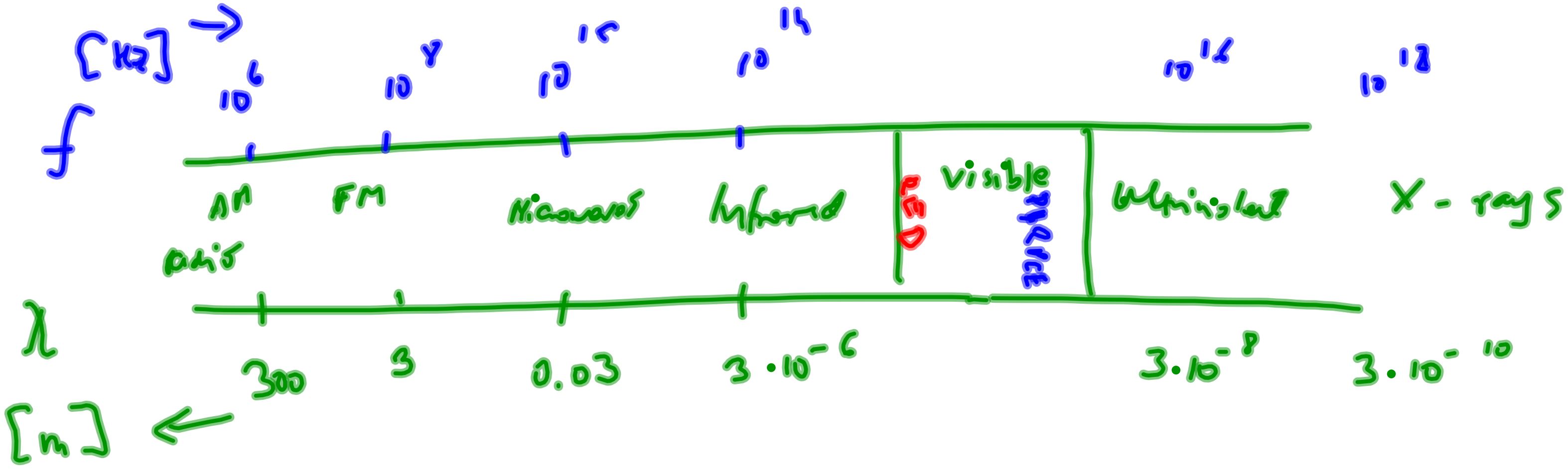
Electromagnetic waves

- light waves: "self-sustaining oscillations of the electromagnetic field"
- displacement, amplitude is an electric or/and magnetic field
- self-sustaining means that they don't require a medium to travel
  - $c = 3 \cdot 10^8$ m/s in vacuum
  - wavelengths λ are very small



$$v = \frac{\omega}{k} = \frac{\lambda}{T} \qquad \boxed{v = f\cdot\lambda}$$

- index of refraction

$$n = \frac{c}{v}$$

Electromagnetic spectrum

Q If the speed of light wave changes as it enters for example glass (n changes, v changes), what happens to light wave's λ & f ?

$$v = \lambda \cdot f \qquad n = \frac{c}{v}$$

either λ or f changes

The frequency of a wave is always a frequency of the source. It does not change as the wave moves from one medium to another.

The same is true for all electromagnetic waves.

Ex.

Orange light with $\lambda = 600\,nm$ comes upon 1 mm thick glass. What is the light speed in the glass? What is λ in glass?

$n = 1.5$

$\lambda_{air} = 600\,nm$

$v = ?$

$\boxed{v = f \cdot \lambda}$

$\boxed{v = \frac{c}{n}} = \frac{3 \cdot 10^8\,m/s}{1.5} =$

$= 2 \cdot 10^8\,m/s$

$\lambda_{glass} = \frac{v_{glass}}{f}$

note f doesn't change

$\lambda_{glass} = v_{glass} / f$

$\lambda_{air} = c / f$

$f = \dfrac{c}{\lambda_{air}}$

$\lambda_{air} = 600 \cdot 10^{-9}$ m

$v = \dfrac{c}{n}$

$f\lambda = \dfrac{c}{n}$

$V = \dfrac{c}{n}$

$\lambda_{glass} = v_{glass} \cdot \dfrac{\lambda_{air}}{c} = \dfrac{\lambda_{air}}{n_{glass}} = \dfrac{600 \cdot 10^{-9} \text{ m}}{1.5}$

$\dfrac{1}{n_{glass}}$

$= 400$ nm

$4 \cdot 10^{-7}$ m

power of the wave is the rate at which in J/s the wave transforms energy.

[J/s = W]   [W] Watt

- loudspeaker emits 2 W of power it means that energy in a form of sound wave is radiated at the rate of 2 J/s

Intensity

$$I = \frac{P}{a}$$

P - power

a - area

$$I \sim A^2$$

A is Amplitude

intensity is proportional to the square of its amplitude

$[\,W/m^2 \text{ or } J/sm^2\,]$

Wave equation in 1 D

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$y(x,t)$

y - displacement

D or y

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

$$\frac{\partial^2 \text{☺}}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \text{☺}}{\partial t^2}$$

<u>Ex.</u>

Verify that $☺(x,t) = A\sin(kx - \omega t)$ satisfies the wave equation.

$$\frac{\partial^2 ☺}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 ☺}{\partial t^2}$$

v wave speed

$$\frac{\partial ☺}{\partial t} = -\omega A\cos(kx - \omega t)$$

$$\frac{\partial^2 ☺}{\partial t^2} = -\omega^2 A\sin(kx - \omega t)$$

$$\frac{\partial^2 \odot}{\partial x^2} = -k^2 A \sin(kx - \omega t)$$

$$\boxed{\frac{\partial^2 \odot}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \odot}{\partial t^2}}$$

wave eqn

$$\frac{\partial^2 \odot}{\partial t^2} = -\omega^2 A \sin(kx - \omega t)$$

from wave eqn

$$\cancel{-} k^2 \cancel{A} \sin\cancel{(kx - \omega t)} = \frac{1}{v^2} \cdot (\cancel{-}\, \omega^2 \cancel{A}) \sin\cancel{(kx - \omega t)}$$

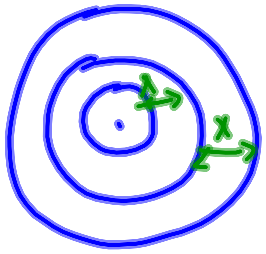
$$k^2 = \frac{\omega^2}{v^2} \qquad v^2 = \frac{\omega^2}{k^2} \checkmark$$

The Doppler effect

An interesting effect occurs when the wave source is moving relative to the observer or the other way around. Ex - pitch of ambulance siren drops as it passes by

If source is at rest & v is the wave speed

we get uniform radiation of waves



λ'_b ↗

v

f lower

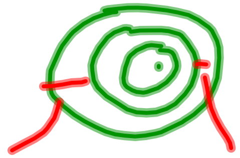
B

λ smaller

v the same

⇓

A



f_b

λ'_f

f higher

But when the source moves, wave crests are bundled in the direction in which the source is moving & λ decreases in front & increases behind the source

source

– v does not change! $v = \lambda \cdot f$

Let's analyze

T', λ wavelength when source is stationary

?', λ' — " — moving at

speed u

through a medium where the wave speed is

v

The moving source will emit λ but also travel the distance $u \cdot T$ in the same T period.

Therefore the distance between wavecrests as seen by an observer in front of the moving source is:

$$\lambda' = \lambda - ut \quad \text{in front !}$$

– we look at $t = T$

$$t = T = \frac{\lambda}{v} \qquad \lambda' = \lambda - uT$$

$$\lambda' = \lambda - \frac{u}{v}\lambda$$

$$= \lambda\left(1 - \frac{u}{v}\right)$$ source approaching the observer

$$\lambda' = \lambda\left(1 + \frac{u}{v}\right)$$ source receeding from the observer

– in terms of frequency $v = \lambda f$

$$\lambda' = v/f' \qquad \lambda = \frac{v}{f}$$

$$f' = \frac{f}{1 + u/v}$$ for receding source

$$f' = \frac{f}{1 - u/v}$$ for approaching source

λ smaller in approaching source

f bigger —"—

$$f' = f\left(1 + \frac{u}{v}\right)$$ observer approaching a stationary source

$$f' = f\left(1 - \frac{u}{v}\right)$$ observer moving away from the source