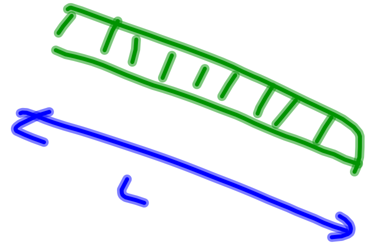
Uniformly charged rod $\qquad \lambda = \frac{Q}{L}$



$$E_{rod} = \frac{1}{4\pi\varepsilon_0} \frac{|Q|}{r\sqrt{r^2 + L^2/4}}$$

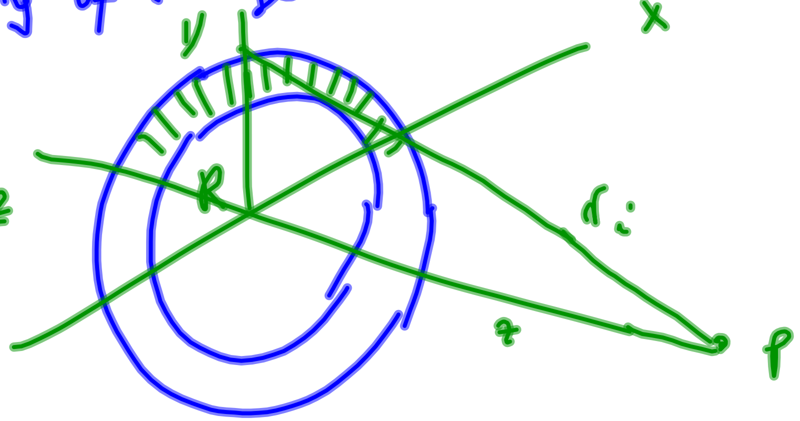
What if the rod is infinite? $L \to \infty$

$$E_{rod} = \frac{1}{4\pi\varepsilon_0} \frac{\overset{\lambda}{|Q|}}{r \cdot L/2} \frac{1}{\sqrt{\underbrace{\cancel{\frac{r^2}{L^2/4}}}_{0} + 1}}$$

$L \to \infty$

$$E_{line} = \frac{1}{4\pi\varepsilon_0} \frac{2|\lambda|}{r}$$

Ring of charge



$$\left(E_{ring}\right)_z = \frac{1}{4\pi\varepsilon_0} \frac{z\,Q}{\left(z^2 + R^2\right)^{3/2}}$$

↑

if point P is on axis z

For N rings => disk

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

$$E_{disk} = \frac{\eta}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{1 + R^2/z^2}}\right]$$

$z \to \infty \qquad E_{disk} = 0$

$z >> R$ but not $\infty \qquad \frac{R^2}{z^2} << 1$

Binomial approx. $(1+x)^n = 1 + nx \quad x \ll 1$

$$1 - \left(1 + \frac{R^2}{z^2}\right)^{-1/2} = 1 - \left[1 - \frac{1}{2}\frac{R^2}{z^2}\right]$$

$$E_{disk} = \frac{\eta}{2\varepsilon_0} \cdot \frac{1}{2}\frac{R^2}{z^2} = \frac{Q}{2\varepsilon_0 \cdot \pi \cancel{R^2}} \frac{1}{2}\frac{\cancel{R^2}}{z^2} = \frac{Q}{4\pi\varepsilon_0 z^2}$$

point charge

$$E_{disk}(R \to \infty) = \frac{\eta}{2\varepsilon_0} = \text{constant}$$

$$E_{sphere} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \quad \text{for } r \geq R$$

Gauss's law

Electric potential

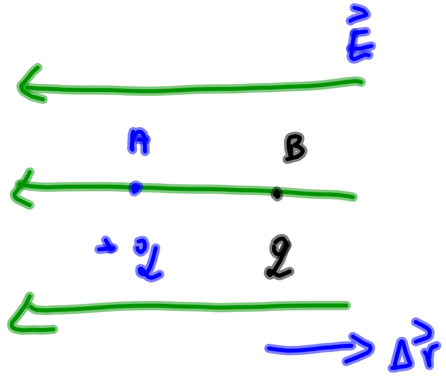
Potential energy difference ΔU_{AB} is the negative of the work W_{AB} done by a conservative force $\vec{F}$ on an object moved from point A to point B (Electric force is conservative)

$$\Delta U_{AB} = U_B - U_A = -W_{AB} = -\int \vec{F} \cdot d\vec{r}$$

- scalar product $\vec{F} \cdot d\vec{r} = |\vec{F}||d\vec{r}| \cos\theta$

- if the force doesn't vary $W = \vec{F} \cdot \Delta\vec{r}$

Ex.



uniform electric field

- move charge $+q$ from A to B

- uniform electric field $\Rightarrow$ constant force

$W = ?$ $\Delta U_{AB} = ?$

$$\Delta U_{AB} = -W_{AB} = -\vec{F} \cdot \Delta\vec{r} = -q\vec{E} \cdot \Delta\vec{r} =$$

$$= -qE\Delta r \cos\theta = [\theta = 180°] = qE\Delta r$$

- pushing a positive charge against the electric field is like pushing a car up the hill. PE increases, if we let go of the charge q goes back to A just like gravity would pull the car back.

– move a charge $2q$

$$\Delta U_{AB} = 2qE\Delta r$$

It is convinient to consider potential energy per unit charge

$$\Delta U_{AB} = -\int_A^B \vec{F}\cdot d\vec{r} = \left[\vec{F} = q\vec{E}\right] = -q\int_A^B \vec{E}\cdot d\vec{r} \quad \Big| : q$$

$$\boxed{\Delta V_{AB} = \frac{\Delta U_{AB}}{q} = -\int_A^B \vec{E}\cdot d\vec{r}}$$

electric potential difference

- special case of uniform electric field

$$\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{r}$$

$$\Delta V_{AB} = -\vec{E} \cdot \Delta\vec{r}$$

- special case when $\vec{E}$ uniform & $\vec{E}$ & $\Delta\vec{r}$ are in opposite directions

$$\Delta V_{AB} = -E\Delta r \cos 180^\circ = E\Delta r$$

- potential difference can be positive & negative
- doesn't depend on path but on 2 points A & B

$V \sim \frac{U}{q}$ $[J/C]$ important enough to get its own unit

$[V]$ Volt

Voltage - potential difference

- the car having 12V battery => it means the battery does 12 J of work on every Coulomb of charge.

- [eV] electronvolts → the energy gained by a particle carrying one elementary charge when it moves through a potential difference of 1 V

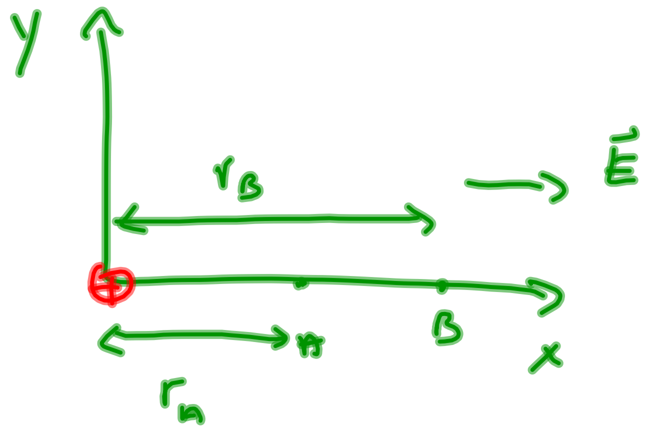
$e = 1.6 \cdot 10^{-19} C$

$1 eV = 1.6 \cdot 10^{-15} J$ not the SI unit

The potential of a point charge

$$\vec{E} = k \frac{q}{r^2} \hat{r}$$

$\Delta V_{AB} = ?$



$$\Delta V_{AB} = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r} =$$

$$= -kq \int_A^B \frac{d\vec{r}}{r^2} \hat{r}$$

$$= -kq \int_A^B \frac{dr}{r^2}$$

As we are moving in $+\hat{r}$ direction

$d\vec{r} = \hat{r}\, dr$

$\vec{E}$ not uniform as it varies with distance

$$\Delta V_{AB} = -kq\left(-\frac{1}{r}\right)\Bigg|_{r_A}^{r_B} = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\frac{1}{r_B} < \frac{1}{r_A}$$

if $r_B > r_A$

$\Delta V_{AB} < 0$ for $q > 0$

> 0 $q < 0$