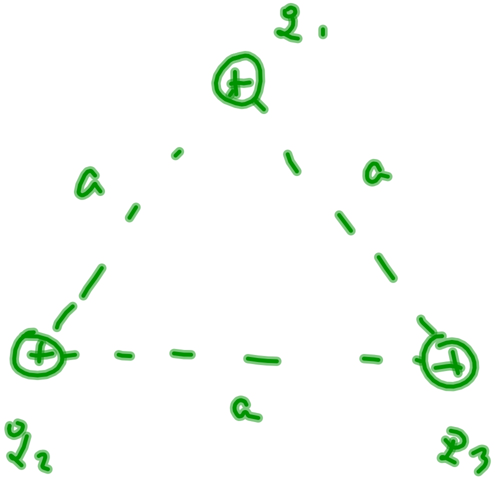
Capacitance & Capacitors

- energy storage of charged conductors



What is energy stored in this configuration?

We start by assuming that charges are widely separated & bring them closer.

1) it takes no work to bring in charge Q_1 (no electic field)

2) bringing in Q_2 means that we are working against Q_1's electic field potential (point charge) $V = k\frac{Q}{r}$

V_1 (due to Q_1 at location where Q_2 will come) $= k\frac{Q_1}{a}$

- remember that potential V is energy per unit charge; given the charge q_2 the work needed to bring in q_2 :

$$W_2 = q_2 V_1 = k \frac{q_1 q_2}{a}$$

3) bringing in q_3 – experiences E from both q_1 & q_2

$$W_3 = k \frac{q_1 q_3}{a} + k \frac{q_2 q_3}{a}$$

$$W_{tot} = W_2 + W_3 = k \frac{q_1 q_2}{a} + k \frac{q_1 q_3}{a} + k \frac{q_2 q_3}{k}$$

$W = U \Leftarrow$ electric field is conservative
force ....

$W < 0$ – that means that it took work to separate charges

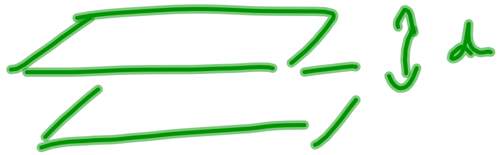
– watch for the sign of the charges

U does not depend on which charge you brought in first

Pair of electrical conductors - equal but opposite charges - CAPACITORS

$R \to \infty$

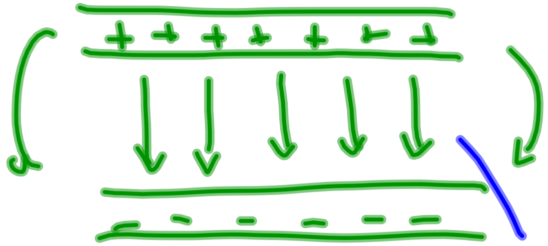
$$E_{disk} = \frac{\eta}{2\varepsilon_0}$$



$$E = \frac{\eta}{2\varepsilon_0} + \frac{\eta}{2\varepsilon_0} = \frac{\eta}{\varepsilon_0}$$

$$\eta = \frac{Q}{A}$$

A surface



E uniform

$E = \frac{Q}{\varepsilon_0 A}$ inside

$E = 0$ outside

If E is uniform potential $V = E \cdot d$

$$V = \frac{Qd}{\varepsilon_0 A}$$

$Q \sim V$

$$\boxed{Q = \frac{\varepsilon_0 A}{d} V}$$

C coeff. of proportionality

$$Q = C \cdot V$$

$$C = \frac{\varepsilon_0 A}{d}$$

CAPACITANCE

$C = \frac{Q}{V}$ $\left[\frac{C}{V}\right]$ $[F]$

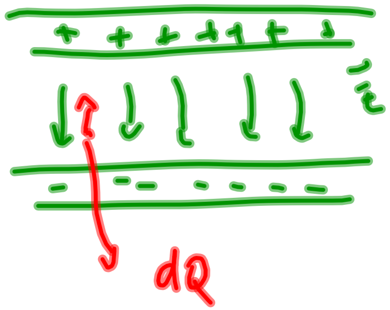
Michael Faraday (19^{th})

1 F is large capacitance - mostly μF or pF

μ: 10^{-6}, p: 10^{-12}

Energy storage in capacitors

- imagine moving a small charge dQ from negative to positive plate of capacitor where there is a potential difference between the plates



The work needed to do this

$$dW = V\,dQ$$

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the additional charge increases E → increases V

$C = \frac{Q}{V}$

$Q = V \cdot C$

$dQ = C\,dV$

$dW = C\,V\,dV$

– if we start with uncharged capacitor & start transfering charge we'll to the total work

$$W = \int dW = \int_0^V C \cdot V\,dV = \frac{1}{2} C V^2$$

- that work is stored as PE U

$$\boxed{U = \frac{1}{2} C V^2}$$ energy of capacitor

- we can measure V directly with voltmeter
or not :)

- keep in mind that the capacitor overall is neutral
even though we call it charged

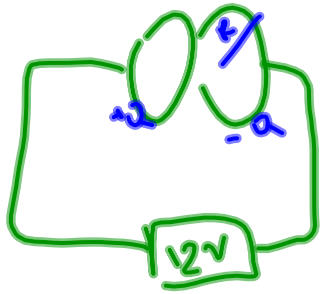
Ex

A capacitor consists of 2 circular metal plates of radius $R = 12\,cm$ separated by $d = 5.0\,mm$.

a) find its capacitance

b) find the charge on the plates

c) find the stored energy when the capacitor is connected to a 12V battery



$R = 12 \cdot 10^{-2}\ m$

$d = 5 \cdot 10^{-3}\ m$

$V = 12\ V$

a) $C = \frac{\varepsilon_0 A}{d} = [A = \pi R^2]$

$= \frac{\varepsilon_0 \pi R^2}{d} = 80\ pF$

b) $Q = CV = 960\ pC$

c) $U = \frac{1}{2} C V^2$

$= 5760\ pJ$

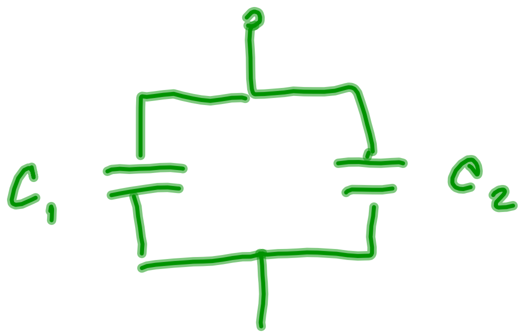
So far we assumed air between plates, but most have insulating materials or dielectrics in between that act to reduce E & therefore V between the plates. The factor by which they decrease is dielectric constant K.

$$C = K \frac{\varepsilon_0 A}{d} \qquad K \sim 2-10$$

Connecting capacitors

- to achieve capacitance or working voltage that might not be available in single capacitor

- 2 simple ways
  - in parallel
  - in series



—||— standard circuit symbol

same ΔV when connected in parallel
(connected to the same wire)

- moving charge from negative to positive

$$C = \frac{Q}{V}$$

$$Q_1 = C_1 V_1 = C_1 \cdot V$$

$$Q_2 = C_2 V_2 = C_2 \cdot V$$

$$Q = Q_1 + Q_2 = C_1 V + C_2 V = (C_1 + C_2) \cdot V$$

$$Q = \qquad C \cdot V$$

In parallel $\quad C = C_1 + C_2 + \ldots\ldots$

C increases

In series - carry the same charge Q, voltages can be different

$$V_1 = \frac{Q}{C_1}$$

$$V = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$V_2 = \frac{Q}{C_2}$$

$$V = \qquad = \frac{Q}{C}$$

In series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\cdots$$