Remember

Potential of point charge $\quad V = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

The zero of potential

- only potential difference has physical significance
- it is convinient to define a zero potential $\Rightarrow$

  we say that potential V at some point P

  is pot. diff. between zero point & P

Assumption – take a zero of potential at $\vec{r} \to \infty$

$$V_{AB} = V(B) - V(A)$$
$$= kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

as $r_A \to \infty$ $\frac{1}{r_A} \to 0$ $V(A) = 0$

$$\boxed{V_{\infty r} = V(r) = \frac{kq}{r}}$$

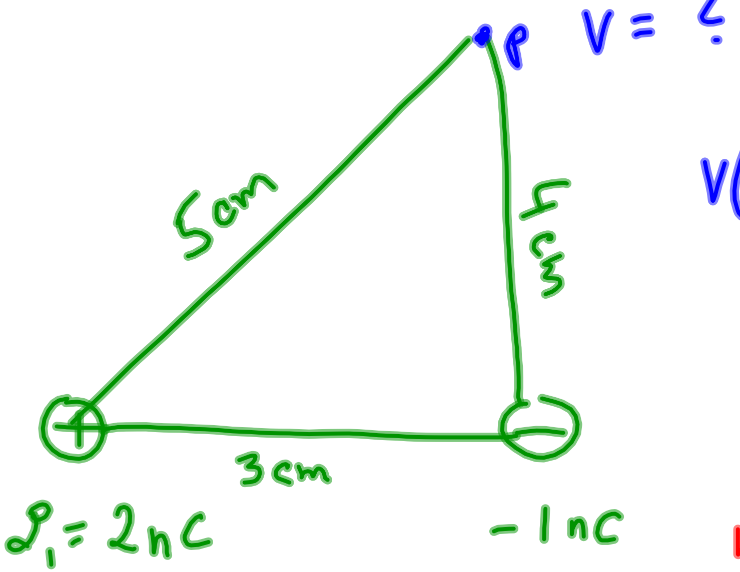
point charge potential

potential difference superposition

$$V(P) = \sum_{i=1}^{N} \frac{k q_i}{r_i}$$

watch for units!

Ex.



$$V(P) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} = 135\ V$$

no angles

A bit on units

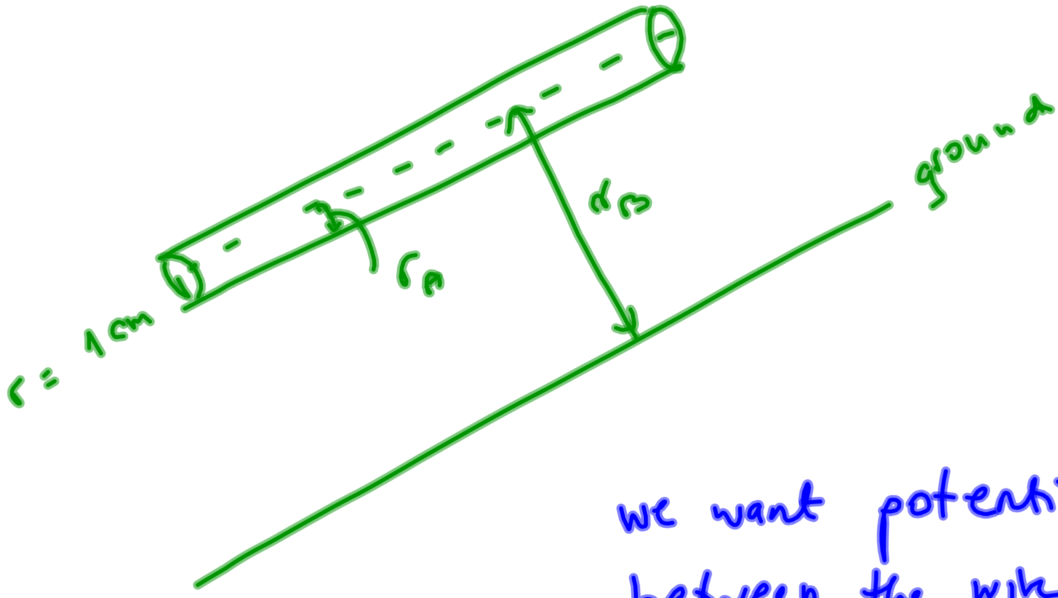
$$V \; [V] \sim \frac{PE}{q} \quad \left[\frac{J}{C}\right] \quad \left[\frac{Nm}{C}\right]$$

$$\frac{N}{C} \overset{?}{=} \frac{V}{m} \qquad \left[\frac{\cancel{\frac{Nm}{C}}^{\,V}}{m}\right] = \left[\frac{V}{m}\right]$$

Ex.

A long, straight power line wire has a radius 1cm & carries the charge density $\lambda = 2.6\,\mu C/m$.

Assuming no other charges are present, what's the potential difference between the wire & the ground 22 m below?



$r_A = 1\,cm =$
$= 0.01\,m$

$r_B = 22\,m$

we want potential difference between the wire surface & the ground

- long, straight wire $\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$

- electric field varies with a position (not uniform) $\Rightarrow$ need an integral

$\vec{r} = \hat{r} \cdot r$

$\hat{r} \cdot \hat{r} = 1$

$$\Delta V = - \int \vec{E} \cdot d\vec{r}$$

$$\Delta V_{AB} = - \int_{r_A}^{r_B} \frac{\lambda \, \hat{r}}{2\pi\varepsilon_0 r} \, d\vec{r} = - \frac{\lambda}{2\pi\varepsilon_0} \int_{r_A}^{r_B} \frac{\hat{r} \, d\vec{r}}{r}$$

dr

$$= - \frac{\lambda}{2\pi\varepsilon_0} \int_{r_A}^{r_B} \frac{dr}{r} = - \frac{\lambda}{2\pi\varepsilon_0} \ln r \Big|_{r_A}^{r_B} =$$

$$\Delta V_{AB} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_B}{r_A} = +\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_A}{r_B} =$$

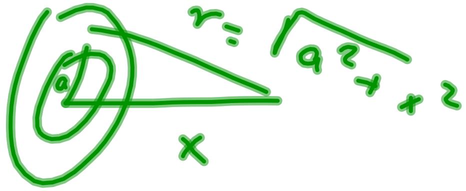
$$\left(\ln r \Big|_{r_A}^{r_B} = \ln r_B - \ln r_A = \ln\frac{r_B}{r_A} \right)$$

$$= -360\ \mathrm{kV}$$

Ex:1.

$$\boxed{E_x = \frac{kQx}{(x^2+a^2)^{3/2}}}$$

we defined as E_z



$u = x^2 + a^2$

$du = 2x\,dx$

$x\,dx = \frac{du}{2}$

$V = ?$

$$V = -\int E_x\,dx = -kQ\int \frac{x}{(x^2+a^2)^{3/2}}dx =$$

$$= -kQ\int \frac{du}{2u^{3/2}} = -\frac{kQ}{2}\int \frac{du}{u^{3/2}} = [u^{-3/2}]$$

$$= -\frac{kQ}{2}\frac{u^{-3/2+1}}{-3/2+1} + C = -\frac{kQ}{\cancel{2}}\frac{u^{-1/2}}{-1\cancel{/2}} + C = kQu^{-1/2}$$

$$\boxed{V = kQu^{-1/2} = kQ\frac{1}{\sqrt{x^2+a^2}}}$$

Finding the electric field from potential

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$$E = ?$$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{x^2 + a^2}}$$

$$\vec{E} = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{x^2+a^2}}\right)\hat{i} + \cancel{\frac{\partial}{\partial y}\left(\frac{1}{\sqrt{x^2+a^2}}\right)\hat{j}} + \cancel{\frac{\partial}{\partial z}\left(\frac{1}{\sqrt{x^2+a^2}}\right)\hat{k}} \right] = \frac{-Q}{4\pi\varepsilon_0} \frac{-\cancel{2}x\left(\cdot\frac{1}{\cancel{2}}\right)}{(x^2+a^2)^{3/2}} =$$

$$E_x = \frac{Qx}{4\pi\varepsilon_0 (x^2+a^2)^{3/2}} = k\frac{Qx}{(x^2+a^2)^{3/2}}$$

$$V = 2x^2 + 3y^3 + 5z^2$$

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right)$$

$$= -\left[4x\hat{i} + 9y^2\hat{j} + 10z\hat{k} \right)$$