Ex

A police siren has $f = 550\,\text{Hz}$ as the car approaches you, $450\,\text{Hz}$ after it passed. How fast is police car traveling?

$$f_a' = \frac{f}{1 - \frac{u}{v}}$$

$$f_r' = \frac{f}{1 + \frac{u}{v}}$$

$u = ?$

$v = 343\ \text{m/s}$

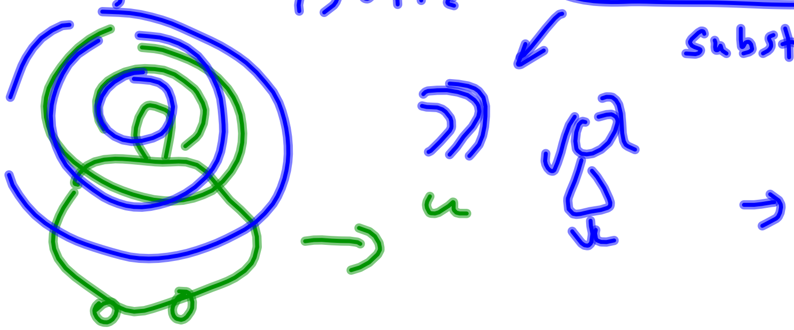
$f' = 550\,\text{Hz}$

$$f = \left(1 - \frac{u}{v}\right) f_a'$$

$$f = \left(1 + \frac{u}{v}\right) f_r'$$

subtract

$f' = 450\,\text{Hz}$



$$0 = (f_r' - f_a') + \frac{u}{v}(f_r' + f_a')$$

$f_a' = 550\,Hz$

$f_r' = 450\,Hz$

r - receding

a - approaching

$f = 495\,Hz$

$u = 34.3\,m/s$

The Doppler effect for light waves

- waves

$$\lambda_r = \sqrt{\frac{1+u/c}{1-u/c}}\,\lambda \qquad v = \lambda \cdot f$$

$$\lambda_a = \sqrt{\frac{1-u/c}{1+u/c}}\,\lambda$$

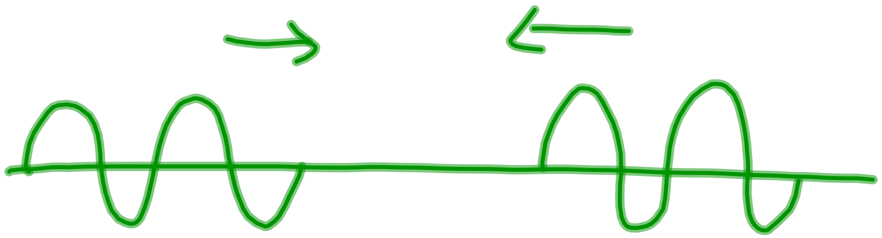
- remember
  blue & red
  red shift

we use Einstein theory

galaxies

moving away

Interference



2 waves meet each other

- what happens ?

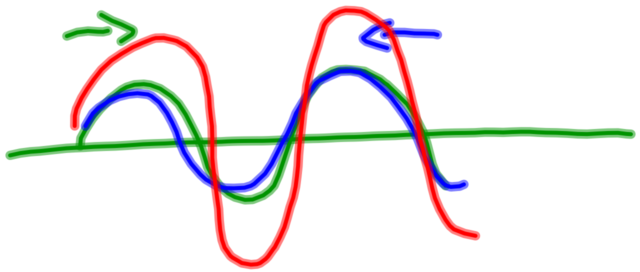
They interfere !

Most of the time their displacements ($y(x,t)$ or $D(x,t)$)

add up => superposition principle

2 opposite situations of interference

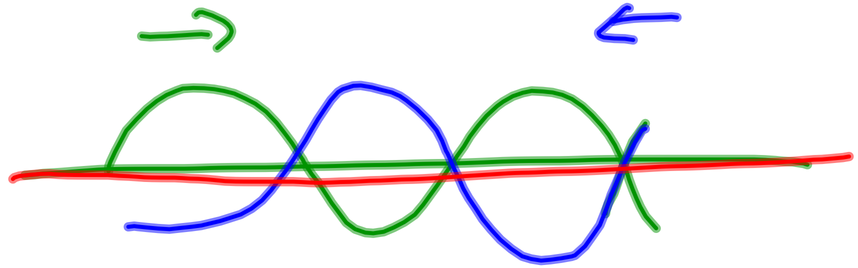
1)



constructive interference

- the crests coincide & so do the troughs
- the resulting wave is for a moment twice as big

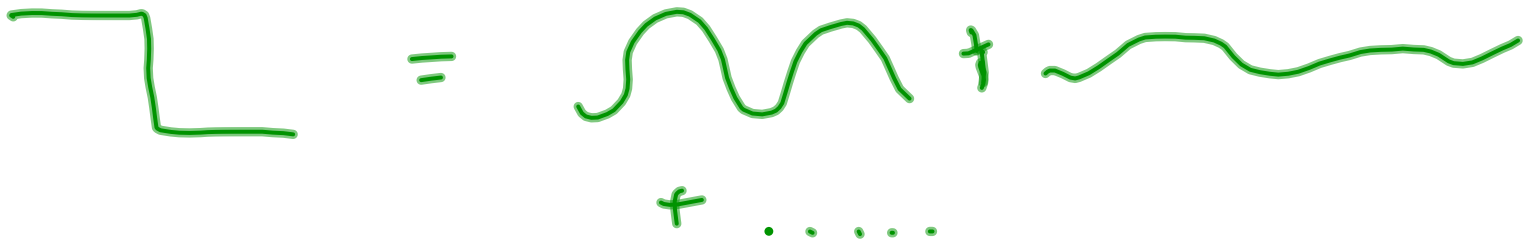
2)



the waves cancel each other

destructive interference

Note: Fourier analysis = any periodic wave can be written as a sum (superposition) of simple harmonic waves



Standing waves

- called standing because they stand "still"
  - water waves in confined space
  - musical instruments
  - experiment (movie)

Standing waves on a string clamped at both ends

- can be described mathematically as a superposition (sum) of 2 waves travelling in opposite directions & reflecting at the ends

wave 1 $\quad y_1(x,t) = A\cos(kx - \omega t)$ propagating in +x direction

$$y_2(x,t) = (-A)\cos(kx + \omega t)$$ - x direction

↓ as it is reflected

Superposition of those 2 waves

$$y(x,t) = y_1(x,t) + y_2(x,t) = A\left[\cos(kx-\omega t) - \cos(kx+\omega t)\right]$$

$\cos(\alpha-\beta) - \cos(\alpha+\beta)$

$$= -2A\sin\left[\frac{1}{2}\left(kx - \cancel{\omega t} + kx + \cancel{\omega t}\right)\right]\sin\frac{1}{2}\left(\cancel{kx} - \omega t - \cancel{kx} - \omega t\right)$$

$$= -2A\sin kx\,\underline{\sin(-\omega t)}$$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$$= \underbrace{2A\sin kx}_{\text{amplitude at given } x}\ \underbrace{\sin\omega t}_{}$$ oscillation in y direction

Because the string is clamped at both ends the amplitude at the ends must be zero

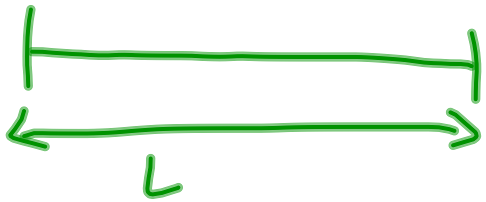
$2A \sin kx = 0$ at $x = 0$

$\boxed{2A \sin k \cdot L = 0}$ at $x = L$

$k \cdot L = m\pi$ $m = 1, 2, 3 \ldots.$

$k = \frac{2\pi}{\lambda}$

$\frac{2\cancel{\pi}}{\lambda} \cdot L = m\cancel{\pi}$

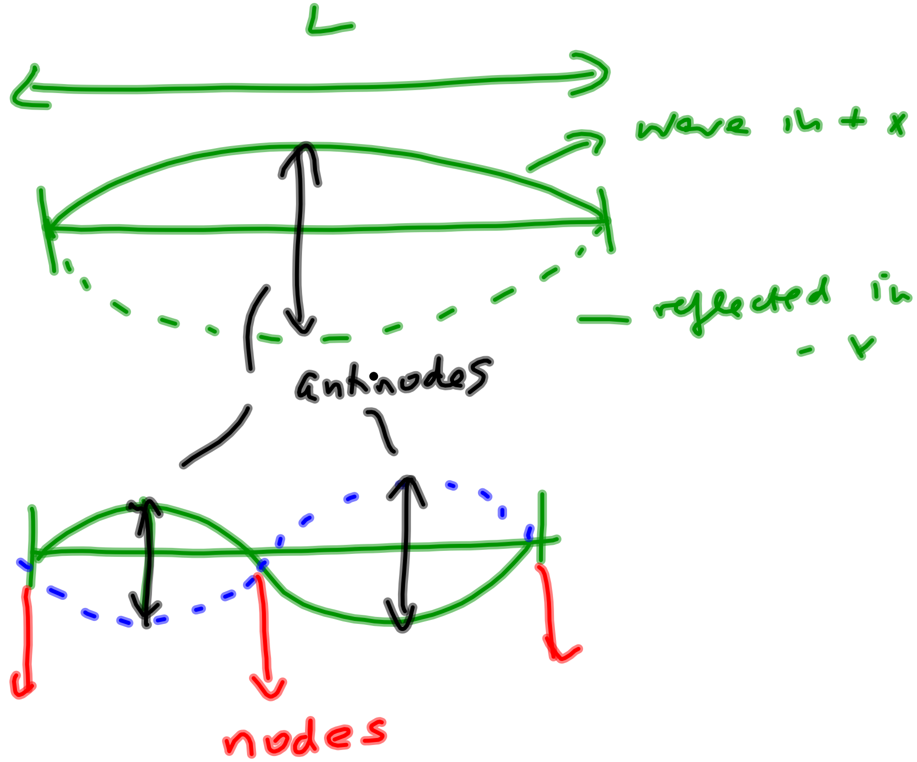


$$L = \frac{m\lambda}{2}$$

$m = 1, 2, 3, \ldots$ mode number

for $m = 1$ $\qquad \lambda = 2L$

$m = 2$ $\qquad \lambda = L$



Given a particular string length L we limit the allowed standing waves to a discrete set of wavelengths that we call harmonics. $n = 1$ gives fundamental mode while the rest are called overtones.

Nodes - points where the string doesn't move
destructive interference

Antinodes - A is max
- constructive interference

In terms of frequency:

$$v = f \cdot \lambda$$

rigid lid $\lambda = \dfrac{2L}{m}$

$$f = \frac{v}{\lambda} = \frac{v}{2L} \cdot m$$

fundamental f_1 $\quad f_1 = \dfrac{v}{2L} \quad (m=1)$

$f_m = m \cdot f_1$ harmonics

Beats

- when there is slightly different frequency involved