Electric potential

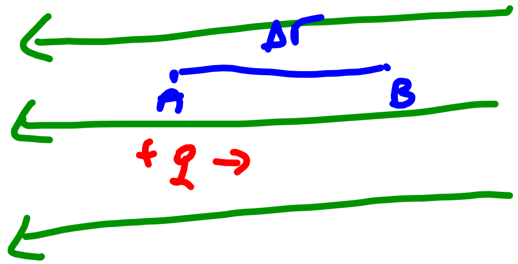
Potential energy difference ΔU_{AB} is the negative of the work W_{AB} done by a conservative force $\vec{F}$ on an object moved from point A to point B.

$$\Delta U_{AB} = U_B - U_A = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{r}$$

$$\vec{F}\cdot d\vec{r} = |\vec{F}||d\vec{r}|\cos\theta$$

$$W_{AB} = \int_A^B \vec{F}\cdot d\vec{r}$$

- if the force doesn't vary $W = \vec{F}\cdot\Delta\vec{r}_{AB}$



move charge $+q$ from A to B

$W = ?$

$\Delta U_{AB} = ?$

$$\Delta U_{AB} = -W_{AB} = -\vec{F}\cdot\Delta\vec{r} = -q\vec{E}\cdot\Delta\vec{r} =$$

(uniform field => constant force)

$$= -qE\Delta r\cos 180^\circ = qE\Delta r$$

makes sense?

Pushing a positive charge against the electric field is like pushing a car up the hill.

PE increases

- if we let go the charge q goes back to A just like the gravity would pull the car down

$$\Delta U_{AB} = q E \Delta r$$

- move charge $2q$

$$\Delta U_{AB} = 2q E \Delta r$$

It is convenient to consider potential – energy per unit charge

$$\Delta U_{AB} = -\int_A^B \vec{F}\cdot d\vec{r} = \left[\vec{F} = q\vec{E}\right] = -q\int_A^B \vec{E}\cdot d\vec{r} \;\Big|\, :q$$

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q} = -\int_A^B \vec{E}\cdot d\vec{r}$$

electric potential difference

The electric potential difference from A to B is the potential energy change per unit charge in moving from A to B.

Special case of uniform field

$$\Delta V_{AB} = -\vec{E} \cdot \Delta\vec{r}$$

Special case when $\vec{E}$ & $\Delta\vec{r}$ are in opposite directions

$$\Delta V_{AB} = E\Delta r$$

- potential diff. can be positive & negative
- doesn't depend on the path but on 2 points A & B

$$V \sim \frac{U}{Q} \quad \left[\frac{J}{C}\right]$$

VOLTAGE
potential difference

however V is important enough to have its own unit [V] Volt

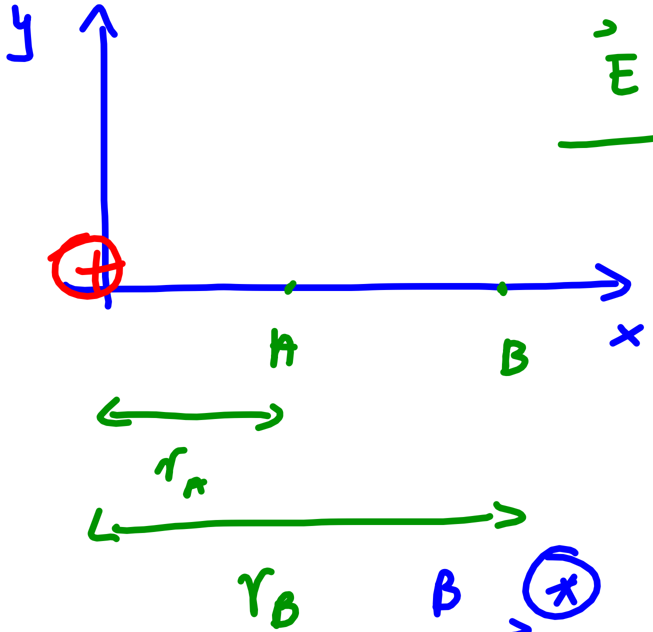
- the car having 12 V battery => it means the battery does 12 J of work on every Coulomb of charge

The potential of point charge

$$\vec{E} = k\frac{q}{r^2}\hat{r} \quad (*)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$\Delta V_{AB} = \,?$$



$\vec{E}$ not uniform $\longrightarrow$ varies with distance

$$\Delta V_{AB} = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{r} \quad (*)$$

$$\Delta V_{AB} = -kq \int_A^B \frac{d\vec{r}}{r^2}\,\hat{r}$$

we are moving in $\hat{r}$ direction

$$d\vec{r} \cdot \hat{r} = dr$$

$$= -kq \int_A^B \frac{dr}{r^2}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$= (-)kq\left((-)\frac{1}{r}\right)\Bigg|_A^B$$

$$= kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$r_B > r_A \quad q > 0$

$\Delta V_{AB} < 0$

The zero of potential

- only potential differences have physical significance
- it is convenient to define a zero potential => we say that potential V at some point P is potential difference between zero point & P

$$V_{AB} = V(B) - V(A)$$

- earth "grounded" zero potential

Assumption: take zero of potential at ∞ as

$$V \sim \frac{1}{r} \qquad r_A \to \infty, \; \frac{1}{r_A} \to 0$$

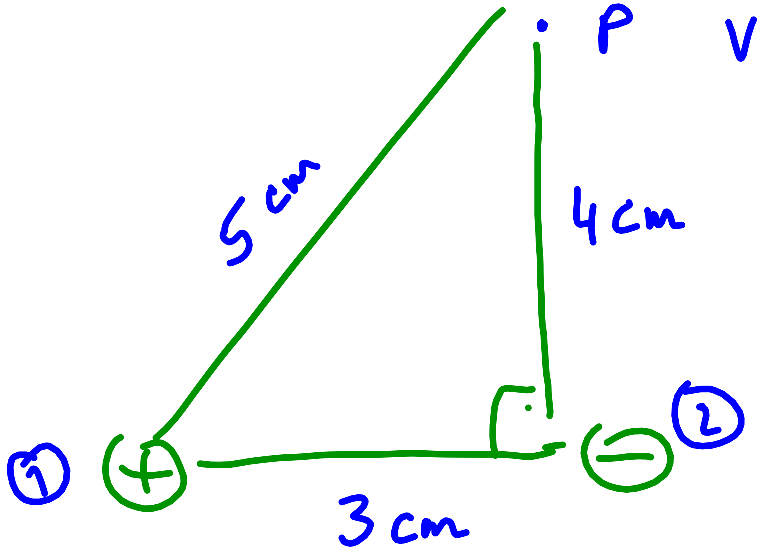
$$V(A) = 0$$

$$V_{\infty r} = V(r) = \frac{kq}{r}$$ point charge potential

what if we have more than 1 charge?

$$V(P) = \sum_{i=1}^{n} \frac{kq_i}{r_i}$$ potential difference

superposition



$V(P) = ?$

2 nC   −1 nC

$$V = k\frac{q_1}{r_1} + k\frac{q_2}{r_2}$$

$$= 135\ V$$

$q_1 = 2 \cdot 10^{-9}\ C$ $r_1 = 5 \cdot 10^{-2}\ m$

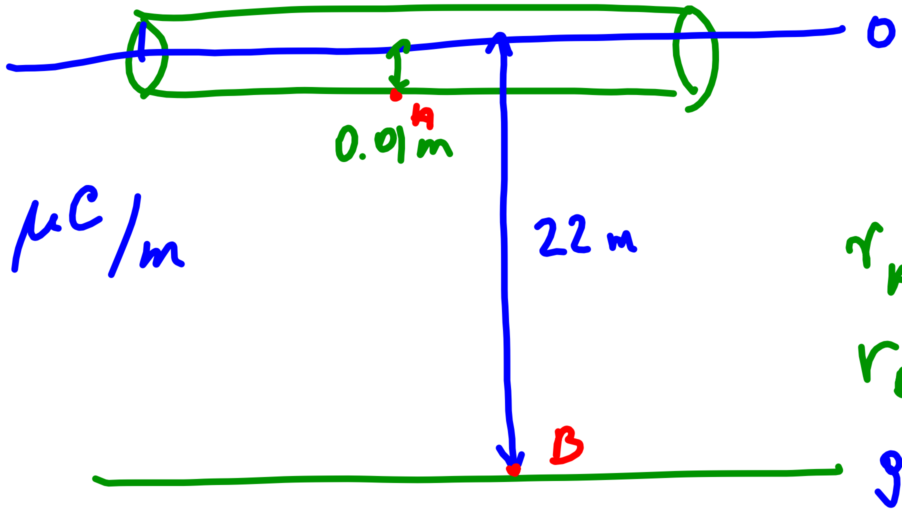
$q_2 = -1 \cdot 10^{-9}\ C$ $r_2 = 4 \cdot 10^{-2}\ m$

Potential difference for rod

line charge density λ

Surface ... η

$r = 1\,\text{cm}$



$\lambda = 2.6\,\mu C/m$

$r_A = 0.01\,m$

$r_B = 22\,m$

$$\Delta V_{AB} = -\int \vec{E} \cdot d\vec{r} \qquad d\vec{r} = \hat{r}\, dr$$

for very long rod $\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$

$$\Delta V_{AB} = -\int_{r_A}^{r_B} \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}\cdot\hat{r}\, dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{dr}{r} =$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\ln r\Big|_{r_A}^{r_B} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_A}{r_B} = -360\ \text{kV}$$

Finding the electric field from potential

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right]$$

$$V = \text{☺} \cdot x^2 \cdot \text{☹} \cdot \ln z \cdot y^2 \cdot \ln y \cdot e^{z^2}$$

$$E_x = -\frac{\partial V}{\partial x}\hat{i} = -2x \cdot \ \ldots \ldots$$