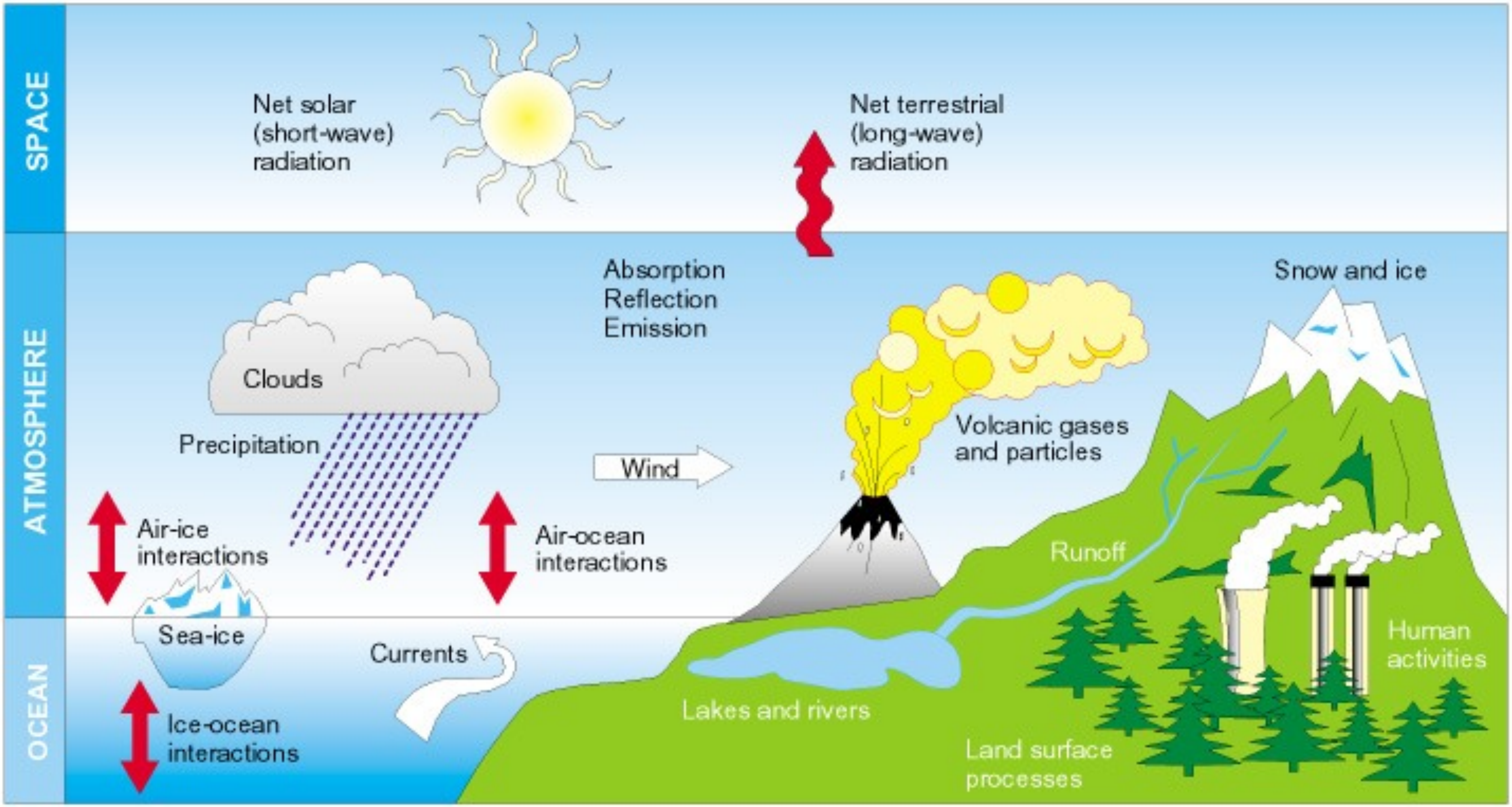
Diagnostic Modelling

Modelling techniques for diagnosing large scale dynamical responses in the atmosphere

Nick Hall
LEGOS / Univ. Toulouse, France

SWAP workshop, Croatia, May 2010

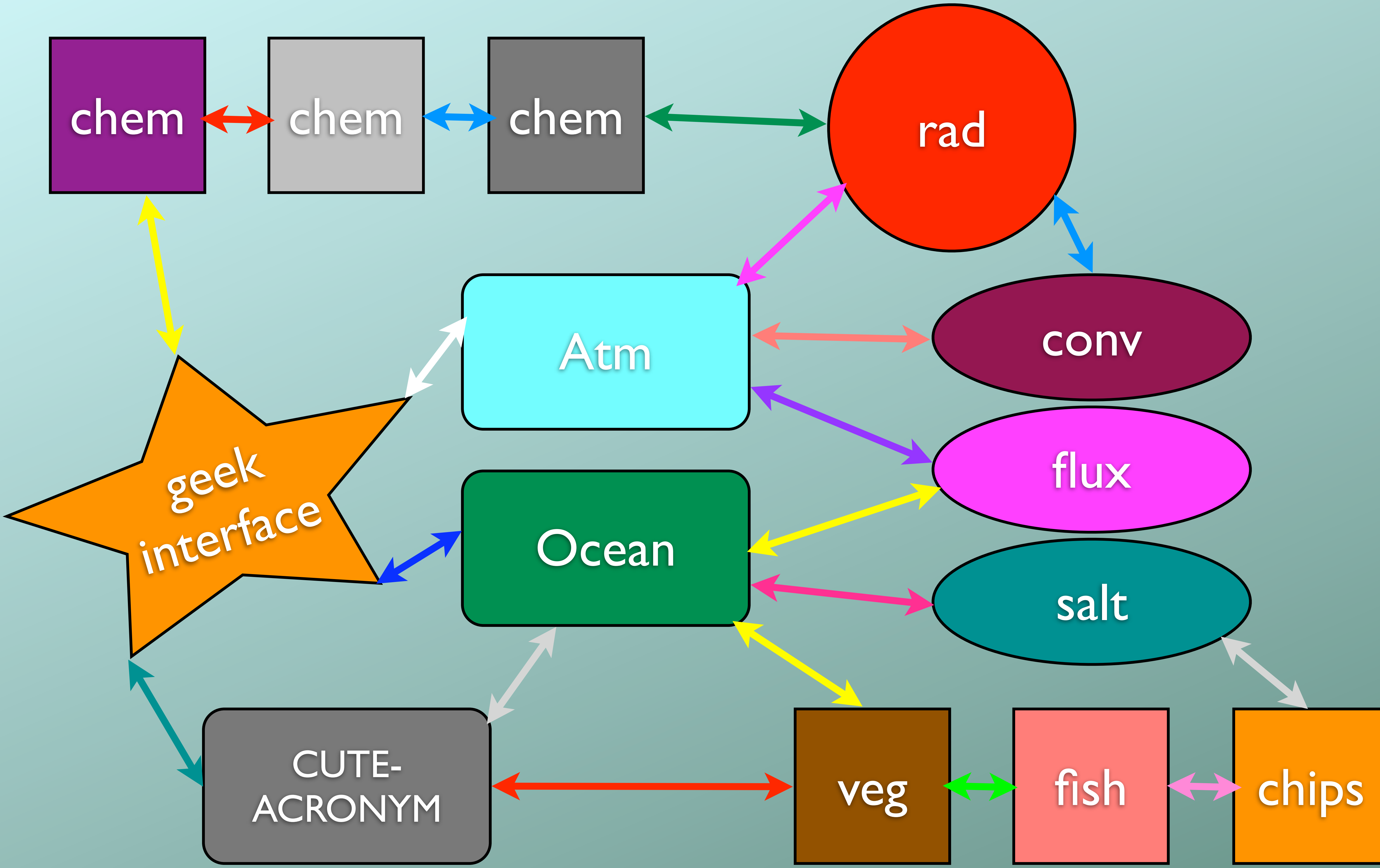
The Climate System



courtesy N. Noreiks, L. Bengtsson, MPI

AV/Global/0101

a model ...


chem
chem
chem
rad
Atm
conv
flux
geek interface
Ocean
salt
CUTE-ACRONYM
veg
fish
chips

model development

applications of simple models

- *generalized budget for systems with transients*
- *stability analysis*
- *response to forcing anomalies*
- *simulating transients*
- *driving anomalies with data*

generalized development

Generic equation for the development of a nonlinear system

$$\frac{\partial q}{\partial t} = Lq - \nabla .vq + f$$

introduce a state vector $\Phi = (v, q, ...)$

this becomes

$$\frac{d\Phi}{dt} = L\Phi + \Phi^{\dagger} Q\Phi + f$$

where $\Phi^{\dagger}$ is the diagonal matrix that contains the elements of Φ

basic state separation: 1

specify a basic state that is a solution of the equations

$$\Phi = \Phi_0 + \Phi_1$$

$$f = f_0 + f_1$$

$$\frac{d\Phi_0}{dt} = 0$$

$$\frac{d\Phi}{dt} = L\Phi + \Phi^{\dagger} Q\Phi + f$$

and

$$L\Phi_0 + \Phi_0^{\dagger} Q\Phi_0 + f_0 = 0$$

so

$$\frac{d\Phi_1}{dt} = L\Phi_1 + \Phi_1^{\dagger} Q\Phi_0 + \Phi_0^{\dagger} Q\Phi_1 + \Phi_1^{\dagger} Q\Phi_1 + f_1$$

or to put it another way

$$\frac{d\Phi_1}{dt} = L_0\Phi_1 + O(\Phi_1^2) + f_1$$

ADVANTAGE: eddy development independent of separation

DISADVANTAGE: basic state unrealistic so nonlinear term large - linearization of questionable relevance

basic state separation: 2

specify a realistic basic state (for example, the time mean flow)

$$\Phi = \overline{\Phi} + \Phi' \qquad f = \overline{f} + f' \qquad \overline{\frac{d\Phi}{dt}} = 0 \qquad \frac{d\Phi}{dt} = L\Phi + \Phi^{\dagger} Q\Phi + f$$

but this time

$$L\overline{\Phi} + \overline{\Phi}^{\dagger} Q\overline{\Phi} + \overline{f} \neq 0$$

mean advection

in fact we have

transient eddy forcing

$$L\overline{\Phi} + \overline{\Phi}^{\dagger} Q\overline{\Phi} + \overline{\Phi'^{\dagger} Q\Phi'} + \overline{f} = 0$$

ADVANTAGE: realistic basic state so meaningful linearization possible

DISADVANTAGE: linear development equation not independent of time mean transient "forcing"

and

$$\frac{d\Phi'}{dt} = L\Phi' + \Phi'^{\dagger} Q\overline{\Phi} + \overline{\Phi}^{\dagger} Q\Phi' + \left[\Phi'^{\dagger} Q\Phi' - \overline{\Phi'^{\dagger} Q\Phi'}\right] + f'$$

or to put it another way

$$\frac{d\Phi'}{dt} = L_{mean}\Phi' + \left[O(\Phi'^2) - \overline{O(\Phi'^2)}\right] + f'$$

a perturbation model

So much for theory.
How do we solve these equations ? We use a dynamical model.
We can appeal to data to deduce the appropriate forcing functions:

data

$$\frac{d\Phi}{dt} = L\Phi + \Phi^{\dagger} Q\Phi + f(t)$$

model

$$\frac{d\Psi}{dt} = L\Psi + \Psi^{\dagger} Q\Psi + g$$

We now define *g* using data, so that if we initialize the model with $\overline{\Phi}$ it will not develop.

so

$$g = -L\overline{\Phi} - \overline{\Phi}^{\dagger} Q\overline{\Phi}$$

We can easly find *g* by integrating the unforced model from $\overline{\Phi}$ for just one timestep.

From the time-mean budget equation we also see that this definition of *g* gives

$$g = \overline{f} + \overline{\Phi'^{\dagger} Q\Phi'}$$

So this forcing represents the time mean diabatic forcing plus the mean “transient eddy forcing”.
These are the two processes that maintain the time-mean circulation.

stability analysis

With our data-derived forcing, for small perturbations, integrating

$$\frac{d\Psi}{dt} = L\Psi + \Psi^{\dagger} Q \Psi + g$$

is equivalent to integrating $$\frac{d\Psi'}{dt} = L_{mean}\Psi'$$

We use the dynamical model to analyse the linear growth problem for normal modes of L_{mean}

$$L_{mean} e_n(x,y,z) = \lambda_n e_n(x,y,z)$$

for a single mode

$$\lambda_n = \sigma + i\omega \qquad \Psi_n = e_n(x,y,z)e^{(\sigma+i\omega)t}$$

in general e_n is complex so the solution takes the form

$$\Psi' = \left[A(x,y,z)\sin\omega t + B(x,y,z)\cos\omega t\right] e^{\sigma t}$$

is the time-mean circulation unstable ?

$$\sigma < 0$$

Farrell (1982)
Whitaker and Sardeshmukh (1998)

$$\sigma \approx 0$$

Stone (1978)
Hall and Sardeshmukh (1998)

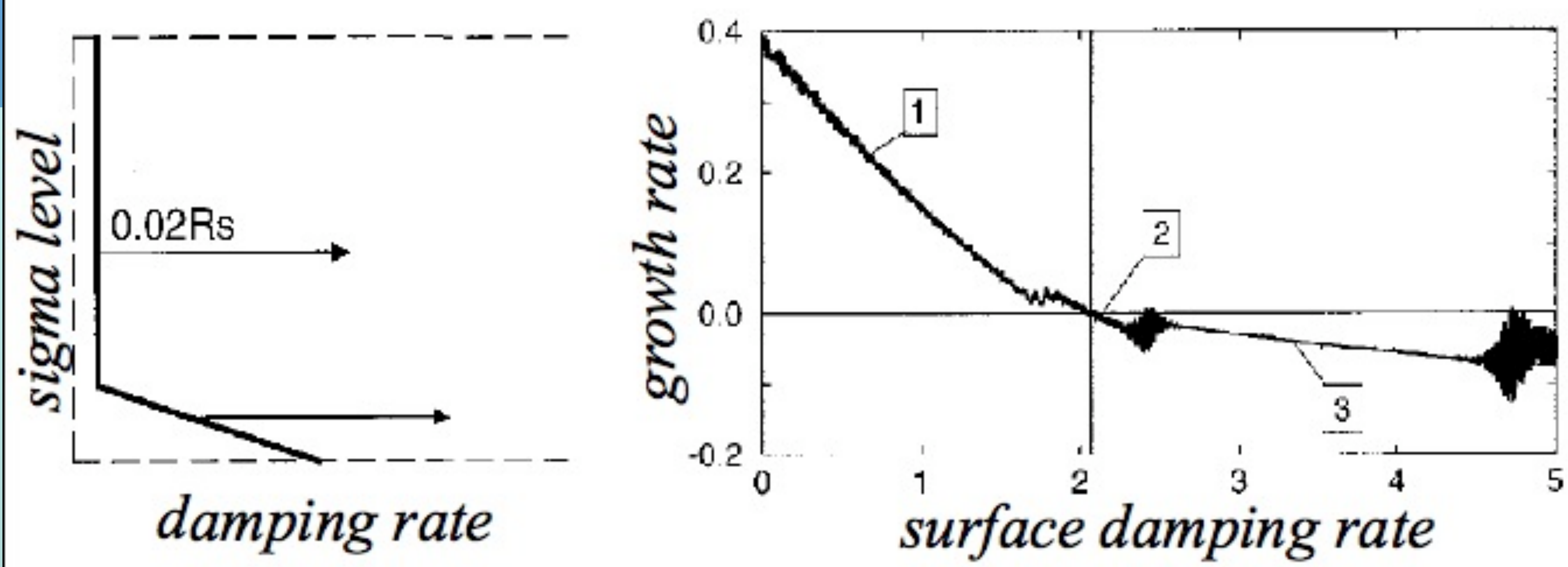
$$\sigma > 0$$

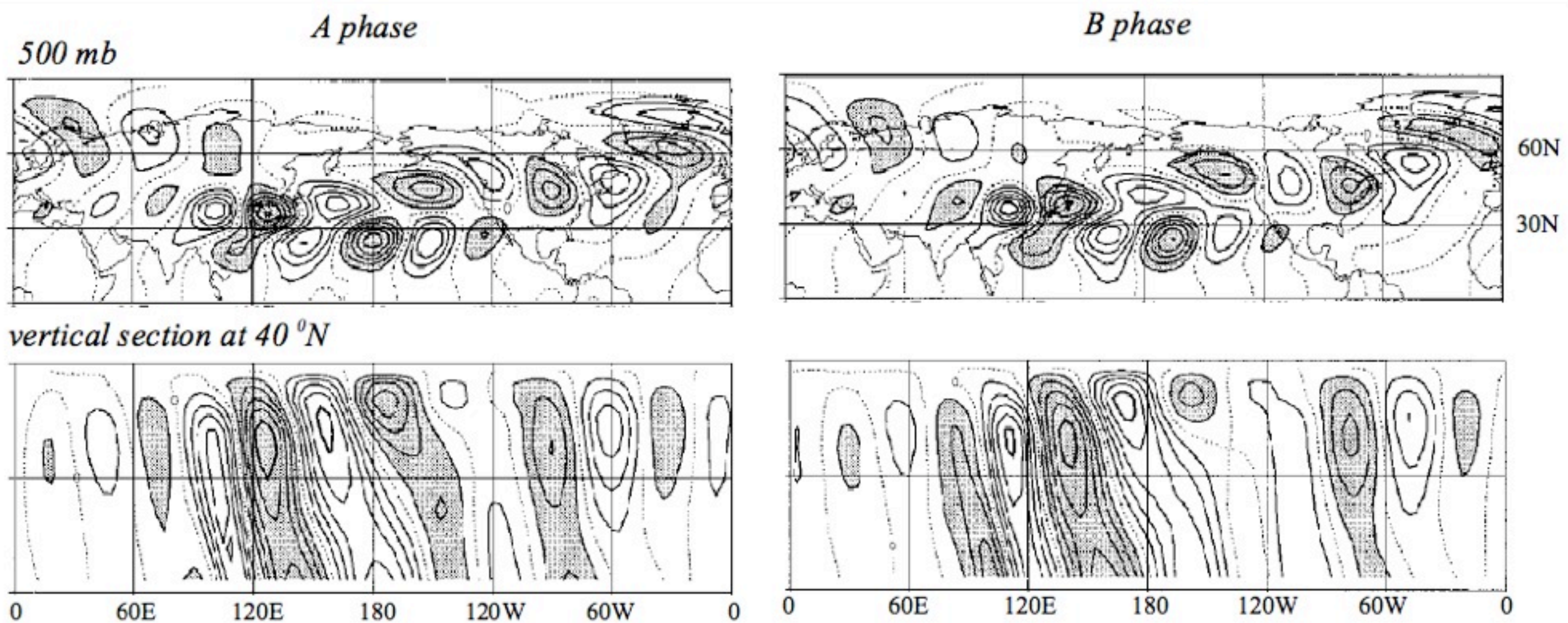
Charney (1947)
Eady (1949)
Simmons and Hoskins (1978)

The midlatitude storm tracks

effect of low-level damping on modal growth

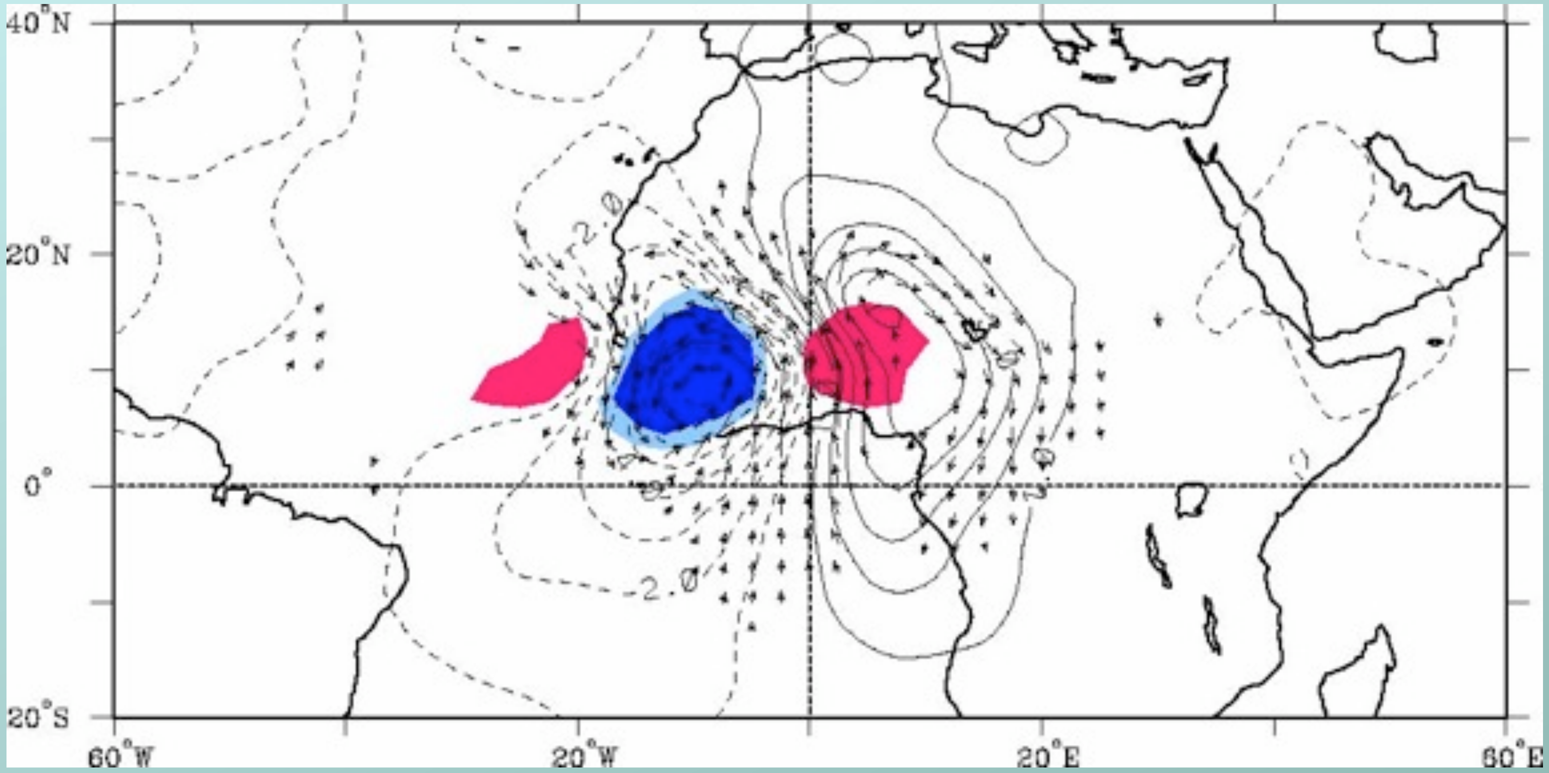
Hall and Sardeshmukh (1998)



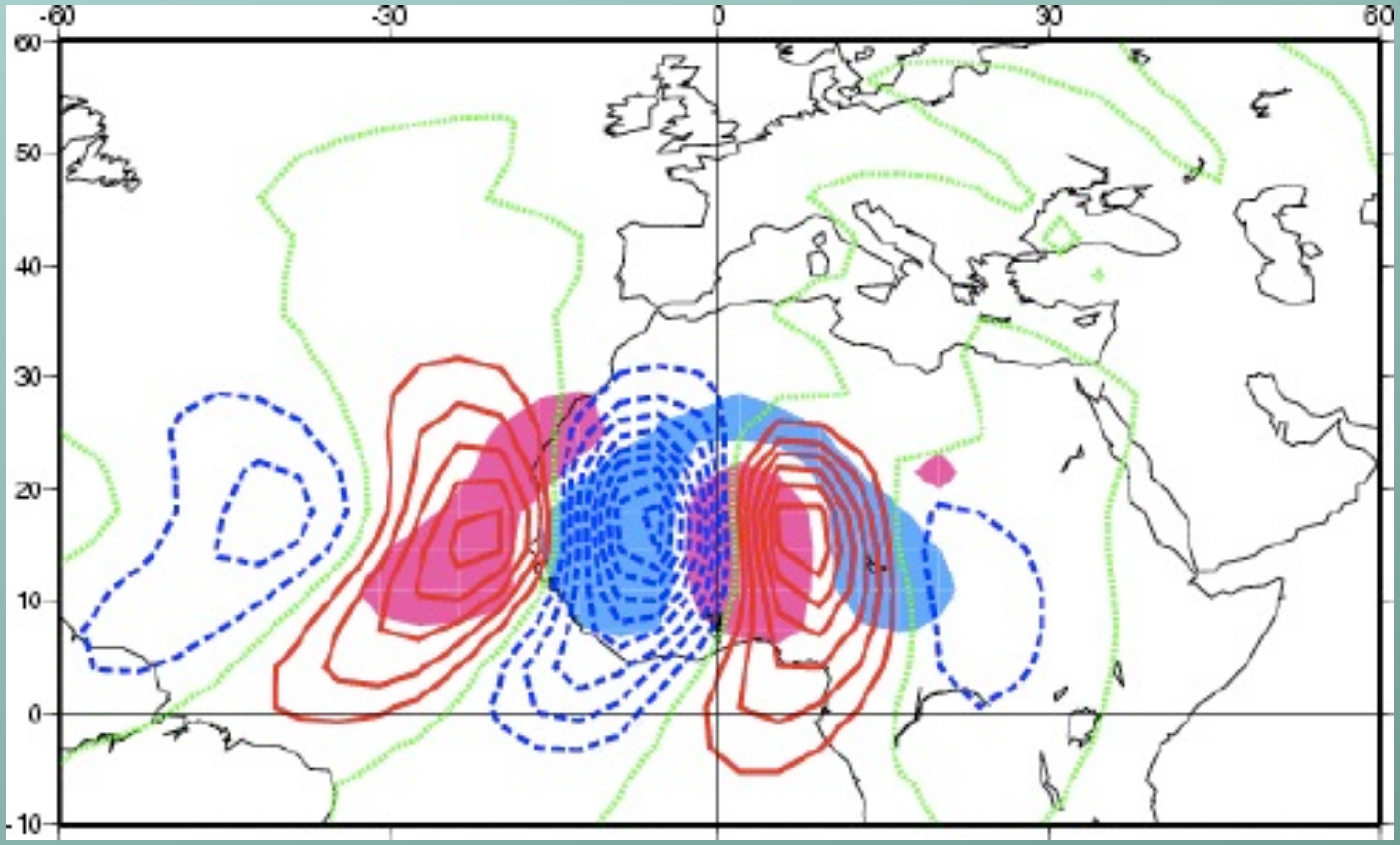


African easterly waves

Observations



Model



response to forcing

We can use our dynamical model to find the response to a perturbation forcing *f’*

$$\frac{d\Psi'}{dt} = L_{mean}\Psi' + f'$$

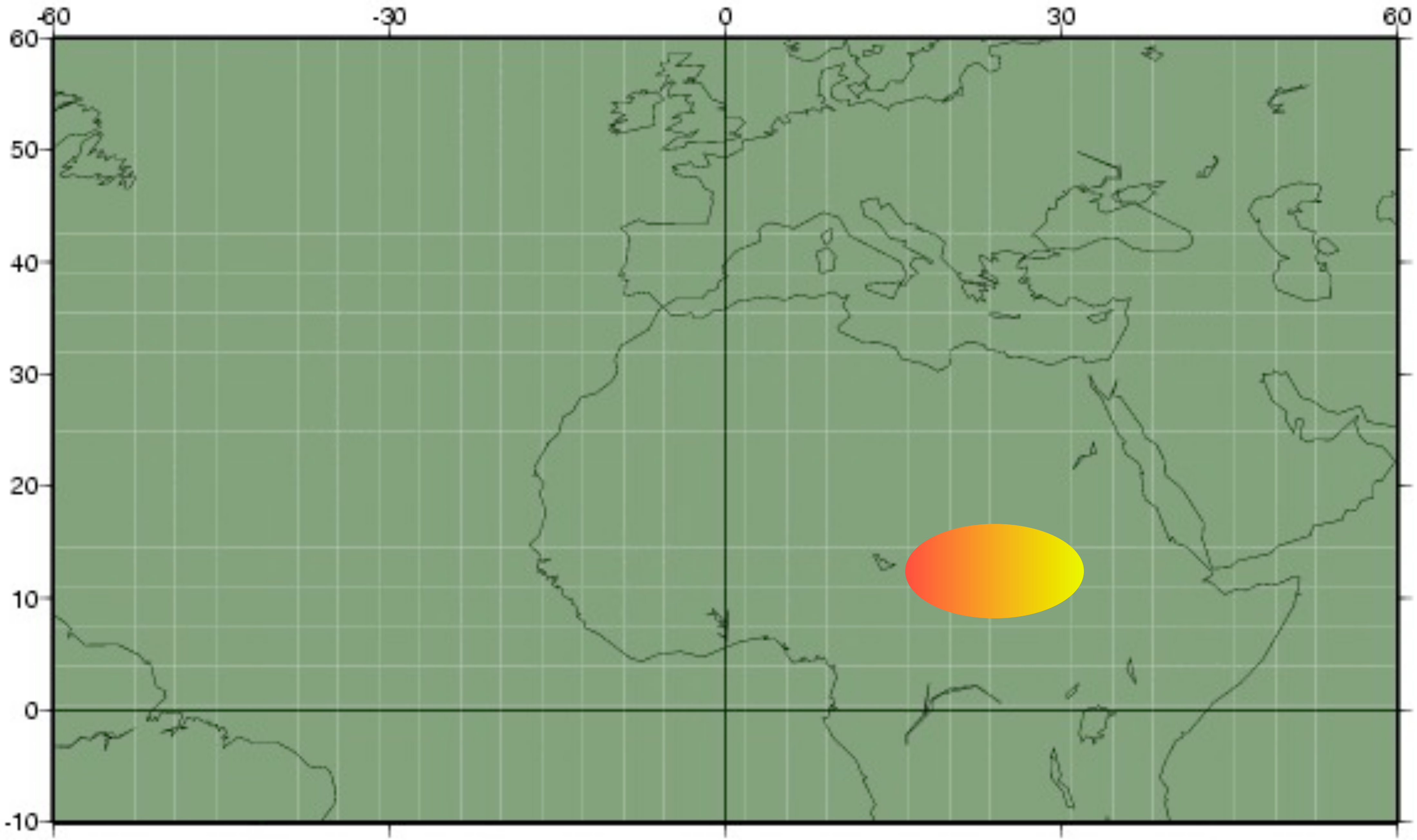
(and if we keep *f’* small the response is linear)

Start with another example from African easterly waves. This time we use a (convective) heating anomaly as *f’*, to trigger a response. The response still looks like the normal mode that we found before. But it decays in time.

Initial value problems

- If normal mode solutions are neutral they can tell us about efficient structures.
- But we still lack a complete theory for the generation and intermittence of AEW events.

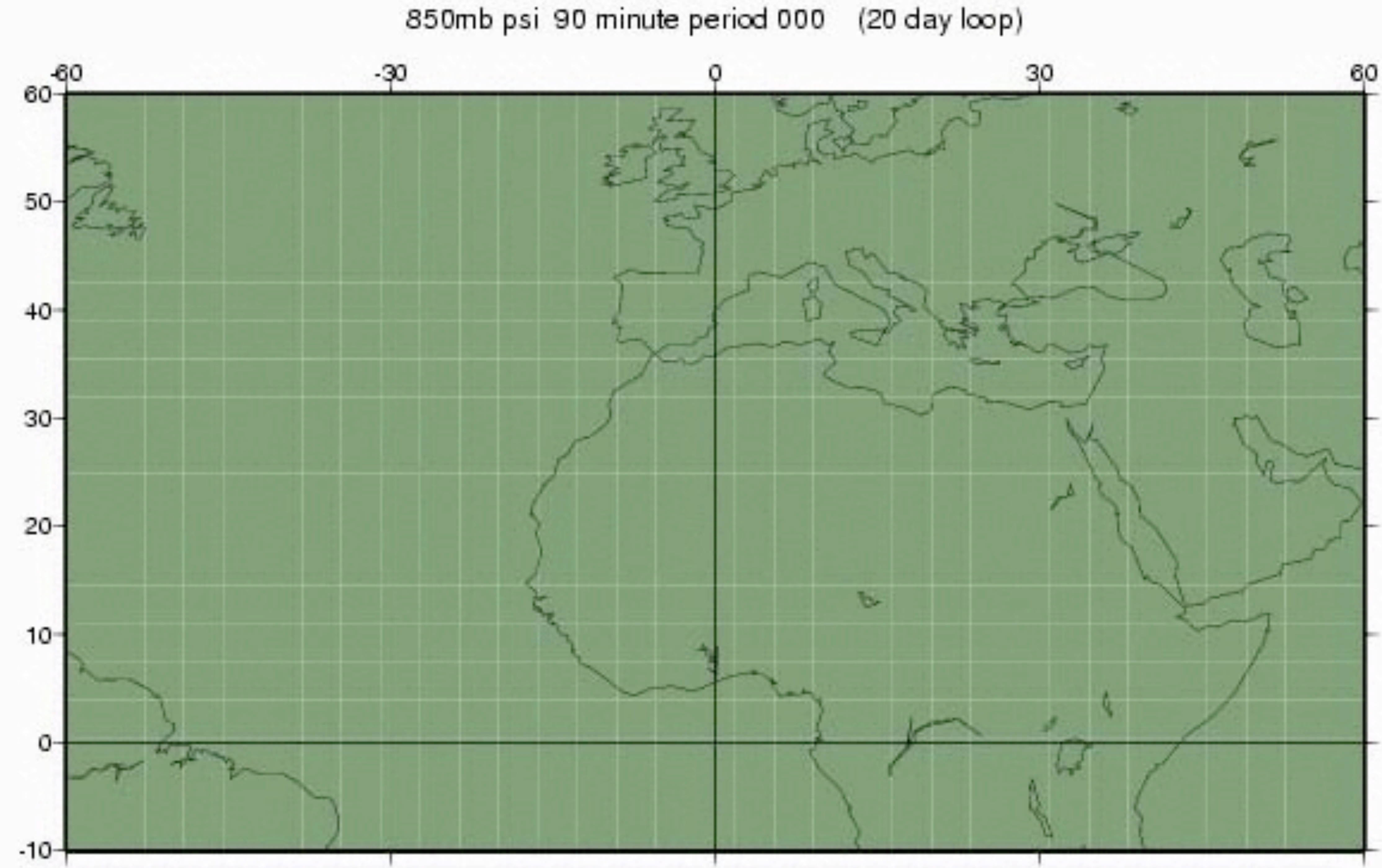
heat for one day in an elipse with a cosine squared bell shaped distribution and various vertical profiles



Initial

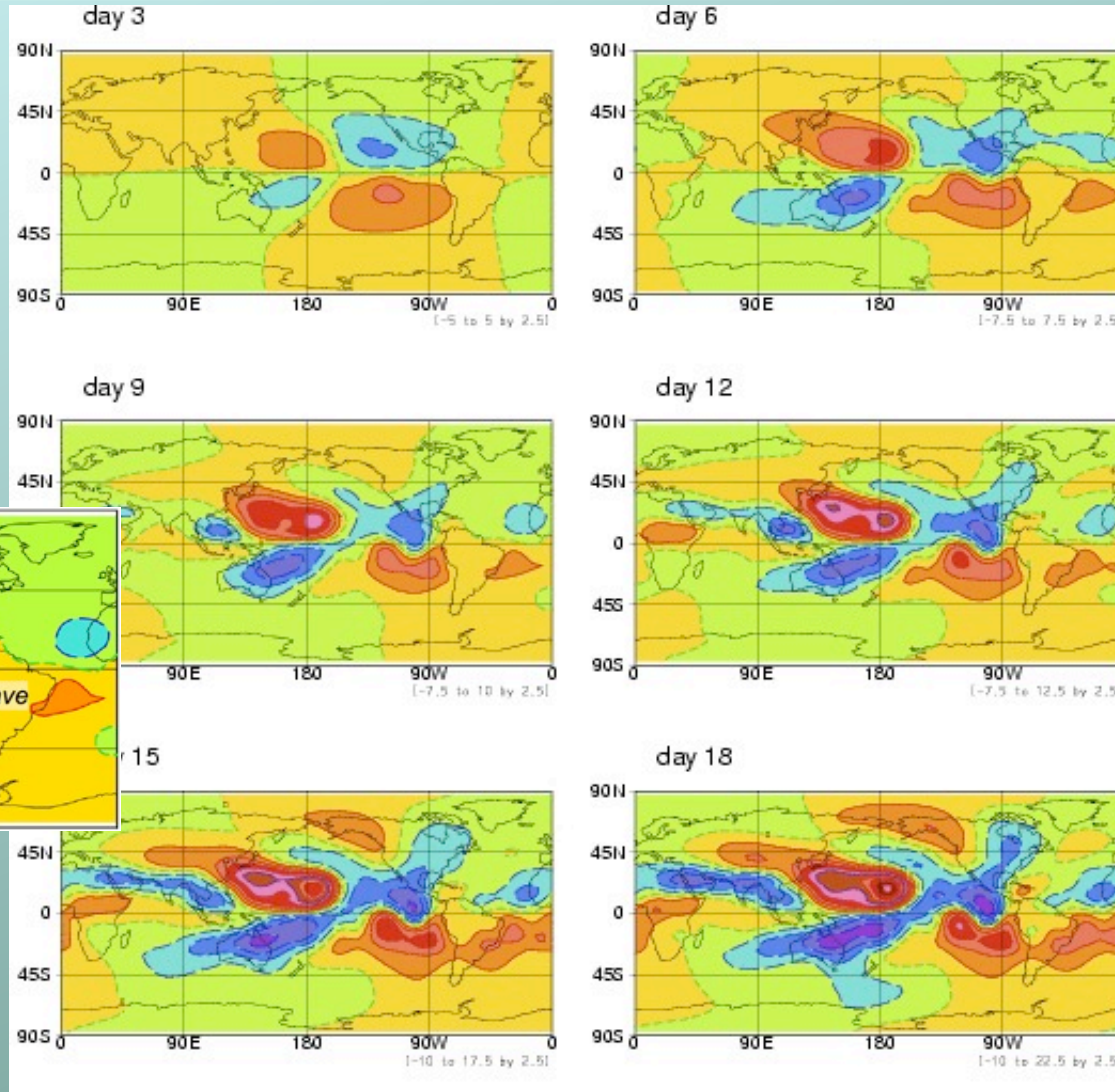
- If normal m
- But we still

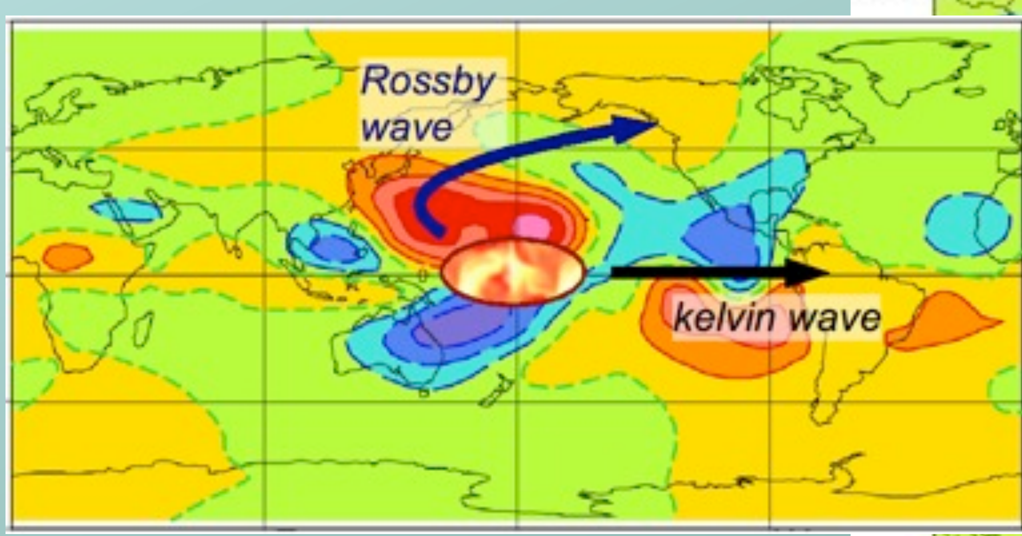
heat for one day in an elipse with a cosine squared bell shaped distribution and various vertical profiles



response to El Niño

The remote response to a steady tropical forcing involves radiation of Rossby and Kelvin waves (see Lisa's talk)





explicit transients: a simple GCM

Let's reconsider the definition of our forcing function *g*.
Recall the development equations:

data

$$\frac{d\Phi}{dt} = L\Phi + \Phi^{\dagger} Q\Phi + f(t)$$

model

$$\frac{d\Psi}{dt} = L\Psi + \Psi^{\dagger} Q\Psi + g$$

If we set $g = \overline{f}$ then this is the same as setting $g = -L\overline{\Phi} - \overline{\Phi}^{\dagger} Q\overline{\Phi} - \overline{\Phi'^{\dagger} Q\Phi'}$

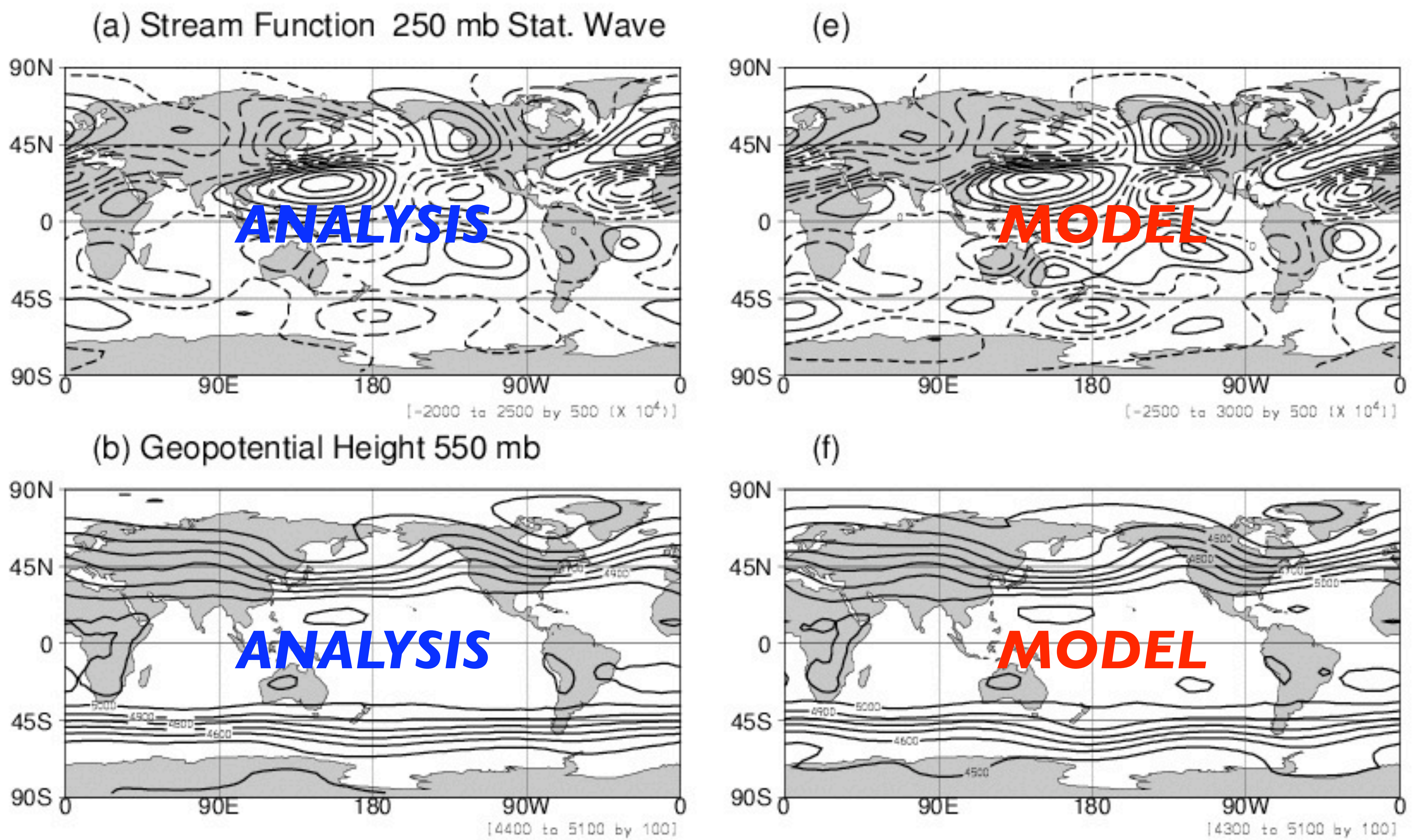
i.e. we have subtracted out the time-mean "forcing" due to the transients.

Again, we can calculate this forcing by initializing the unforced model from a series of values of Φ and then taking the time-average.

If the model is now initialized with $\overline{\Phi}$ it will develop in time. In fact we hope it will develop its own explicit transient activity. And we hope that it will be realistic. But there is no guarrantee that this "simple GCM" will have a realistic climatology. The only thing that is guarranteed is that:

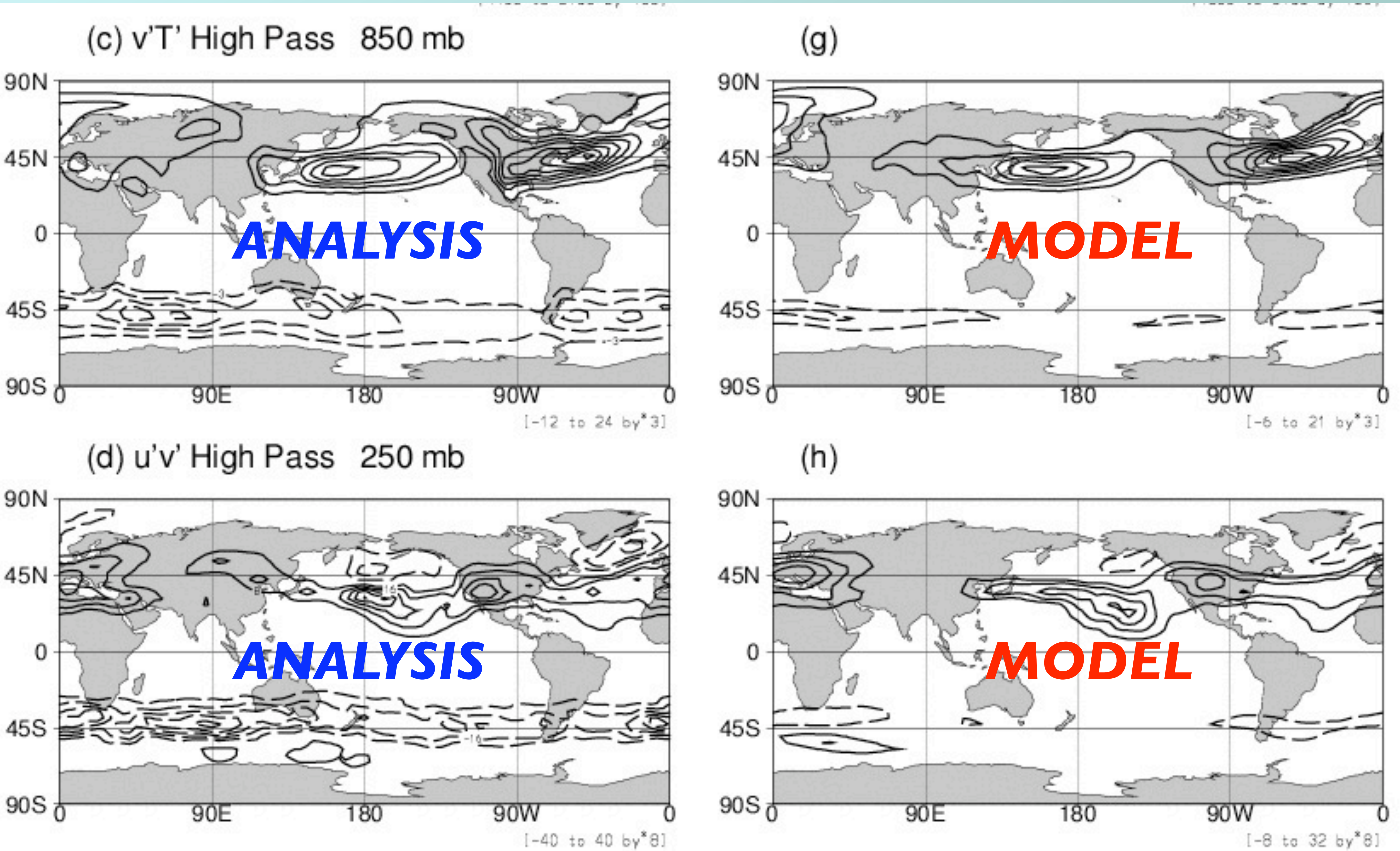
$$L\overline{\Psi} + \overline{\Psi}^{\dagger} Q\overline{\Psi} + \overline{\Psi'^{\dagger} Q\Psi'} = L\overline{\Phi} + \overline{\Phi}^{\dagger} Q\overline{\Phi} + \overline{\Phi'^{\dagger} Q\Phi'}$$

it works !



(a) Stream Function 250 mb Stat. Wave
ANALYSIS
[-2000 to 2500 by 500 (X 10^4)]
(e)
MODEL
[-2500 to 3000 by 500 (X 10^4)]
(b) Geopotential Height 550 mb
ANALYSIS
[4400 to 5100 by 100]
(f)
MODEL
[4300 to 5100 by 100]
90N
45N
0
45S
90S
0
90E
180
90W

it works !



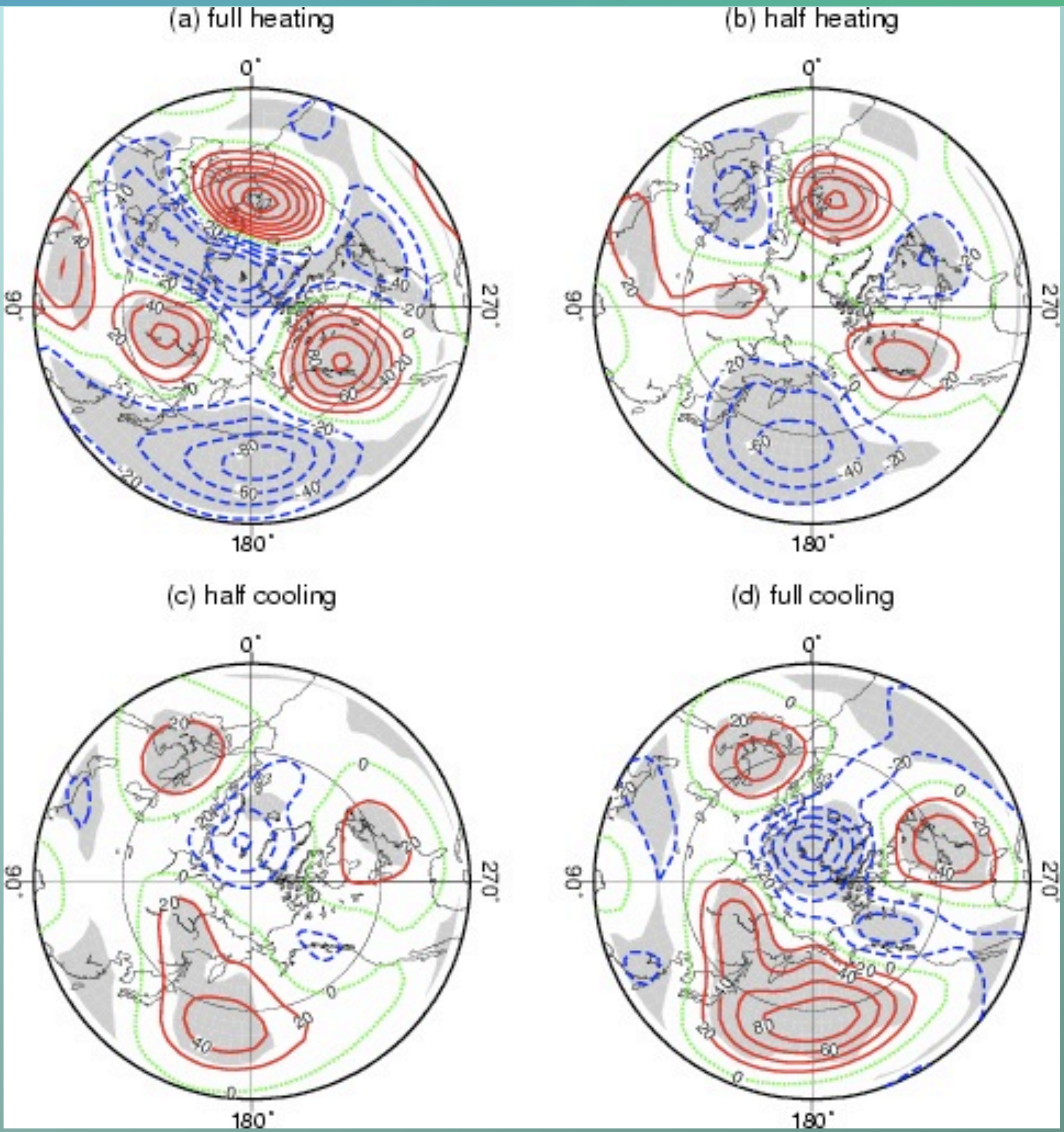
(c) v'T' High Pass 850 mb
(g)
90N
45N
0
45S
90S
0
90E
180
90W
ANALYSIS
MODEL
[-12 to 24 by*3]
[-6 to 21 by*3]
(d) u'v' High Pass 250 mb
(h)
ANALYSIS
MODEL
[-40 to 40 by*8]
[-8 to 32 by*8]

asymmetry of the mean response

Perform long integrations with equal and opposite heating (El Niño) and cooling (La Niña).

Look at the difference between them and a control integration.

The asymmetry comes from dynamical nonlinearity: differences in the transient feedback.



time-independent solutions

It would be nice to get a time independent solution - i.e. a solution of

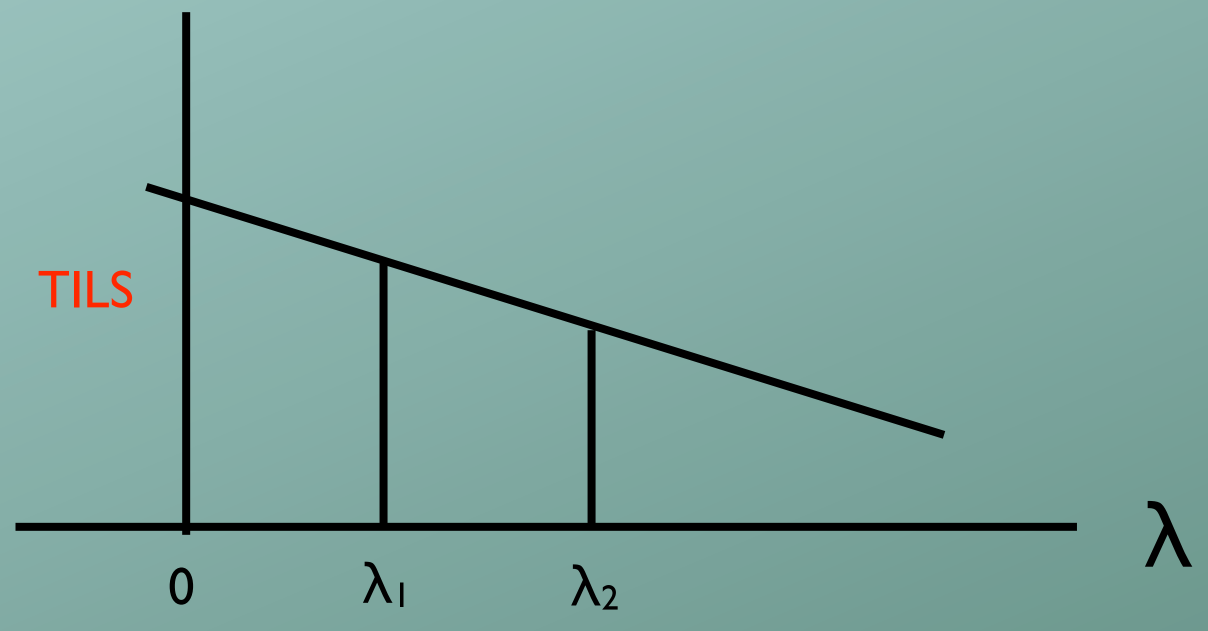
$$L_{mean}\Psi' + f' = 0$$ to compare with the equilibrated GCM reponse.

Not easy if L_{mean} is unstable (i.e. has positive values of σ). If this is the case any integration of the model will end up with a growing mode that dominates, and is unrelated to the forcing *f'*.

We can stabilize L_{mean} by subtracting a multiple of the identity matrix *I*. This does not affect the modal structure of *L*. We can then find the time indepenent solution by integration of

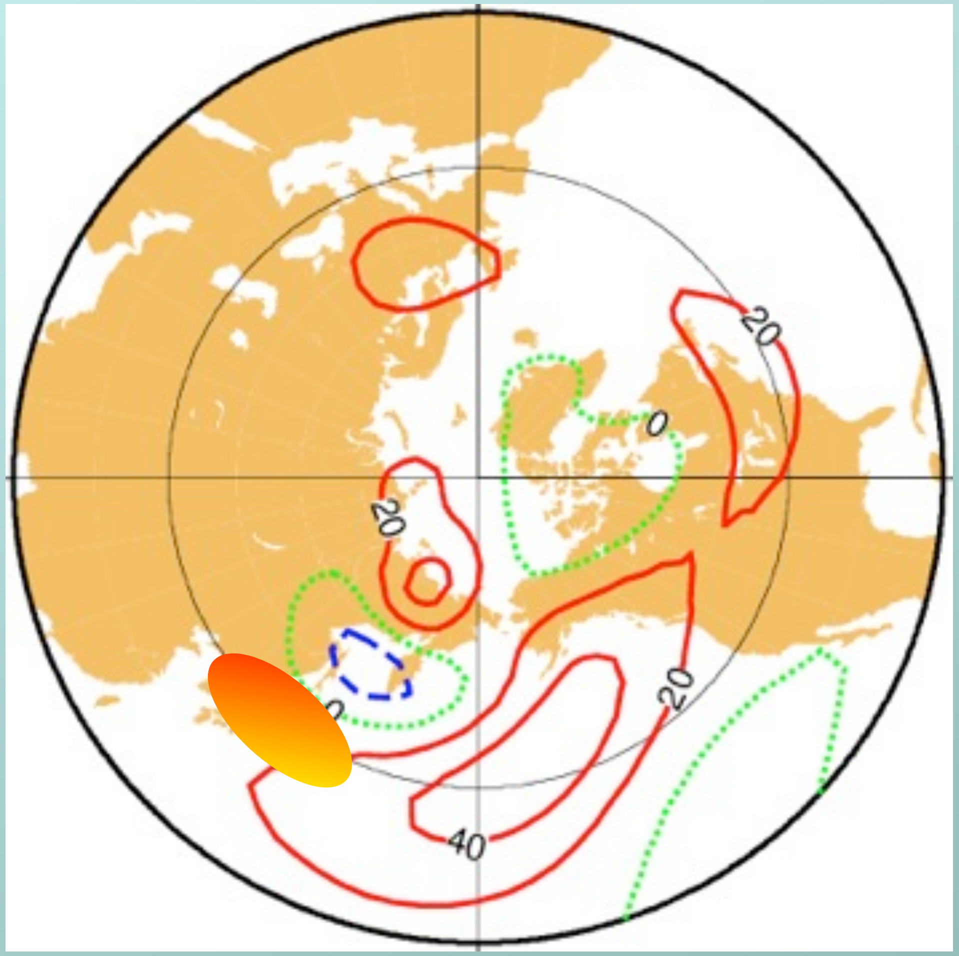
$$\frac{d\Psi'}{dt} = (L_{mean} - \lambda I)\Psi' + f'$$

and then extrapolate back to *λ = 0* to get our time independent linear solution.

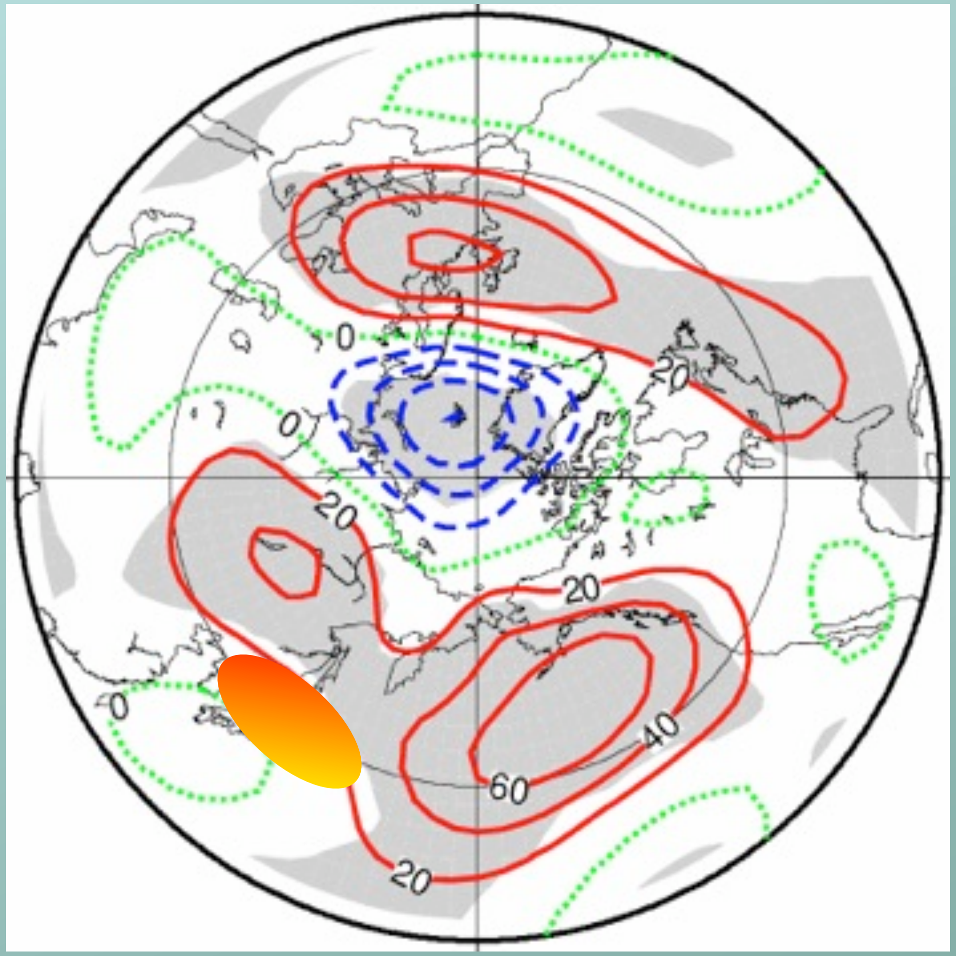


application to a midlatitude SST anomaly

500 mb geopotential height response to a heating anomaly over a midlatitude SSTA



Linear response using perturbation model

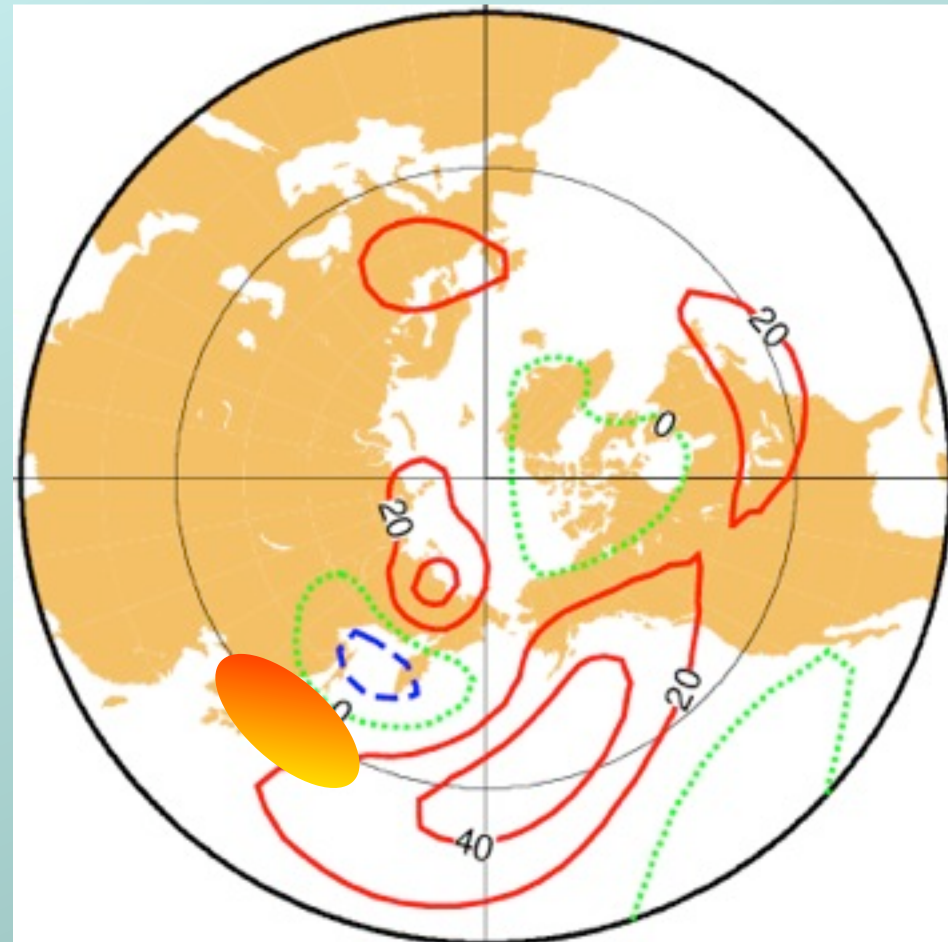


Simple GCM mean response

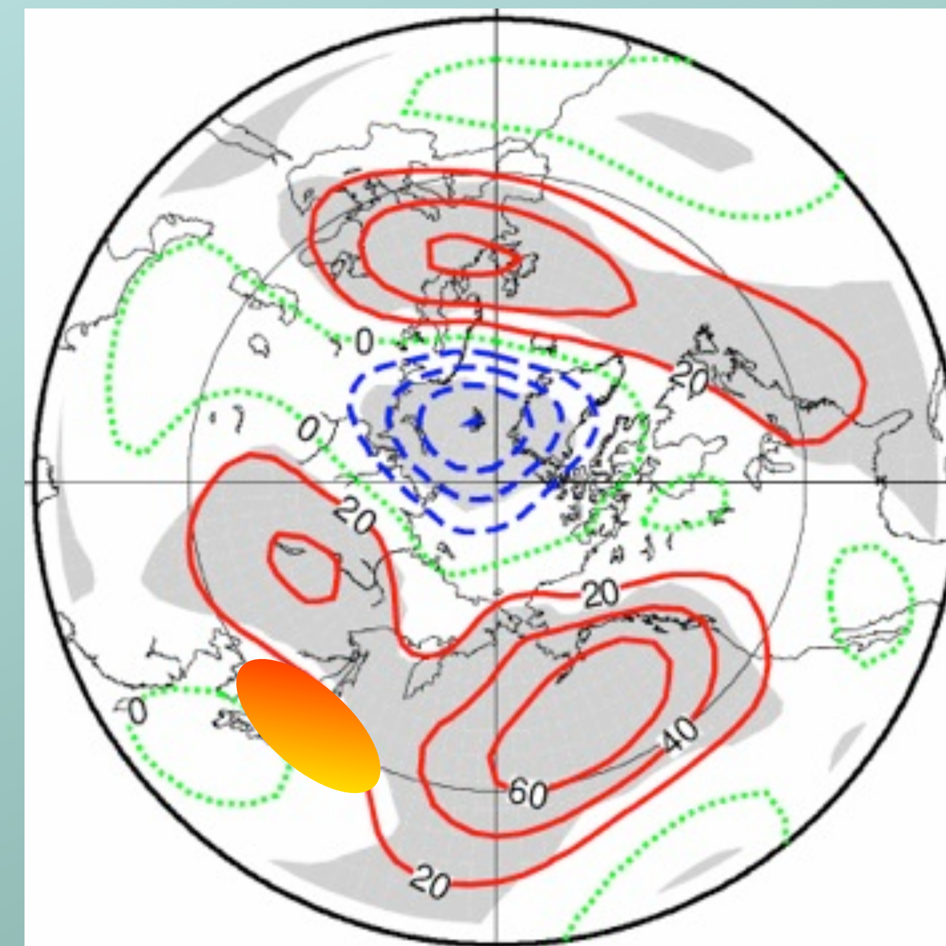
$$\frac{d\Psi'}{dt} = L_{mean}\Psi' + f'$$

application to a midlatitude SST anomaly

500 mb geopotential height response to a heating anomaly over a midlatitude SSTA

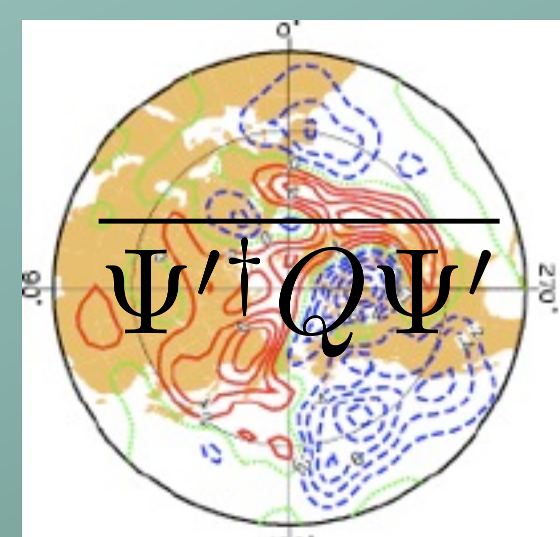


Linear response using perturbation model



Simple GCM mean response

$$\frac{d\Psi'}{dt} = L_{mean}\Psi' + f'$$

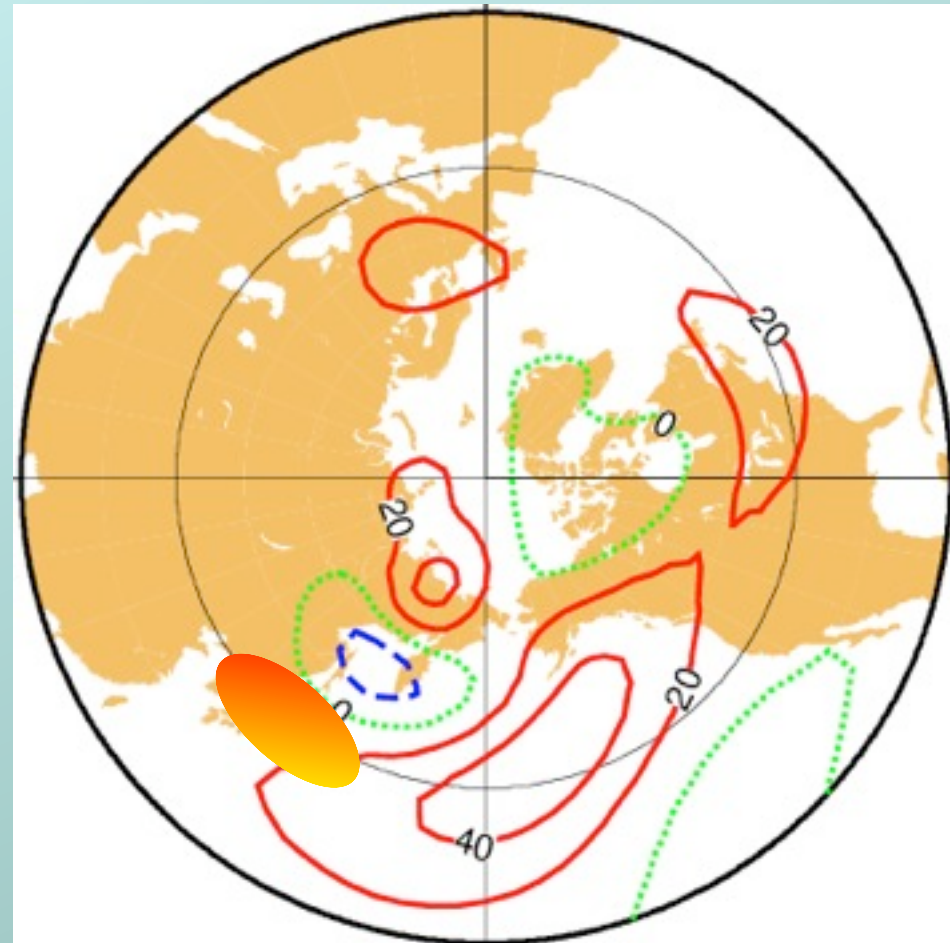


$$\overline{\Psi'^{\dagger} Q \Psi'}$$

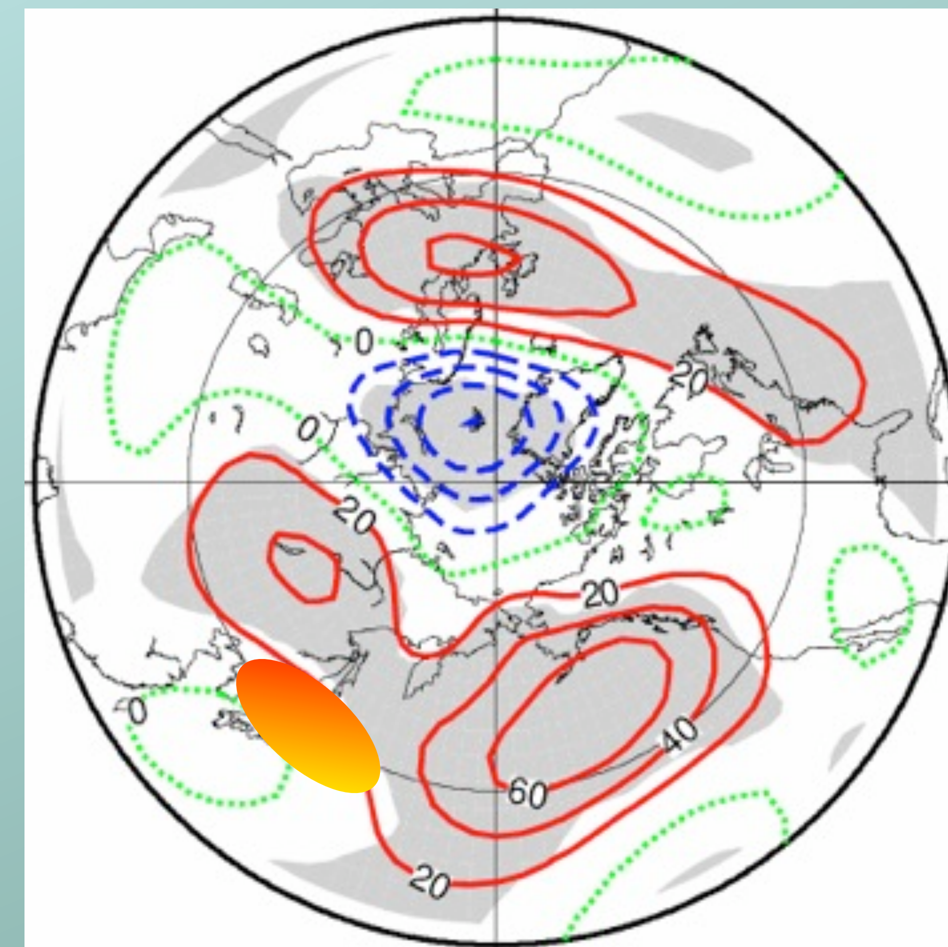
deduce transient forcing term

application to a midlatitude SST anomaly

500 mb geopotential height response to a heating anomaly over a midlatitude SSTA

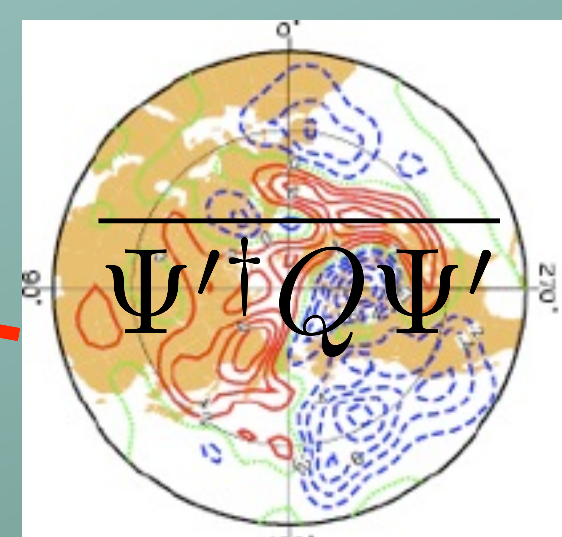


Linear response using perturbation model



Simple GCM mean response

deduce transient forcing term



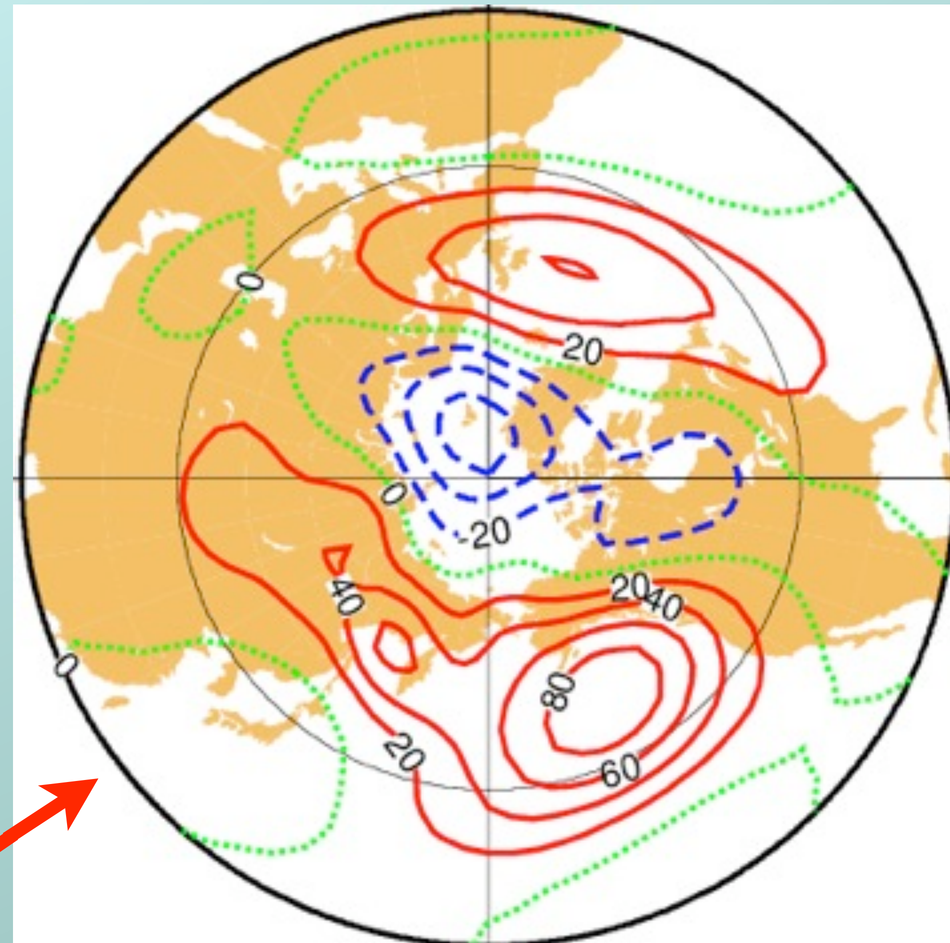
$$\overline{\Psi'^{\dagger} Q \Psi'}$$

add it in here

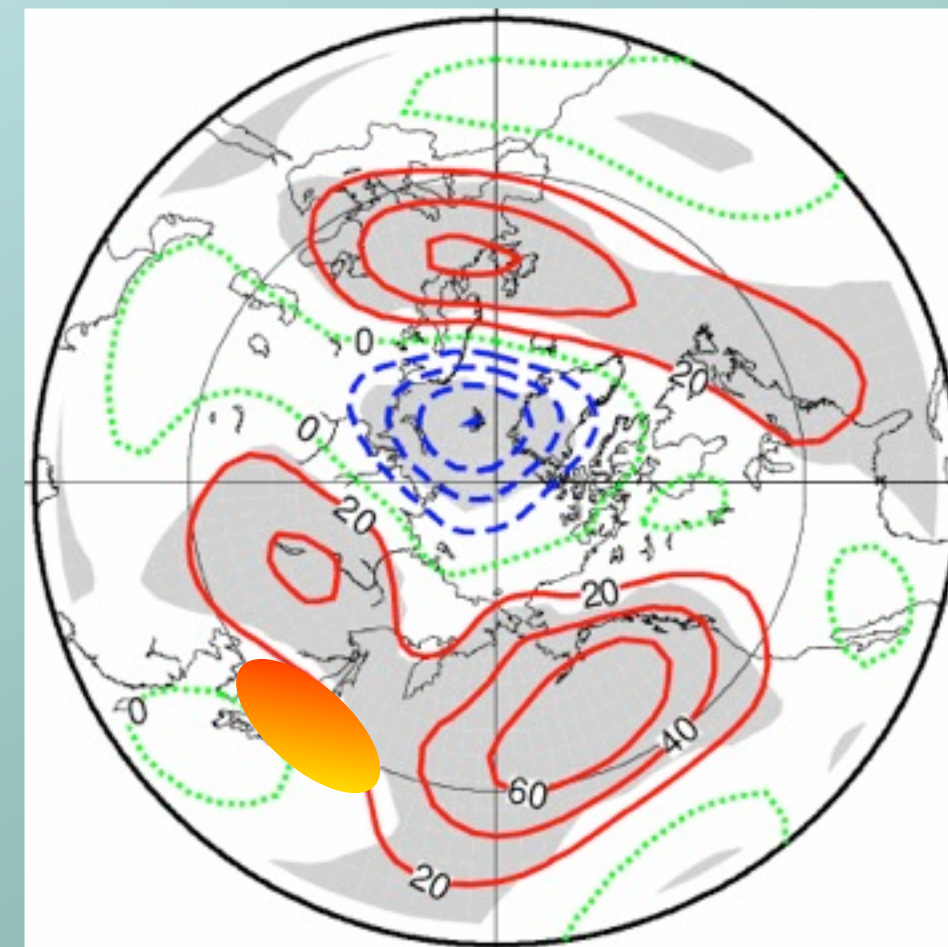
$$\frac{d\Psi'}{dt} = L_{mean}\Psi' + f'$$

application to a midlatitude SST anomaly

500 mb geopotential height response to a heating anomaly over a midlatitude SSTA



Linear response using perturbation model

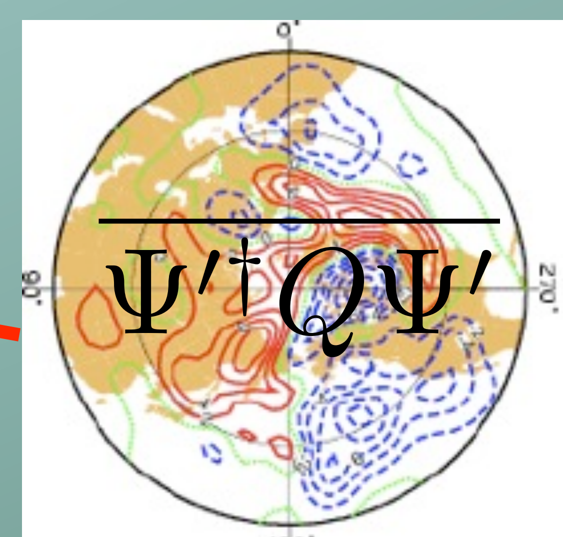


Simple GCM mean response

get this

$$\frac{d\Psi'}{dt} = L_{mean}\Psi' + f'$$

add it in here



$$\overline{\Psi'^{\dagger} Q \Psi'}$$

deduce transient forcing term

nudge nudge

Another way of forcing a model is to push it towards a desired climatology in a restricted region, and look at the effect on the solution outside that region. This is called nudging.

$$\frac{d\Psi}{dt} = L\Psi + \Psi^{\dagger} Q\Psi + g + \left(\frac{\Phi_n - \Psi}{\tau}\right)$$

Nudging involves an additional constant forcing term and a damping term.

In a linear experiment, the appropriate model is:

$$\frac{d\Psi'}{dt} = L_{mean}\Psi' + \epsilon\left(\frac{\Phi_n - \overline{\Phi}}{\tau}\right) - \frac{\Psi'}{\tau}$$

This can be useful technique for diagnosing climate anomalies or simulating other people's GCMs with a simple model.

effect of the tropics on the extratropics in 2000

time independent
linear solutions

nuging the tropical
band in different
regions