Continuing with Electric field

Superposition principle applies to $\vec{F}$ & $\vec{E}$

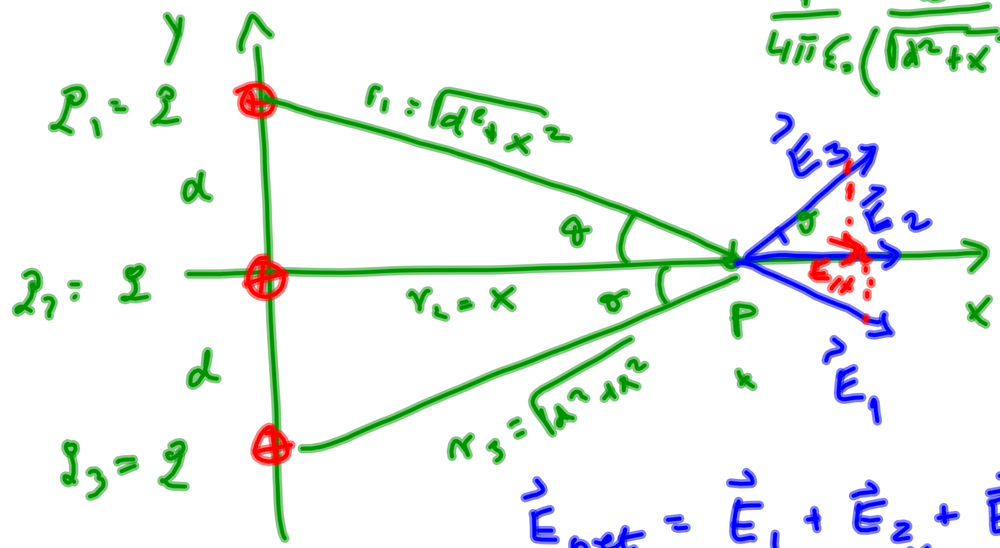
$$\vec{F}_{on\,q} = \vec{F}_{1\,on\,q} + \vec{F}_{2\,on\,q} + \ldots$$

$$\vec{E}_{net} = \frac{F_{1\,on\,q}}{q} + \ldots\ldots$$

$$\vec{E}_1 + \vec{E}_2 + \ldots$$

3 positive equal charges q are located on y axis at $y=0$ & $y=\pm d$. What is the electric field at a point on x axis?

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{\left(\sqrt{d^2+x^2}\right)^2}\cos\theta = E_{1x} = E_{3x}$$



all ys cancel

$$E_{2x} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

$$\vec{E}_{net_x} = \vec{E}_{1_x} + \vec{E}_{2_x} + \vec{E}_{3_x}$$

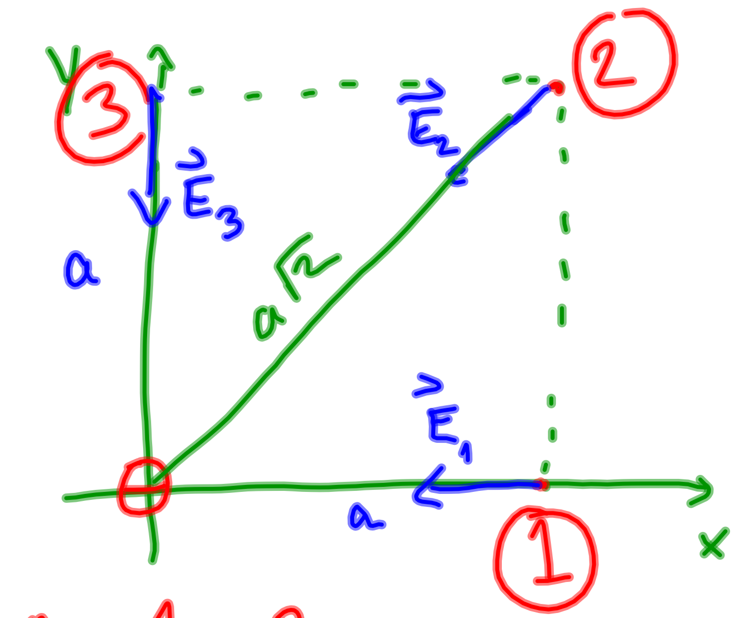
$$E_{1x} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2+d^2}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2+d^2}\frac{x}{\sqrt{x^2+d^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2+d^2)^{3/2}}$$

$$(E_{net})_x = \frac{2}{4\pi\varepsilon_0}\frac{qx}{(x^2+d^2)^{3/2}} + \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

$$(E_{net})_y = 0$$

$$E_{net} = (E_{net})_x$$



$q = -1\ nC$

What is $\vec{E}$ at points 1, 2 & 3

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{|q|}{a^2}\hat{i}$$

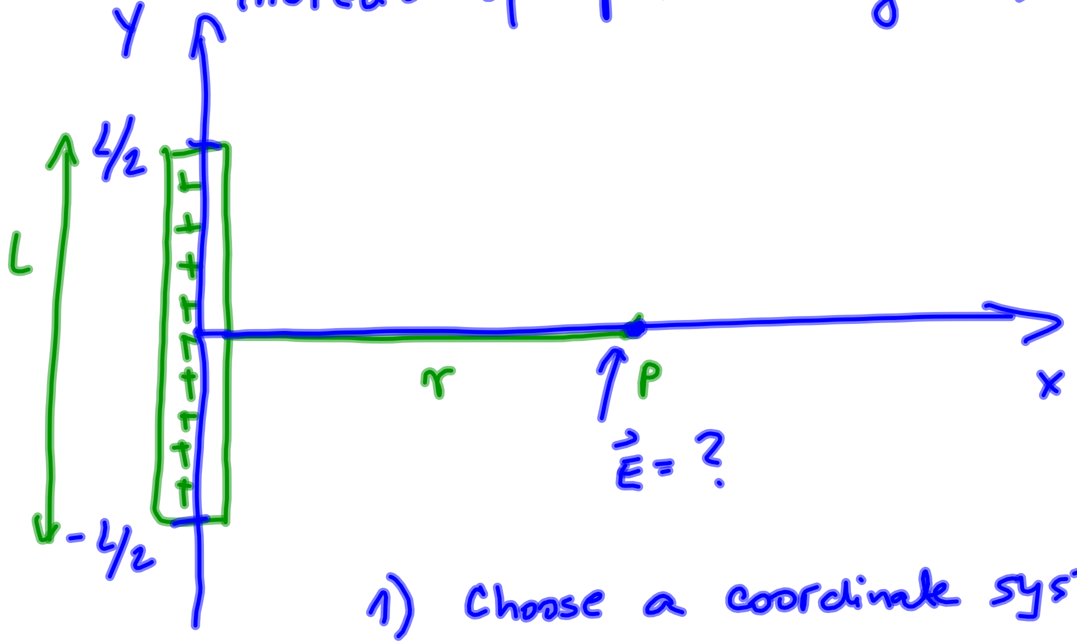
$$\vec{E}_3 = -|\vec{E}_1|\,\hat{j}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{(a\sqrt{2})^2}\cdot\left[-\cos 45^\circ\,\hat{i} - \sin 45^\circ\,\hat{j}\right]$$

$$\vec{E}_2 = -\text{☺}\,\hat{i} - \text{☹}\,\hat{j}$$

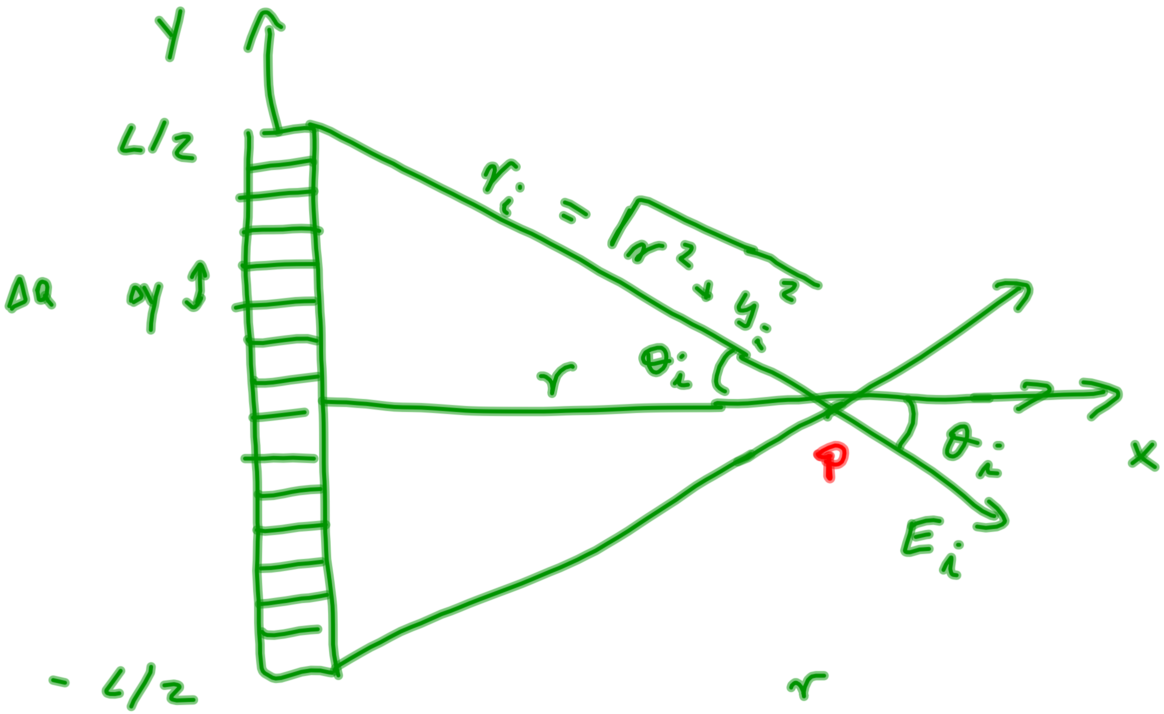
$$|\vec{E}_2| = \sqrt{\text{☺}^2 + \text{☹}^2}$$

Instead of point charge => continuous charge



1) Choose a coordinate system with an origin at the center of the rod

2) Divide rod in N small segments of length Δy & average charge ΔQ



linear charge density

$$\lambda = \frac{Q}{L}$$

2 small segments of length Δy & charge

$$\Delta Q = \lambda \Delta y$$

$$\cos\theta_i = \frac{r}{\sqrt{r^2 + y_i^2}}$$

$$E_{ix} = E_i \cos\theta_i$$

more on step 2 - each of the segments of charge can be modeled as a point charge

Step 3 Choose segment i & draw a field vector of charge & find the symmetric y component so they can cancel out

$$E_{ix} = E_i \cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{r_i^2}\cos\theta_i =$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{r^2+y_i^2}\cdot\frac{r}{\sqrt{r^2+y_i^2}} = \frac{1}{4\pi\varepsilon_0}\frac{r\,\Delta Q}{(r^2+y_i^2)^{3/2}}$$

$$E_x = \sum_{i=1}^{N} (E_i)_x = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{r\,\Delta Q}{(r^2 + y_i)^{3/2}} =$$

$$\Delta Q = \lambda \Delta y = \frac{Q}{L} \Delta y$$

$$\lambda = \frac{Q}{L}$$

$$= \frac{Q/L}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{r\,\Delta y}{(r^2 + y_i)^{3/2}}$$

each segment becomes infinitesimal length $\Delta y \to dy$

y_i becomes contionous variable

$$\boxed{\begin{array}{lcl} N & \rightarrow & \infty \\ \Delta y & \rightarrow & dy \\ y_i & \rightarrow & y \\ \sum_{i=1}^{N} & \rightarrow & \int_{-L/2}^{L/2} \end{array}}$$

$$E_x = \frac{Q/L}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{r\,dy}{(r^2+y^2)^{3/2}}$$

remember r is constant

Table

$$\int\frac{dy}{(r^2+y^2)^{3/2}} = \frac{y}{r^2(r^2+y^2)^{1/2}}$$

$$E_x = \frac{Q/L}{4\pi\varepsilon_0} \frac{\cancel{r}\, y}{r^{\cancel{2}}(r^2+y^2)^{1/2}} \Bigg|_{-L/2}^{L/2} =$$

$$= \frac{Q/\cancel{L}}{4\pi\varepsilon_0 r}\left[\frac{\cancel{L}/2}{(r^2+L^2/4)^{1/2}} - \frac{(-\cancel{L}/2)}{(r^2+L^2/4)^{1/2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{r\sqrt{r^2+L^2/4}}$$

$$E_{rod} = E_x$$