

# Linear Response Functions

## A Tool for Model Analysis and Comparison

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- The trouble with modeling
- Specific example
- A solution
- Two Applications

# The trouble with modeling

- Models differ by author, theory, implementation, etc.
- They may not give consistent results.
- Simplified (toy) models are abstract.
- Computing limitations separate large scale from the small and vice versa.

# Specific Example: Testing a Hypothesis

# A simple model

## Testing a hypothesis

Idea! Two instability mechanisms drive convectively-coupled waves (CCWs) :

Moisture-Stratiform Instability(MSI)<sup>1</sup>

relies on moisture variations in mid-troposphere

Direct-Stratiform Instability(DSI)<sup>2</sup>

relies on top-heavy mean convective heating profile

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<sup>1</sup>(Kuang, Z., 2008).

<sup>2</sup>i. e., stratiform instability (Mapes, B. E., 2000).

# CSRM simulations of CCWs

Testing a hypothesis

Are these mechanisms in a cloud system-resolving model (CSRM)?

Simulations using two different wind profiles:

Radiative-Convective Equilibrium (RCE)

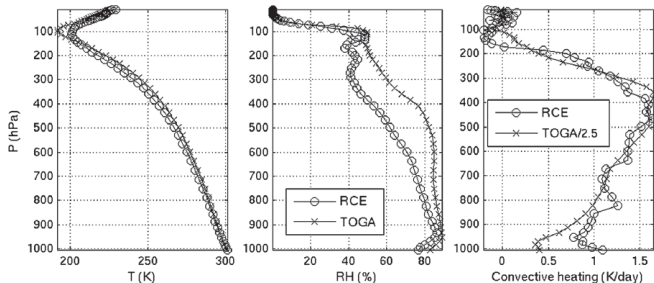
mean vertical velocity is zero

TOGA-COARE<sup>3</sup> observational data

mean vertical velocity is nonzero

# Mean profiles for RCE and TOGA-COARE cases

Testing a hypothesis



(Kuang, Z., 2010)

# CSRM is coupled to a linearized wave model

## Testing a hypothesis

Domain averaged  $T$ ,  $q$  anomalies comprise the CSRM virtual temperature profile

$$\overline{T'}(z), \overline{q'}(z) \rightarrow T'_V(z),$$

which then informs the linearized vertical velocity perturbation

$$T'_V(z) \rightarrow w'(z),$$

which in turn alters  $T$ ,  $q$  in the CSRM

$$w'(z) \rightarrow T(x, y, z), q(x, y, z)$$



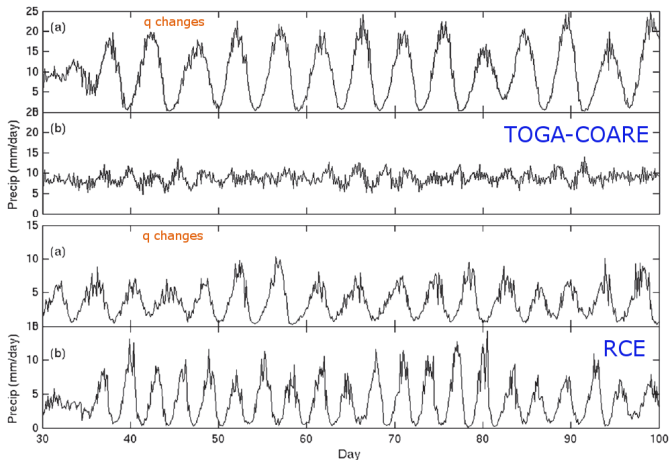
# CCW activity (rain rate) for each simulation

## Testing a hypothesis

In the second row of each case, the MSI *cannot* occur.

Then, if wave activity is present, our hypothesis  $\implies$  DSI.

But DSI  $\implies$  top-heavy mean heating profile!



# A Solution: Linear Response Functions

# What are they?

## Linear response functions

- Distilled version of an atmospheric model
- Describe how the model responds to small<sup>4</sup> anomalies in  $T$  and  $q$  (or other variables of interest)
- Allow an apples-to-apples comparison of structurally different models
- Reasonably approximate the model's convective response
- Illustrate model sensitivities not seen in the mean convective response

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<sup>4</sup>within the linear regime

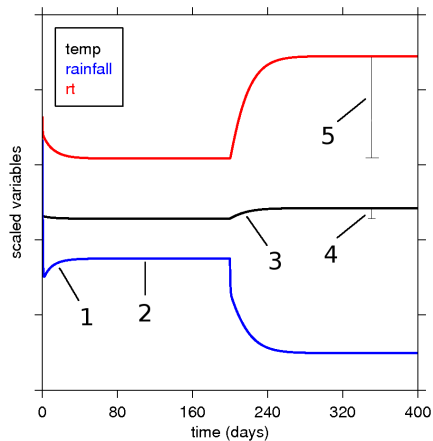
# How to derive them?

## Linear response functions

- The model is run to statistical equilibrium
- A tendency is applied to each variable of interest ( $T$  and  $q$ )
- The model is run until a new equilibrium is reached
- The difference between final (mean) profiles of  $T$  and  $q$  gives anomalous states
- The prescribed tendencies give the magnitude of the anomalous convective response

# Vectors of the response matrix

## Linear response functions



- 1 Initial model behavior
- 2 Statistical Equilibrium
- 3 Tendency is applied in  $T$
- 4 Anomalous  $T$
- 5 Anomalous  $q$

# The response matrix

## Linear response functions

$$\mathbf{Y} = \mathbf{M}\mathbf{X}$$

$\mathbf{Y}$  - Columns are convective tendencies for  $T$  and  $q$

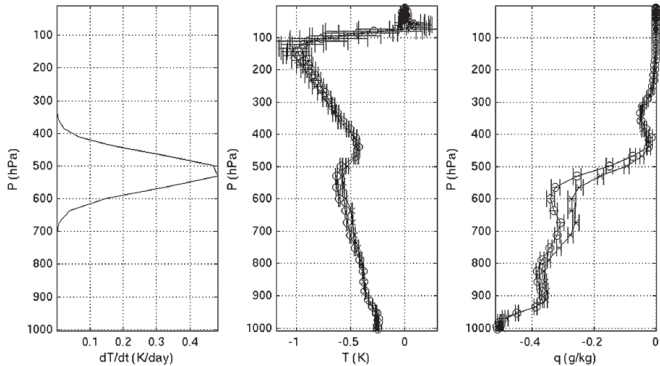
$\mathbf{X}$  - Columns are anomalous states of  $T$  and  $q$

$\mathbf{M}$  - Linear response matrix

$$\frac{d\vec{X}}{dt} = \mathbf{M}\vec{X}$$

# Equilibrium profiles

## Linear response functions



(Kuang, Z., 2010)

## Two Experiments:

- 1) Recall the DSI mystery!



# Summary

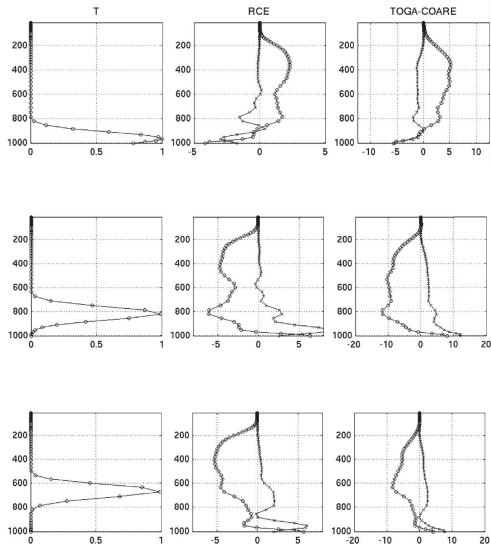
Recall the DSI mystery!

The CSRM exhibits direct-stratiform instability in the RCE case even though its mean convective heating profile was nearly identical to the TOGA-COARE case (which had no DSI).

This makes no sense according to the hypothesis, unless the RCE case exhibits a more top-heavy heating profile. Does it?

# Recall the DSI mystery!

Comparison of RCE and TOGA-COARE responses



Two Experiments:

2) Parameterization Analysis

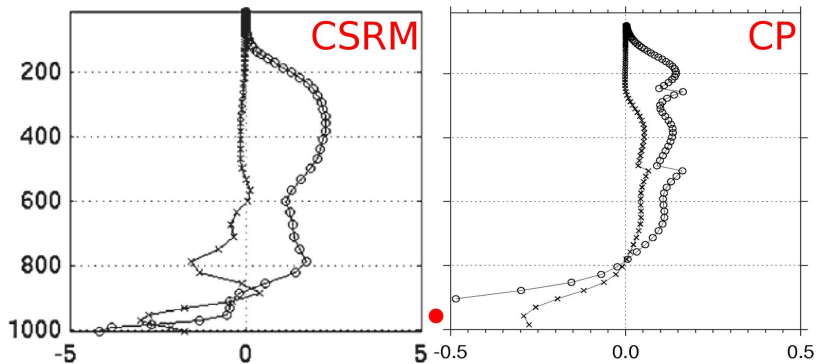
# Concept and information

## Parametrization Analysis

- Compare a cloud parameterization model to the CSRM (explicit convection)
- Learn something from the comparison?
- Diabat3 - toy cumulus parametrization (Raymond, 1994)

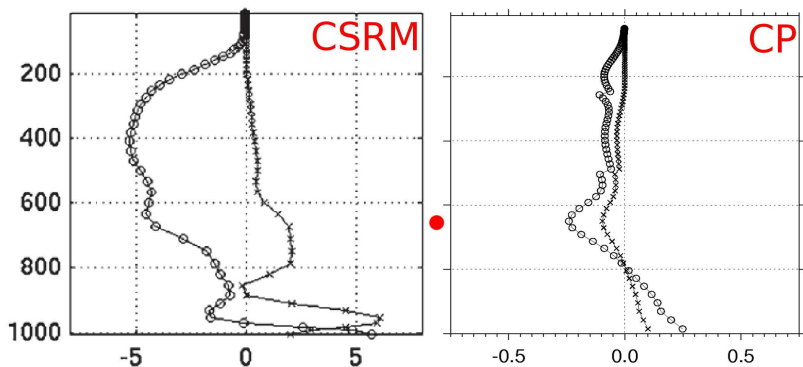
# T anomaly near surface

## Parametrization Analysis



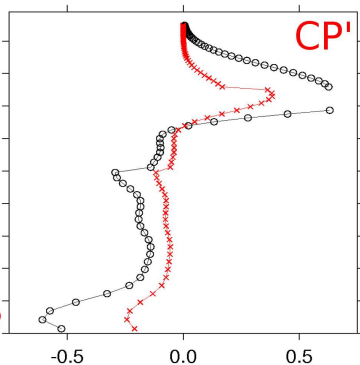
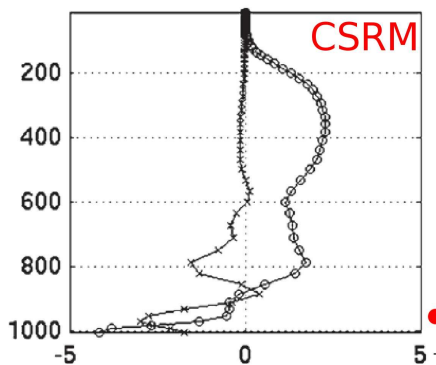
# T anomaly at mid-troposphere

## Parametrization Analysis



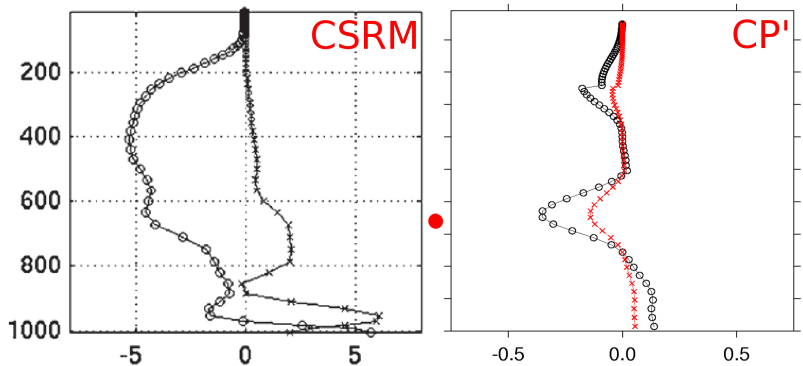
# T anomaly at surface (again)

## Parametrization Analysis



# T anomaly at mid-troposphere (again)

## Parametrization Analysis





# Conclusions

- Atmospheric modeling presents many challenges.
- Linear response functions can be used to compare and analyze the convective response of varied model types.
- They can be used to elaborate or fine-tune hypotheses based on simple models.
- They can be used to bring models into closer agreement.

- Emanuel, K., 2007: *Quasi-equilibrium dynamics of the tropical atmosphere*, in *The Global Circulation of the Atmosphere*, edited by T. Schneider and A. H. Sobel, pp. 186–218, Princeton University Press, Princeton, NJ.
- Kuang, Z., 2010: Linear Response Functions of a Cumulus Ensemble to Temperature and Moisture Perturbations and Implications for the Dynamics of Convectively Coupled Waves. *J. Atmos. Sci.*, **67**, 941-962.
- Kuang, Z., 2008: A Moisture-Stratiform Instability for Convectively Coupled Waves. *J. Atmos. Sci.*, **65**, 834-854.
- Raymond, D. J., 2001: A New Model of the Madden–Julian Oscillation, *J. Atmos. Sci.*, **58**, 2807-2819.