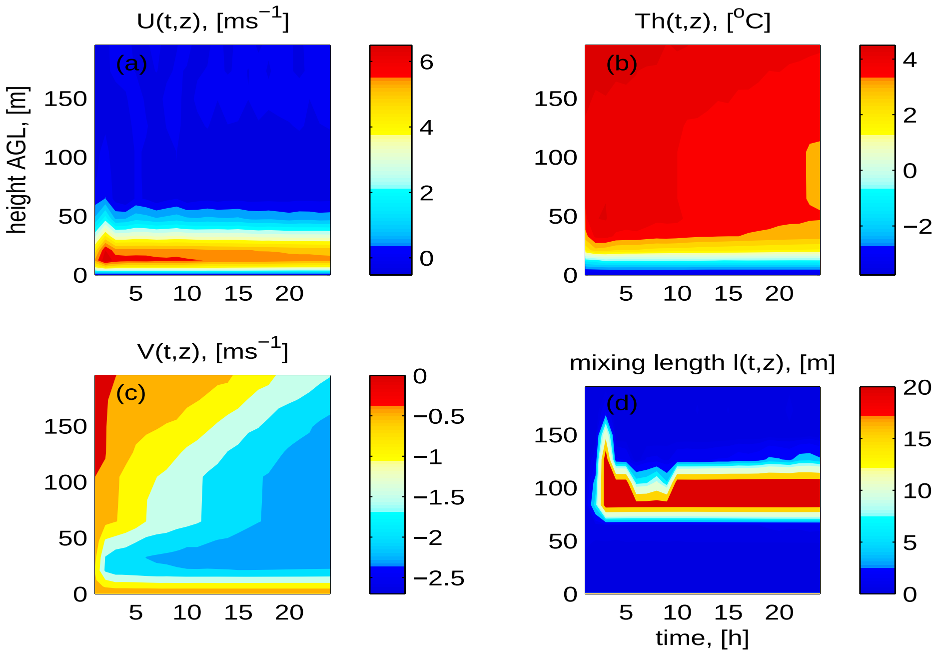

Figure 2. Displaying details from Fig. l, dash-dotted, concurring to pure katabatic wind: (a) the downslope wind-component U, (b) the potential temperature, (c) the slope parallel (Coriolis-induced) wind component V, (d) the turbulent mixing length-scale. Note that (a) and (b) correspond to dash-dotted curves on the left and right panels in Fig. 1, respectively.

Figure 2d shows the relevant mixing length-scale based on (1), allowing for the whole flow field in Fig. 2 (concurring to the calibrated Prandtl model [5]), i.e. its switch in Fig. 1 from solid to dash-dotted solutions. Typical values of this master length-scale in Fig. 2d are less than a couple of meters, often only a few decimetres, except in the upper part of the SABL, z ~ (100 ± 20) m, that is weakly stratified (see Fig. 2b) in the presence of sheared flow (Fig, 2a and 2c); there the mixing length is ≤ 20 m, Fig. 2d. It is the LLJ and its shear determining the turbulence properties, not e.g. a distance from the surface; this is also in a qualitative agreement with [17, 18]. Those modelers trying to describe the VSABL with e.g. Blackadar type of the mixing length-scale, will never be able to represent katabatic flows that often govern the VSABL properties. Moreover, the modelers deploying even a more sophisticated local length-scale, e.g. "z-less" length-scale based or related to Ozmidov scale, like above (1), will also often fail because of excluding the most relevant time-scale, i.e. the wind shear, explicitly in the length-scale. In other words, even a mixing length-scale based on Ozmidov scale should be accompanied with another scale sensing shearing processes more explicitly in the SABL, e.g. as in (1).

Some authors assess the turbulence energy parameterization, by e.g. involving the concept of total turbulent energy [12, 13]; some others find sufficient improvements in the SABL modeling already by changing only a mixing length-scale formulation [5, 7]. The author finds enough amusements and scientific challenge just by reformulating and rescaling the relevant turbulent length-scale, Fig. 1, almost as adjustable as a "turbulence rubber gum", within the accepted concepts of atmospheric turbulence closure single-point modeling. After returning to Fig. 2c, we will discuss a possibility that an NWP model's lowest level is (still) higher than the LLJ. Finally, we will re-derive and propose a slightly different but more generalized expression than (1) for the "z-less" mixing length-scale.

Simple katabatic flows, e.g. as those already shown (i.e. hydrostatic and Boussinesq, quasi-1D, without large-scale pressure gradient, all for constant: slope, surface potential temperature deficit and roughness), if persistent enough, like those over long glaciers during the polar night, may produce a permanent effect on the whole troposphere [9, 10], also see Fig. 2c. Under such persistent katabatic forcing, the cross-slope wind component V is induced due to the Coriolis effect [10, 11, 14]; V diffuses upwards without a well-defined spatio-temporal scale, Fig. 2c. Hence, this might affect, in principle, the whole troposphere, all the way up to the polar vortex (after ~ 180 days of polar night), which is not an intuitive result. To make this statement more convincing, we compare the V component from MIUU model with the analytical solution [10], Fig. 3. The latter asymptotic solution is based on the *WKB* method (the letters coming from the last names of the method's promoting scientists), which is an elegant singular perturbation technique, checked for the Prandtl model and against appropriate observations [5, 10, 27]. This asymptotic solution, shown on the left panel in Fig. 3, consists of a suitable combination of the error function and exponentially decaying cosine, both having a dimensionless similarity variable given with an integral of height, time and a gradually varying eddy diffusivity/conductivity [10, 14]. The latter feature is checked recently against a new set of measurements and LES data with promising results [15]. Of course, the models cannot agree in certain details, simply because of intrinsic differences in their respective nature, ranging from turbulence parameterizations, spatial dimensions involved, grid-point distribution to the boundary conditions, etc. [5]. Nonetheless, it appears that both models converge in their message about the spatio-temporal evolution of the V-component shown in Fig. 3; likewise, the models agree in the other fields (not shown, also see [5, 10]).

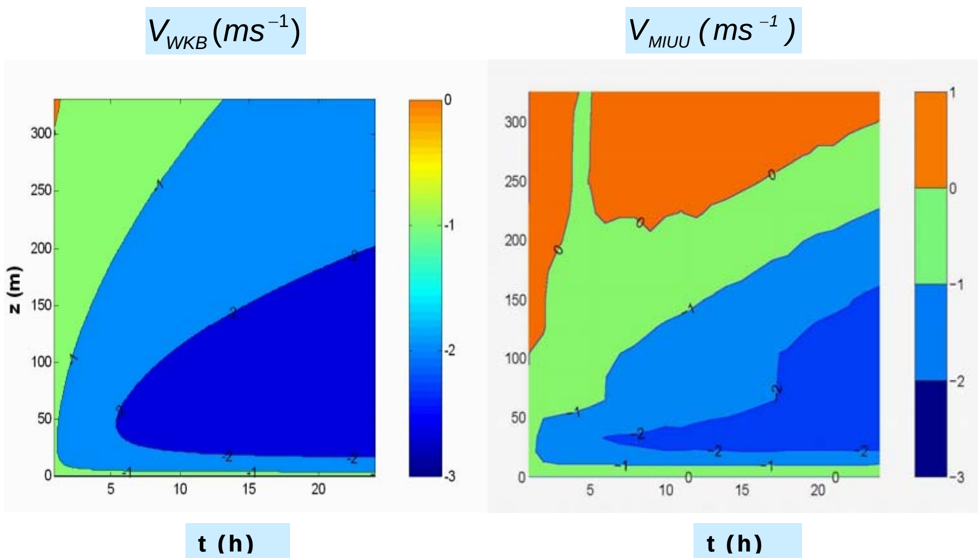

Figure 3. The slope-parallel wind component V as obtained analytically (left) using *WKB* method [10] and (right, as in Fig. 2c) by MIUU mesocale model [5, this study]. Both models show a vertical diffusion of the V component. Certain quantitative differences between the analytical and numerical model arrive from two classes of a multitude of reasons. One is in the underlying model basic assumptions, another is in their technical formulations, e.g. spatio-temporal resolution, etc. [5]. A few details about the simulations: the Coriolis parameter, slope angle, surface potential temperature deficit and background temperature gradient are (f, α, C, $\Delta\theta/\Delta z$) = (10^{-4} s^{-1}, −2.2°, −6.5 °C, $5{\cdot}10^{-3}$ K(km)$^{-1}$). Moreover, in the analytic model only (left), the additional parameters are the Prandtl number, height and the maximum eddy conductivity: (Pr, h, $Kmax$) = (1.1, 200 m, 2 m^{2}s^{-1}); these model details are in [10].

In modeling cases inevitably deploying much too poor spatial resolution, to resolve the LLJ with at least three to four vertical gridpoints, another point is that a katabatic LLJ is also likely to appear below the lowest NWP model level. At the same time, most of NWP models use some version of Monin-Obukhov length, L, to parameterize the near-surface fluxes; this length does not sense any terrain slope (which is increasingly resolved with ever finer resolution in NWP models), i.e. it assumes horizontal homogeneity for the near-surface flow variables. Hence, L, as such, cannot accommodate any direct influence of katabatic (or anabatic) flows. For such situations, a modified Monin-Obukhov length, L_{MOD}, has been recently proposed [7]. It includes a possible bulk effect of katabatic LLJ; the latter height may be estimated from the background flow variables and underlying terrain parameters. This work, as well as a few other contemporary findings [2, 6, 7, 12, 13, 16], give strong evidence that there is no critical Ri pertaining to shifts back and forth between turbulent and laminar geophysical flow regimes. Various turbulent forms exist at various Ri values. To put it simply, historically we did not relate Ri and Pr values for the SABL flows appropriately; this seems now to be quite a settled issue which also discards the existence of critical Ri [3, 12, 13, 30].

2.3. ***Simplified TKE equation and a new generalized "z-less" length-scale***

An extension of (1) follows together with future work remarks. A few plausible derivations for new "z-less" mixing length-scales stem from a few recent works [3, 5, 13], thus allowing for another new result of this study. Let us start with the prognostic equation for TKE under typical simplifying conditions such as horizontal homogeneity, Boussinesq and hydrostatic approximation and the absence of a mean vertical motion:

$$\frac{\partial(TKE)}{\partial t} = -\overline{u'w'}\frac{\partial\bar{u}}{\partial z} + \frac{g}{\theta_0}\overline{w'\theta'} - \frac{\partial}{\partial z}\left[\overline{w'(\frac{p'}{\rho_0} + TKE)}\right] - \varepsilon \,. \qquad (2)$$

The terms have their usual meaning in this well known equation: the local rate of change of TKE on the LHS is balanced by the consecutive terms on the RHS: the shear production, buoyant destruction (in the SABL, while in the CABL this is a source term), transport and redistribution due to pressure- and turbulence-correlations ("fluxes of turbulent fluxes") and viscous dissipation, respectively. Next, we assume a steady-state and neglect transport and redistribution terms. The steadiness assumed also implies here that the mixing length-scale will not remember its own history, i.e. it will be a diagnostic quantity that immediately adjusts to the conditions imposed. Transport and redistribution terms, the square brackets on the RHS of (2), are notoriously difficult both to measure and to model; sometimes these are treated as diffusive-like processes, sometimes are simply leftovers from a bulk budget of the other terms in (2). Furthermore, we parameterize the momentum and heat fluxes in (2) as $K_m\$$ and $K_h\$$, where K_m and K_h are eddy diffusivity and conductivity and $\$$ is (again) the absolute shear. The last term in (2) is parameterized as $\varepsilon = b(TKE)^{3/2}/\Lambda$, where b is an empirical constant and Λ is a new mixing length-scale (Λ replaces l_{STAB} from (1)). Under these simplifications (2) becomes:

$$0 = K_m\$^2 - K_h N^2 - \frac{b}{\Lambda}(TKE)^{3/2}, \qquad (3)$$

signifying that the buoyant destruction and viscous dissipation (last two terms) compete in partitioning TKE after the mechanical/shear production of TKE.

There are a few ways to proceed from (3) in order to estimate Λ, the goal of this subsection. A simple, 1st order closure would assume, based on the absolute shear $\$$: $K_m = a_1\Lambda^2\$$ and likewise $K_h = a_1\Lambda^2\$/Pr$, where a_1 is a model constant and Pr is again turbulent Prandtl number; typically $Pr \geq 1$ in the SABL [7, 10, 12, 13]. A more advanced and, arguably, better parameterization would be a higher-order closure, with a simplest form as $K_m = a_2\Lambda(TKE)^{1/2}$ and likewise $K_h = a_2\Lambda(TKE)^{1/2}/Pr$. When either of these parameterizations is plugged in (3), the following expression for Λ ensues (the first index will be for the 1st order closure, the second index will be for the higher-order closure in $\Lambda_{1,2}$):

$$\Lambda_{1,2} = c_{1,2}\frac{(TKE)^{1/2}}{\$(1 - {}^{Ri}\!/_{\mathrm{Pr}})^{1/(3,2)}}, \qquad (4)$$

where $c_{1,2}$ are appropriate coefficients obtained from b, a_1 or a_2, respectively; moreover, the root exponent in the denominator in (4) is either $1/3$ or $1/2$ for the 1st or the higher-order closure, respectively. While in higher-order closures TKE is most often forecasted, in 1st order schemes it may be only diagnosed. After including an important recent finding about the SABL that

$$Pr \approx 0.8 + 5\,Ri\,, \qquad (5)$$

from [13] into (4), it appears that the denominator in (4) may be justifiably expanded into binomial series since for the SABL (5) gives $max(Ri/Pr) \leq 0.2$. Hence, a newly proposed "z-less" mixing length-scale is (based on a two-term binomial expansion):

$$\Lambda_{1,2} \approx c_{1,2} \frac{(TKE)^{1/2}}{\$}(1 + \frac{Ri}{(3,2)\,\mathrm{Pr}}), \tag{6}$$

which appears as a modification of (1). For generality (5) was not plugged in (6), the latter only needs the asymptotic range of values for the ratio *Ri/Pr*. In fact, there is a whole class of the above parameterizations, between 1st and 2nd order closures, that yield to the same basic formulation: $\Lambda \sim (TKE)^{1/2}/\$$.

If K_m and K_h were parameterized in (3) as $K_m = a_3(TKE/N)$ and $K_h = a_3\, TKE/(PrN)$, respectively (strictly $Ri > 0$), which also makes sense for the VSABL, one would end up, instead of (4), with

$$\Lambda_3 = c_3 \frac{(TKE)^{1/2}}{\$} \frac{Ri^{1/2}}{(1 - Ri/\mathrm{Pr})}, \tag{7}$$

again, due to (5), this allows its binomial series for the denominator's second factor, similar to that in (6). Also note from (4) (or (6)) and (7) that Λ for the 1st order parameterization is somewhat less sensitive to the ratio of *Ri/Pr* than the higher-order closures. Furthermore, we conclude that most of sensible parameterizations, that are between 1st and 2nd order turbulence closures, for the SABL (above the immediate surface sub-layer) are best handled with a "z-less" mixing length scale of type

$$\Lambda = const \frac{(TKE)^{1/2}}{\$} f(Ri, \mathrm{Pr}), \tag{8}$$

where $0 < const < 1$ and $f(Ri, Pr)$ is a relatively simple function, or even a simpler series expansion, already shown for two overall cases to be $\approx 1 + Ri/(3Pr)$, or $1 + Ri/(2Pr)$; while in the third case discussed it is $\approx (Ri)^{1/2}(1+Ri/Pr)$. For both 1st order- and higher-order closure schemes in general, the respective single coefficient entering to the RHS of either (6), (7) or (8) is a priori known number from the respective definitions of eddy diffusivities (see between (3) and (4), or above (7)) in each particular NWP and climate models used. Mesoscale models with advanced higher-order turbulence closures, as e.g. MIUU model [20, 21, 22, 24, 25], usually possess a multiple combination/choices for obtaining eddy diffusivity and conductivity under stable conditions; note that a suitable set of options and entering coefficients is already accommodated implicitly with the proposed Λ. Namely, any combination of these parameterizations discussed end up with (8), i.e. $\Lambda \sim (TKE)^{1/2}/\$$. This generality of Λ is provided by the systematic reduction of the eqn. for *TKE*, (2) toward (3), which still secures a three-term balance used for the estimation of Λ. Aside from the constant in (8), it is indicated that the function multiplying the "z-less" length-scale $(TKE)^{1/2}/\$$, i.e. $f(Ri, Pr)$, is progressively more sensitive to Ri and Pr inter-relation for higher-order closure parameterizations than for the 1st order closure. This finding suggests that higher-order closures should be better in handling the VSABL structures and its turbulence than the 1st order closures because the former ones are more responsive to multiple-scale processes and their variations of Ri and Pr. Preliminary tests with MIUU model show that Λ_2 from (6) behaves in accordance with the expectation, i.e. there is no noticeable difference between the katabatic flow simulations already displayed using (1), and the one obtained with (6), in particular with $\Lambda = 0.2685\ (2\cdot TKE)^{1/2}/\$\cdot(1 + 0.5\cdot Ri/Pr)$, where all relevant coefficients are revealed now. Moreover, test runs are stable even after 30h of simulations. More testing is necessary before the newly proposed length-scale can be reliably used in all types of the SABL, but a further generalization of Λ is already being derived based on a renormalization procedure like that for (6) through (8).

An indirect advantage of the formulation (6) through (8) is that the explicit inclusion of the wind shear, followed by Ri and Pr local numbers, will be more sensible to minute flow variations, than former mixing length expressions for the SABL (e.g. the one above (1), not to mention the prescribed Blackadar scale, etc.). This enhanced Λ sensitivity to shear effects could be instrumental in sensing other, even non-local effects on turbulence, such as buoyancy waves, thus indirectly Λ being susceptible even to the transport/redistribution terms. Further testing is left for new studies and is beyond this analysis. Although (4), (6), (7) and/or (8) may have problems in handling turbulent mixing with the wind shear diminishing faster than $(TKE)^{1/2}$, which is possible in certain strongly-stratified weak-wind conditions, it remains to be seen if this length-scale proposed will bring some practical improvements in modeling VSABL flows. The latter type of VSABL is apparently determined most of its lifetime by unknown dynamics and physics [2, 3, 6, 16, 19]. Without suitable measurements there, we may not even know whether the relatively weak turbulence in the weak-wind VSABL is transported and/or redistributed from elsewhere and then only (partially) destroyed in this VSABL.

3. Concluding remarks

Several aspects of the ABL are reviewed and discussed; the emphasis has been on modeling, in particular, on parameterizing turbulence in the SABL. While the "classical" SABL, that is always weakly stratified (i.e. $Ri \ll \infty$, typically, $0 < Ri \leq 1$), is modeled reasonably well during the last few decades or so, strongly stable cases, i.e. the VSABL (where typically $Ri \gg 1$) is generally not understood well [1, 2, 3, 4, 5, 6, 7]. Various approaches have been envisioned [8, 16, 18, 19]. Here, a pragmatic approach is undertaken. In particular, excessively diffusive and too deep SABL flows, often appearing in numerical models, are discussed in the light of the recently proposed alleviation of this problem [5]. The latter demanded an explicit inclusion of the vertical shear of horizontal wind. A generalization of this proposal is given here in the view of a simplified *TKE* eqn. and a set of subsequent parameterizations for the mixing length-scale.

Long lived SABL over relatively long inclined surfaces often consists of persistent katabatic flows triggering the corresponding cross-slope wind component V due to the Coriolis effect [9, 10, 11, 14]; V diffuses upwards without a steady state. This type of the SABL flow has been modeled analytically, through the calibrated Prandtl model (with vertically varying prescribed eddy diffusivity and conductivity, solved either numerically, or via the *WKB* method), and simulated via MIUU mesoscale model. The results from these two very different models agreed only when the latter model used a more appropriate "z-less" mixing length scale (1).

The new generalized "z-less" mixing length-scale Λ is proposed (4), (7) or (8). It is derived from a couple of most recent studies [3, 5, 13] that indicated a few important shortcomings of the current turbulence parameterizations for the SABL and its turbulence as modeled in NWP, air-chemistry and climate models. It basically states that $\Lambda \sim (TKE)^{1/2}/\$$, almost regardless of the other parameterization details. This new length-scale remains yet to be checked in simulations against observations; a few preliminary tests for the katabatic flows already discussed, but now using the newly proposed Λ_2 from (6), show a promising behavior and agreement with (1).

List of Symbols

a, a_i, b	dimensionless coefficients in turbulence parameterizations	
ABL	atmospheric boundary layer	
α	terrain slope	rad
C	surface potential temperature deficit	^{0}C
CABL	convective ABL	
c_i	coefficients obtained from a_i and b	
ε	dissipation of TKE	m^2s^{-3}
f	Coriolis parameter	s^{-1}
g	acceleration due to gravity	ms^{-2}
h	height of the maximum value of a prescribed eddy diffusivity or conductivity	m
K_h	eddy conductivity	m^2s^{-1}
K_m	eddy diffusivity	m^2s^{-1}

K_{max}	maximum value of a prescribed gradually varying eddy conductivity	m^2s^{-1}
LLJ	low-level jet	
l_{STAB}	"z-less" turbulent mixing length-scale	m
Λ	"new generalized "z-less" mixing length-scale	m
MIUU	Meteorologiska Institutionen Uppsala Universitetet	
N	buoyancy frequency	s^{-1}
NWP	numerical weather prediction	
p'	turbulent fluctuation of pressure	Pa
Pr	Prandtl number	
Ri	gradient Richardson number	
ρ_0	mean density	kgm^{-3}
S	vertical shear of the horizontal wind	s^{-1}
$\$$	$=\mid S \mid$, absolute shear	s^{-1}
SABL	stable atmospheric boundary layer	
t	time	s (or h)
TKE	turbulent kinetic energy	m^2s^{-2}
Θ	potential temperature	K
θ'	turbulent fluctuation of potential temperature	K
U,V	(or with a bar on its top) horizontal mean wind components	ms^{-1}
$u,'v,'w'$	turbulent fluctuations of the wind field	ms^{-1}
VSABL	very (strongly) SABL	
WKB	name of a singular perturbation method	
z	vertical coordinate	m
$\overline{(...)}$	suitable averaging	

Acknowledgement

The author is thankful to Iva Kavčič for reproducing the left panel of Fig. 3; he is also indebted to Dragutin Mihailović and Boro Rajković for recommending the overall frame to this work and more. This study was inspired by collaboration with Danijel Belušić, Sergej Zilitinkevich, Leif Enger, Thorsten Mauritsen, Željko Večenaj and Larry Mahrt; it was supported by EMEP4HR project No. 175183/S30 provided by the Research Council of Norway and by the Croatian Ministry of Science, Education & Sports, projects BORA No. 119-1193086-1311.

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