Linear Response Functions
A Tool for Model Analysis and Comparison

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Overview

- The trouble with modeling
- Specific example
- A solution
- Two Applications
The trouble with modeling

- Models differ by author, theory, implementation, etc.
- They may not give consistent results.
- Simplified (toy) models are abstract.
- Computing limitations separate large scale from the small and vice versa.
Specific Example:
Testing a Hypothesis
Two instability mechanisms drive convectively-coupled waves (CCWs):

Moisture-Stratiform Instability (MSI) \(^1\)
relies on moisture variations in mid-troposphere

Direct-Stratiform Instability (DSI) \(^2\)
relies on top-heavy mean convective heating profile

\(^1\) (Kuang, Z., 2008).
\(^2\) i.e., stratiform instability (Mapes, B. E., 2000).
Are these mechanisms in a cloud system-resolving model (CSRM)?

Simulations using two different wind profiles:

Radiative-Convective Equilibrium (RCE)
mean vertical velocity is zero

TOGA-COARE$^3$ observational data
mean vertical velocity is nonzero

$^3$ Tropical Ocean Global Atmosphere Coupled Ocean-Atmosphere Response Experiment
Mean profiles for RCE and TOGA-COARE cases

Testing a hypothesis

(Kuang, Z., 2010)
CSRM is coupled to a linearized wave model

Testing a hypothesis

Domain averaged $T$, $q$ anomalies comprise the CSRM virtual temperature profile

$$
\overline{T}'(z), \overline{q}'(z) \rightarrow T'_V(z),
$$

which then informs the linearized vertical velocity perturbation

$$
T'_V(z) \rightarrow w'(z),
$$

which in turn alters $T$, $q$ in the CSRM

$$
w'(z) \rightarrow T(x, y, z), q(x, y, z)$$. 
CCW activity (rain rate) for each simulation

Testing a hypothesis

In the second row of each case, the MSI cannot occur. Then, if wave activity is present, our hypothesis $\Rightarrow$ DSI. But DSI $\Rightarrow$ top-heavy mean heating profile!

![Graph showing precipitation changes over days for different cases: TOGA-COARE and RCE.](image)
A Solution:
Linear Response Functions
What are they?
Linear response functions

- Distilled version of an atmospheric model
- Describe how the model responds to small\(^4\) anomalies in \(T\) and \(q\) (or other variables of interest)
- Allow an apples-to-apples comparison of structurally different models
- Reasonably approximate the model’s convective response
- Illustrate model sensitivities not seen in the mean convective response

\(^4\text{within the linear regime}\)
How to derive them?
Linear response functions

- The model is run to statistical equilibrium
- A tendency is applied to each variable of interest \((T\) and \(q\))
- The model is run until a new equilibrium is reached
- The difference between final (mean) profiles of \(T\) and \(q\) gives anomalous states
- The prescribed tendencies give the magnitude of the anomalous convective response
Vectors of the response matrix

Linear response functions

1. Initial model behavior
2. Statistical Equilibrium
3. Tendency is applied in $T$
4. Anomalous $T$
5. Anomalous $q$
The response matrix

Linear response functions

\[ Y = MX \]

Y - Columns are convective tendencies for T and q
X - Columns are anomalous states of T and q
M - Linear response matrix

\[ \frac{d\vec{X}}{dt} = M\vec{X} \]
Equilibrium profiles
Linear response functions

(Kuang, Z., 2010)
Two Experiments:

1) Recall the DSI mystery!
The CSRM exhibits direct-stratiform instability in the RCE case even though its mean convective heating profile was nearly identical to the TOGA-COARE case (which had no DSI).

This makes no sense according to the hypothesis, unless the RCE case exhibits a more top-heavy heating profile. Does it?
Recall the DSI mystery!
Comparison of RCE and TOGA-COARE responses
Two Experiments:

2) Parameterization Analysis
Concept and information
Parametrization Analysis

- Compare a cloud parameterization model to the CSRM (explicit convection)
- Learn something from the comparison?
- Diabat3 - toy cumulus parametrization (Raymond, 1994)
T anomaly near surface
Parametrization Analysis
T anomaly at mid-troposphere
Parametrization Analysis
T anomaly at surface (again)
Parametrization Analysis
T anomaly at mid-troposphere (again)
Parametrization Analysis
Conclusions

- Atmospheric modeling presents many challenges.
- Linear response functions can be used to compare and analyze the convective response of varied model types.
- They can be used to elaborate or fine-tune hypotheses based on simple models.
- They can be used to bring models into closer agreement.

