Governing equations

Momentum eqns

$$\left\{\begin{aligned}
&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega\sin\phi\, v - 2\Omega\cos\phi\, w + F_{friction_x}\\
&\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega\sin\phi\, u + F_{fr\,y}\\
&\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega\cos\phi\, u + F_{fr\,z}
\end{aligned}\right.$$

$$\underbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}}_{\nabla_H \cdot \vec{V}} + \frac{\partial w}{\partial z} = 0$$

Continuity eqn for incompr. fluid

Thermo-dynamic eqns

$$\left\{\begin{aligned}
&\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \text{Heating / Rad Cool.}\\
&\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} + w\frac{\partial q}{\partial z} = \text{Precip / Evap.}
\end{aligned}\right.$$

Momentum eqns in x & y (horizontal) $f = 2\Omega \sin\phi$ Coriolis parameter

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv - 2\Omega\cos\phi\, w + F_{fr}$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_{fr}$$

We want to do scale analysis to find terms that overpower other terms $\rightarrow$ 0^{th} order balance

Synoptic scale disturbances characteristics

$$f \sim 10^{-4}\, s^{-1}$$

$$U = u, v \sim 10\ m/s$$

$$W = w \sim 1\ cm/s$$

10 hPa difference between adjacent synoptic scale features

$$(x,y) \Rightarrow L \sim 10^{6}\ m$$ (horiz scale)

$$H \sim 10^{4}\ m$$ (vertic scale)

$$T \sim \frac{L}{U} = \frac{10^6}{10} = 10^5\ s$$ time scale

$$\frac{\delta p}{\rho} \sim 10^{3}\ m^{2} s^{-2}$$ characteristic horiz. pressure fluctuation

WINNERS :)

Assume $\phi = 45°$
$\sin\phi = \cos\phi$

$$\underbrace{\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv}_{\text{WINNERS}} - \underbrace{2\Omega\cos\phi}_{f} w + F_{fr}$$

$\frac{u}{T}$

Scales as	$\frac{u^2}{L}$	$\frac{\delta p}{\rho}\frac{1}{L}$	$f \cdot u$	$f w$	F_{fr}
Magnitude $[m/s^2]$	10^{-4}	10^{-3}	10^{-3}	10^{-6}	10^{-12}

$\frac{du}{dt}$	$\frac{u^2}{L}$ $\frac{10^2}{10^6}$; $\frac{u}{T}$ $\frac{10}{10^5}$	δp $\frac{10^3\, m^2 s^{-2}}{10^6\, m}$; $-\frac{1}{\rho}\frac{\partial p}{\partial x}$	$10^{-4} s^{-1}$ $10\, m/s$; fv	$10^{-4} s^{-1} \cdot$ $10^{-2}\, m/s$; $2\Omega\cos\phi \cdot w$	10^{-12} ; F_{fr}

WINNERS

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_{fr}$$

Scale $\frac{u^2}{L}$ $\frac{\delta p}{\rho}\frac{1}{L}$ fu F_{fr}

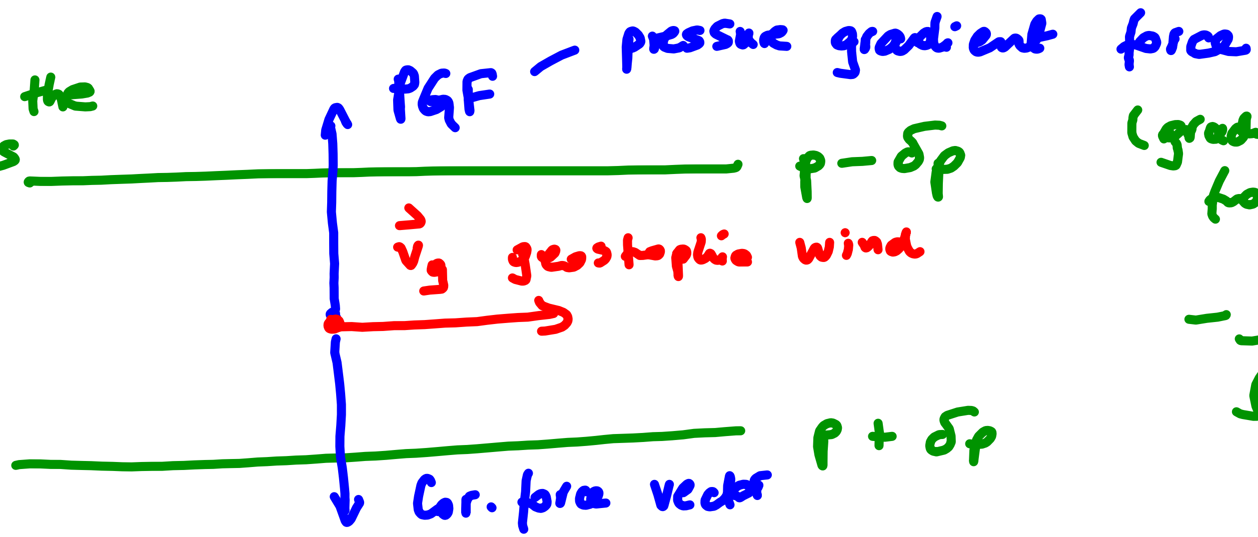
Magnitude m/s^2 10^{-4} 10^{-3} 10^{-3} 10^{-12}

Only 2 terms are 10^{-3}: pressure gradient force & Coriolis force terms. The closest after that is du/dt or dw/dt.

This result implies that as the 0^{th} 1^{st} approximation to the full equations of motion, there is a balance!

The fundamental balance for mid latitude synoptic scale is GEOSTROPHIC BALANCE.

closer the isobars bigger the v_g



(gradient goes from − +)

$$-\frac{1}{\rho}\nabla p$$

$$\vec{v}_g = \frac{1}{\rho f}\hat{k}\times\nabla p \qquad -\frac{1}{\rho}\frac{\partial p}{\partial x} + f v_g = 0 \qquad v_g = \frac{1}{f\rho}\frac{\partial p}{\partial x}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} - f u_g = 0 \qquad u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y}$$

Vertical eqn of motion - scale analysis

WINNERS

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega\cos\phi\, u - g + F_{fr}$$

Scale	$\frac{UW}{L}$ $1/T$	$\frac{P}{\rho H}$	fu	g	F_{fr}
Magnitude m/s^2	10^{-7}	10	10^{-3}	10	10^{-15}
	$\frac{10^{-2}\ m/s}{10^{5}\ s}$	$\frac{10^{5}}{10^{4}}$			

$p = 1000\ hPa = 10^{5}$

$$\boxed{-\frac{1}{\rho}\frac{\partial p}{\partial z} - g = 0}$$

Hydrostatic balance

To the first order the mid latitude atmosphere on Earth is in geostrophic balance in the horizontal & hydrostatic balance in the vertical.

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\nabla_H \cdot \vec{V} = -\frac{\partial w}{\partial z}$$

$$\underbrace{\nabla_H \cdot \vec{V}} = - \frac{\partial w}{\partial z}$$

>0 div.

<0 conv.