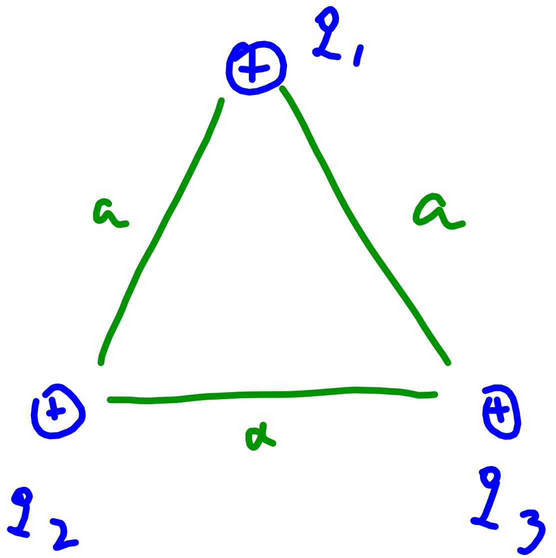
Capacitance & Capacitors

- energy storage of charged conductors

assume: charges are widely spread; we'll bring them closer



what is the energy stored in this configuration?

1) it takes no work to bring in charge q_1 since initially there is no $\vec{E}$ $W_1 = 0$

2) bringing in q_2 means working against q_1's electric field potential of the point charge $V = k\frac{q}{r}$

$$V_1(\text{due to } q_1 \text{ at location where } q_2 \text{ will come}) = k\frac{q_1}{a}$$

- remember that potential V is energy per unit charge ; the work needed to bring in the charge q_2

$$W_2 = q_2 V_1 = k \frac{q_1 q_2}{a} \quad *$$

3) bringing in q_3 -> experiences $\vec{E}$ from both q_1 & q_2

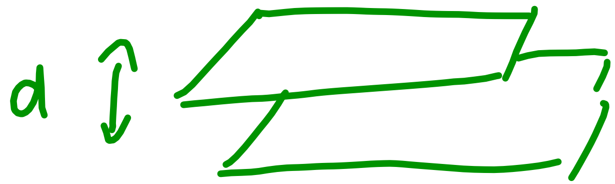
$$W_3 = k \frac{q_1 q_2}{a} + k \frac{q_2 q_3}{a} \quad *$$

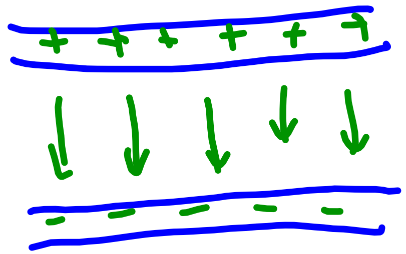
Total work to assemble our charge distribution is

$$W = W_1 + W_2 + W_3 = k\frac{Q_1 Q_2}{a} + k\frac{Q_1 Q_3}{a} + k\frac{Q_2 Q_3}{a}$$

$U \sim W$

Pair of electrical conductors of equal but opposite charges is called a **capacitor.** It is the easiest to analyse parallel plate capacitor





E uniform

$$E_{disk} = \frac{\eta}{2\epsilon_0} \qquad \eta = \frac{Q}{A}$$

$$E = \frac{\eta}{2\varepsilon_0} + \frac{\eta}{2\varepsilon_0} = \frac{\eta}{\varepsilon_0} \qquad \eta = \frac{Q}{A}$$

$$E = \frac{Q}{\varepsilon_0 A}$$ between plates

$$E = 0$$ outside

Potential (E uniform)

$$V = E \cdot d = \frac{Q d}{\varepsilon_0 A} \quad \Big| \cdot \frac{\varepsilon_0 A}{d}$$

$$Q = \frac{\varepsilon_0 A}{d} V$$

C

$$C = \frac{\varepsilon_0 A}{d} \quad \text{CAPACITANCE}$$

$$C = \frac{Q}{V}$$

$Q \sim V$ linearly proportional

coefficient of proportionality depends only on constant ε_0 & geometry of the capacitor (plate area A, spacing between them d

$C = \frac{Q}{V}$ $\left[\frac{C}{V}\right] = [F]$ Farad

in honor of 19^{th} century scientist Michael Faraday

1 F is large capacitance (μF, nF, pF)

Energy storage in capacitors - imagine moving a small charge dQ from negative to positive plate of capacitor when there is V between the plates $\Rightarrow$

the work needed to move the charge $dW = V\,dQ$

the additional charge increases $E \Rightarrow$ increase in dV

from $C = \frac{Q}{V} \Rightarrow dQ = C\,dV$

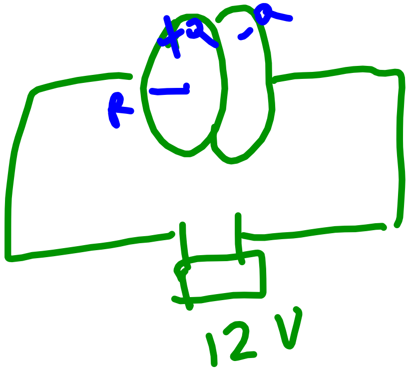
$$dW = C\,dV = V\,dQ = C\,V\,dV$$

$$W = \int dW = \int_0^V C\,V\,dV = \frac{1}{2} C V^2$$

That work is stored as potential energy U

$$\boxed{U = \frac{1}{2} C V^2}$$ energy in capacitor

Ex. A capacitor consists of 2 circular metal plates of radius $R = 12\text{ cm}$ separated by $d = 5\text{ mm}$. Find C, Q, stored energy U when the capacitor is connected to 12 V battery.



$V = 12\ V$

$R = 12 \cdot 10^{-2}\ m$

$d = 5 \cdot 10^{-3}\ m$

$C = ?$

$$C = \frac{\varepsilon_0 A}{d}$$

$A = r^2 \pi$

$Q = CV = 960\ pC$

$C = 80\ pF$

$U = \frac{1}{2} C V^2 = 5760\ pJ$

If we put dielectric (instead of air) between the plates

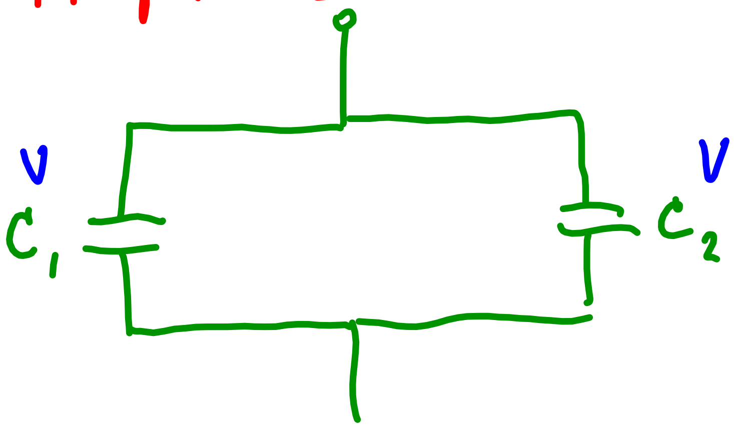
$$C = k \frac{\varepsilon_0 A}{d} \qquad k \sim 2 - 10$$

Connecting capacitors

- to achieve capacitance that might not be available in single capacitor
- 2 simple ways
  - in parallel
  - in series

 standard circuit symbol for C

In parallel



(moving charge from - to +)

Same ΔV when connected in parallel

$Q = C \cdot V$

$C = \frac{Q}{V}$

$Q_1 = C_1 V$

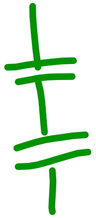
$Q_2 = C_2 V$

$Q = Q_1 + Q_2$

$= (C_1 + C_2) \cdot V$

$C_{eq} = C_1 + C_2 + C_3 + \cdots$

In series $\quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$

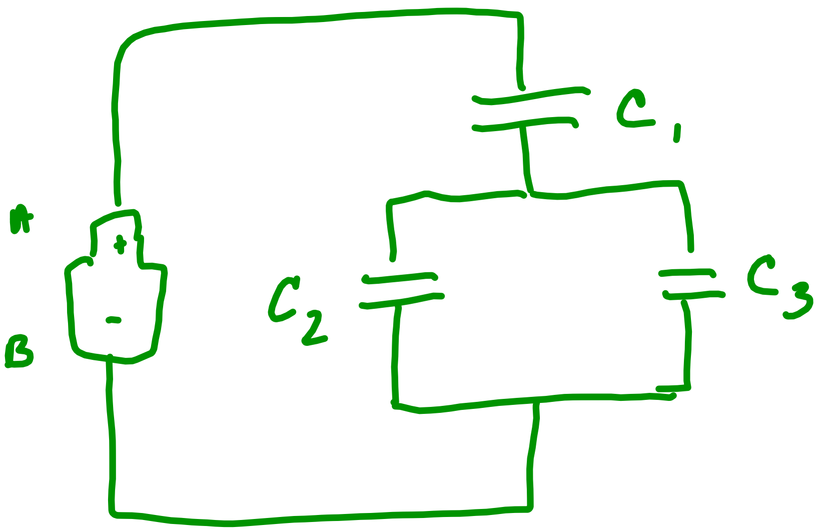


Capacitors in series carry the same charge, voltages are different

$$C = \frac{Q}{V} \qquad V = \frac{Q}{C} \qquad V_1 = \frac{Q}{C_1}$$

$$V_2 = \frac{Q}{C_2}$$

$$V = V_1 + V_2 = Q\underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}_{\frac{1}{C_{eq}}}$$



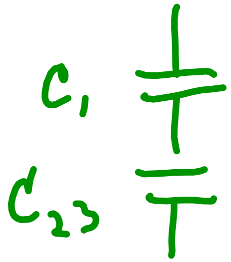
$C_1 = 12\mu F$

$C_2 = 3\mu F$

$C_3 = 1\mu F$

$C = ?$

$\Downarrow$



$C_{23} = C_2 + C_3 = 4\mu F$

$1/C_{123} = 1/C_1 + 1/C_{23}$

$C_{123} = 3\mu F$

What is the voltage on C_1? $V_{AB} = 100\,V$

- start from the end & work back

$C_{123} = 3\mu F$

$V_{AB} = \dfrac{Q_{123}}{C_{123}}$

going to series set-up

the same Q, diff V

$Q_1 = Q_{123}$

$V_{AB} = \dfrac{Q_{123}}{C_{123}} \Rightarrow$

$Q_1 = V_1 C_1$

$V_1 = \dfrac{Q_1}{C_1} = 25V \Leftarrow Q_1 = Q_{123} = 300\mu C$