Waves

- A, λ, T, f

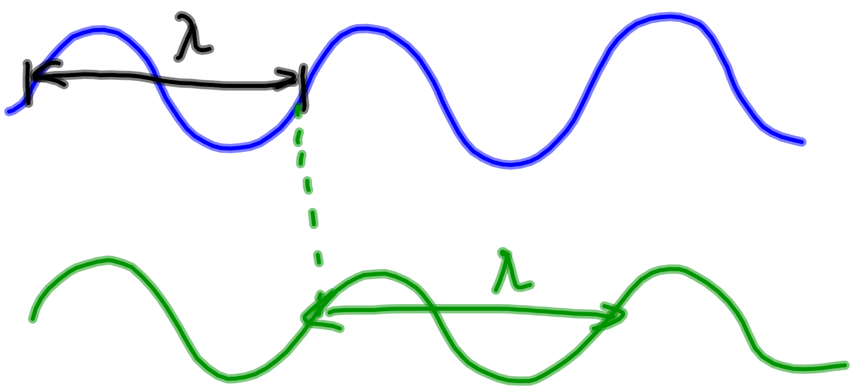
$f = \frac{1}{T}$ [Hz]

angular freq — $\omega = \frac{2\pi}{T} \left[\frac{rad}{s}\right]$

time for one cycle

t = 0

t = T



Wave speed V
- sound 340 m/s
- light in vacuum $3 \cdot 10^8$ m/s

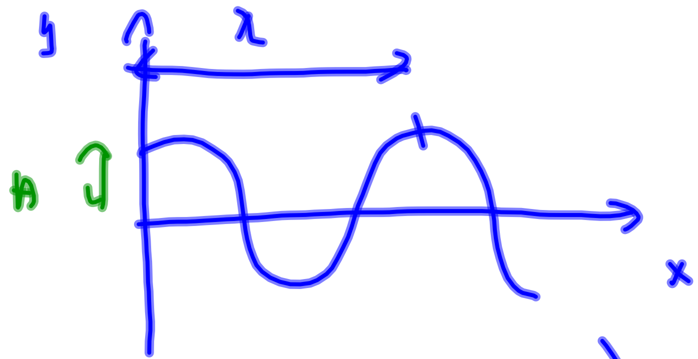
$v, s, t \;\rightarrow\; v = s/t$

$v, \lambda, T \qquad v_{wave} = \frac{\lambda}{T}$

$\boxed{v = \frac{\lambda}{T}} \qquad \boxed{v = \lambda \cdot f}$

Simple harmonic wave

$t = 0$



We choose coordinates so that at $x = 0$ there is a maximum of the wave

$$y(x, t=0) = A \cos kx$$

y wave displacement

A amplitude

k wavenumber

A sinusoidal wave as a function of position

$y(x, t=0) = A\cos kx$

- period of cosine function is 2π

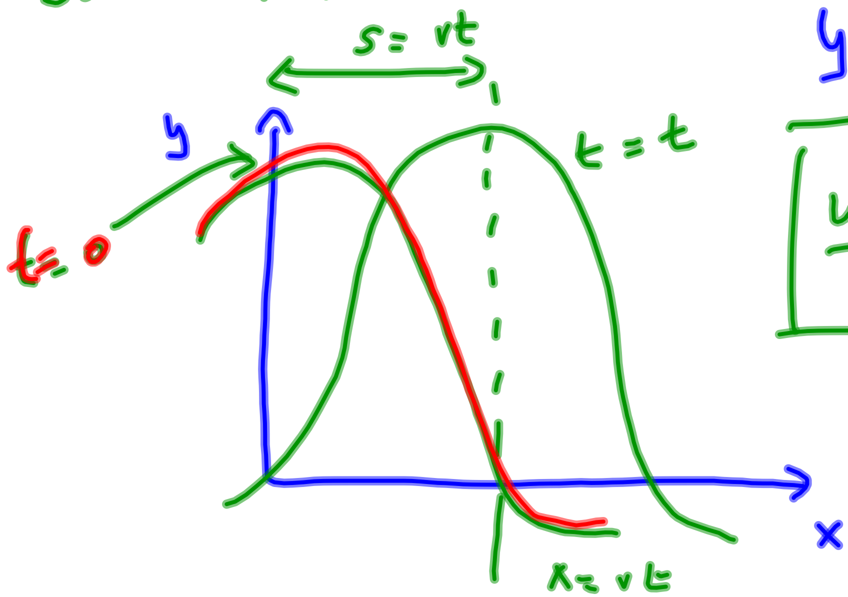
$\boxed{\omega = \frac{2\pi}{T}}$

$x = 0 \qquad kx = 0 \qquad y = A$

$x = \lambda \qquad \underline{kx = 2\pi} \qquad y = A$

$k \cdot \lambda = 2\pi \implies \boxed{k = \frac{2\pi}{\lambda}} \; [m^{-1}]$

To describe a wave moving with the speed v, we replace X with x − vt and we look at some time t



$$y(x, t=0) = A\cos kx$$

$$\boxed{y(x,t) = A\cos[k(x - vt)]}$$

Now we only want to write a sinusoidal wave as a function of ω & not v

$$\omega = \frac{2\pi}{T}$$

$$v = \frac{\lambda}{T} \Rightarrow v = \left(\frac{\lambda}{2\pi}\right)\omega = \frac{\omega}{k} \qquad kv = \omega$$

$1/k$

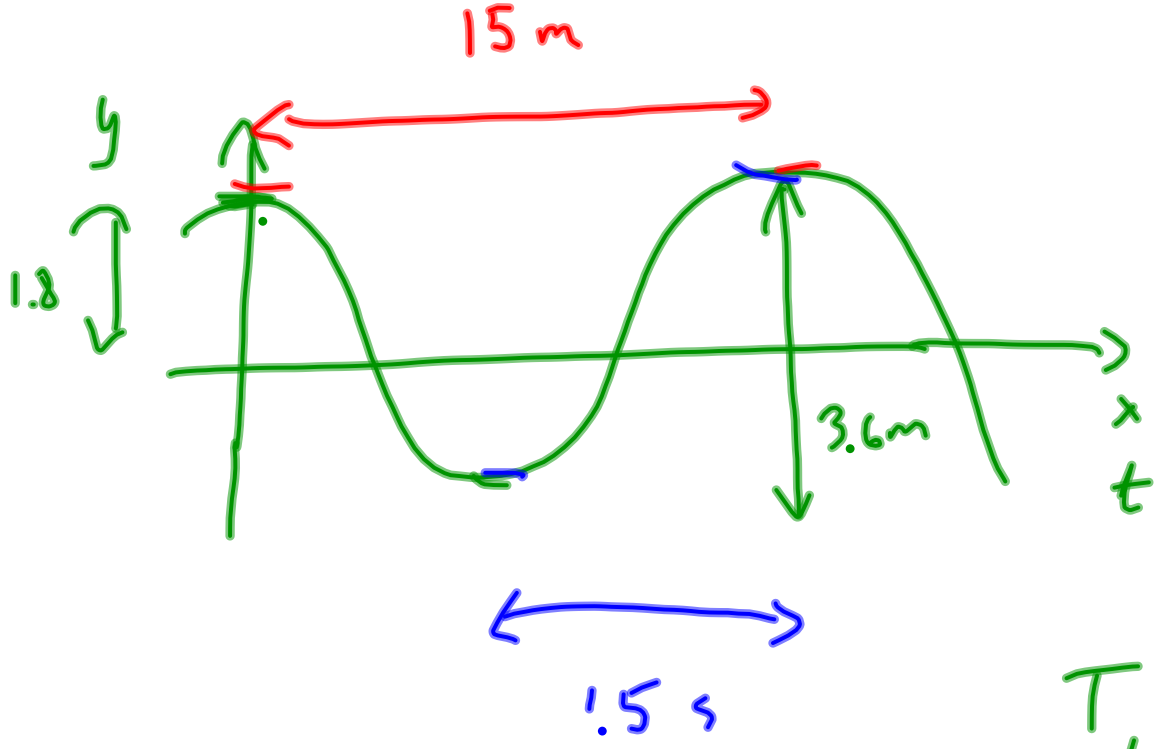
$$y(x,t) = A\cos\left[k(x - vt)\right] = A\cos(\underbrace{kx \mp \omega t}_{\text{wave's phase}})$$

traveling wave in positive x direction

negative

Example

A surfer is on the wave sinusoidal in shape with crests 15 m apart. The distance from through to crest of the wave is 3.6 m and it takes a surfer 1.5 s to cover that distance. Find the wave speed & write wave equation.



$\lambda = 15\ m$

$A = 1.8\ m$

$T = 3\ s$

$v = \frac{\lambda}{T} = 5\ m/s$

$T,\ \lambda$

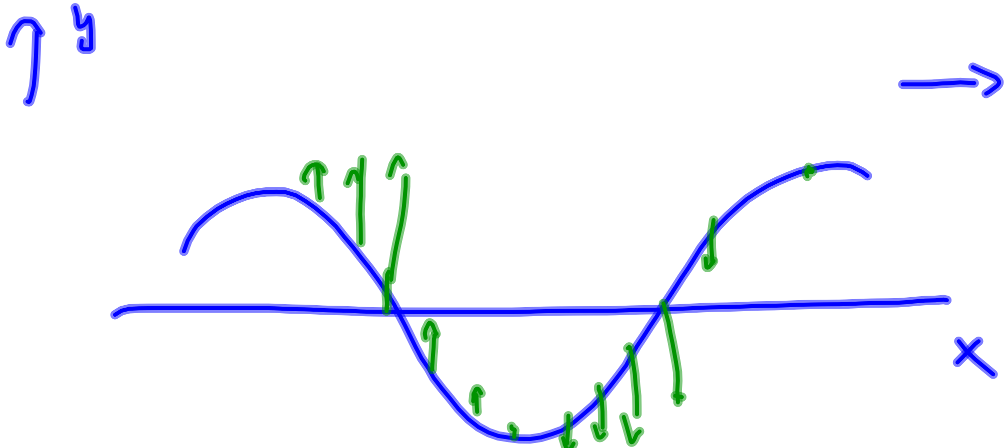
$y(x,t) = A\cos(kx - \omega t)$ $\quad k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T}$

$$k = \frac{2\pi}{15\,m}$$

$$\omega = \frac{2\pi}{3\,s}$$

$$y(x,t) = 1.8\,m \cos\left(\frac{2\pi}{15\,m} x - \frac{2\pi}{3\,s} t \right)$$

Waves on the string



→ velocity of the wave

arrows - velocity of a particle on a string

At $y = 0$ the particle's velocity is max

At turning point the particle's velocity is zero

$$\boxed{y(x,t) = A\cos(kx - \omega t)}$$

velocity of the wave $v = \frac{\lambda}{T}$

particle's velocity $V_y(x,t) = \frac{dy}{dt}$

$$V_y(x,t) = -A\sin(kx - \omega t)\cdot(-\omega)$$

$$= A\omega\sin(kx - \omega t)$$

!

$a_y(x,t) = dv_y/dt$

$$v_y = A\omega \sin(kx - \omega t)$$

$$a_y = -A\omega^2 \cos(kx - \omega t)$$

larger wave speed → particles oscillate more quickly → larger acceleration

- at the crest of the wave where displacement is maximum, so is the acceleration

for string wave

tension force

$$v = \sqrt{\frac{T}{\mu}}$$

μ - coeff.

$$a_{y_{max}} = -\omega^2 A$$

$$= -k^2 v^2 A$$

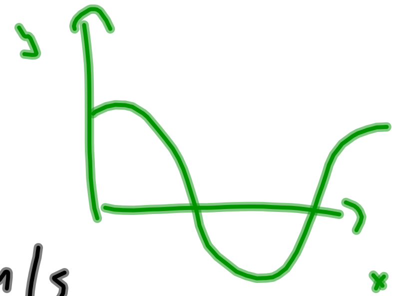
speed of the wave

$$V = \frac{\omega}{k} \qquad \omega = k \cdot v$$

we can relate particle's acceleration & speed of the wave

A very strong string with $\mu = 2\,g/m$ is stretched along x-axis with a tension of 5 N.
At $x = 0$ m it is tied to 100 Hz SHO that vibrates $\perp$ to the string with an amplitude of 2 mm.
The oscillator is at its max. positive displacement at $t = 0$. What is the displacement eqn of the wave? At $t = 5$ ms what is the string's displacement at a point 2.7 m from the oscillator?

$$y(x,t) = A\cos(kx - \omega t)$$



$A = 2\,mm = 0.002\,m$

$f = 100\,Hz$

$\mu = 2\,g/m$

$T = 5\,N$

$\omega = ?$

$k = ?$

$$v = \sqrt{\frac{T}{\mu}} = 50\,m/s$$

$$\omega = \frac{2\pi}{T} = 2\pi \cdot f = 200\pi\ s^{-1}$$

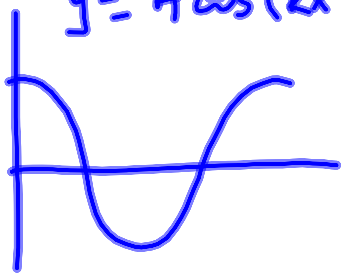
$k = ?$ $\quad k = 4\pi\ m^{-1}$ $\quad \left(k = \frac{\omega}{v}\right)$

$$y(x,t) = 0.002\cos(4\pi x - 200\pi t)$$

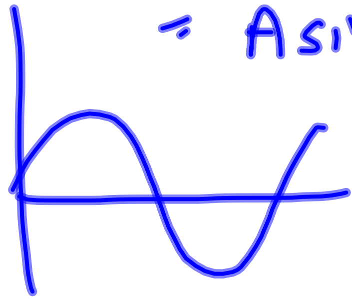
$$y(x=2.7, t=0.005) = \qquad 1.6\cdot 10^{-3}\ m$$

Phase of the wave

$$y = A\cos(kx - \omega t)$$



$$y = A\cos\left(kx - \omega t - \frac{\pi}{2}\right) = A\sin(kx - \omega t)$$



$-\pi/2$ diff in phase

Every wave eqn we can write as $y = A\cos(kx - \omega t + \phi)$

Phase difference between 2 points on a wave

$$\Delta\phi = \phi_2 - \phi_1$$

Sound waves

$V_{sound} = 343$ m/s at air temp.

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

γ ate characteristic to the gas

p pressure

ρ density

- your ears are able to detect sound waves with frequencies between 20 Hz & 20000 Hz

- low freq → low pitch - bass notes
- high -||- high -||-