Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Our goal is to understand

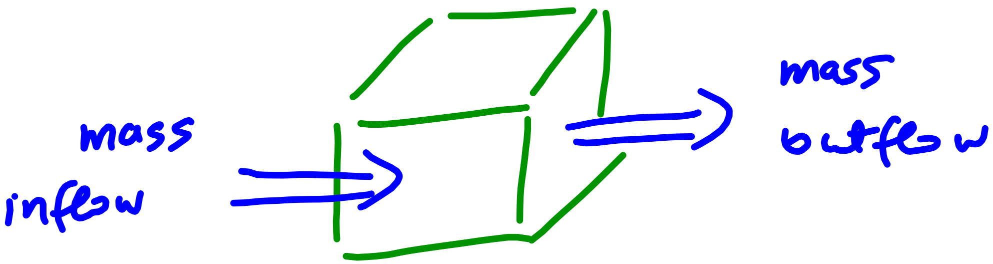
Gauss (1777 - 1855)

Gauss's law – more elegant than Coulomb's law
- more general

Coulomb's law – sum or integral for all present charges
- complicated

To understand Gauss's law we need to understand FLUX

Flux



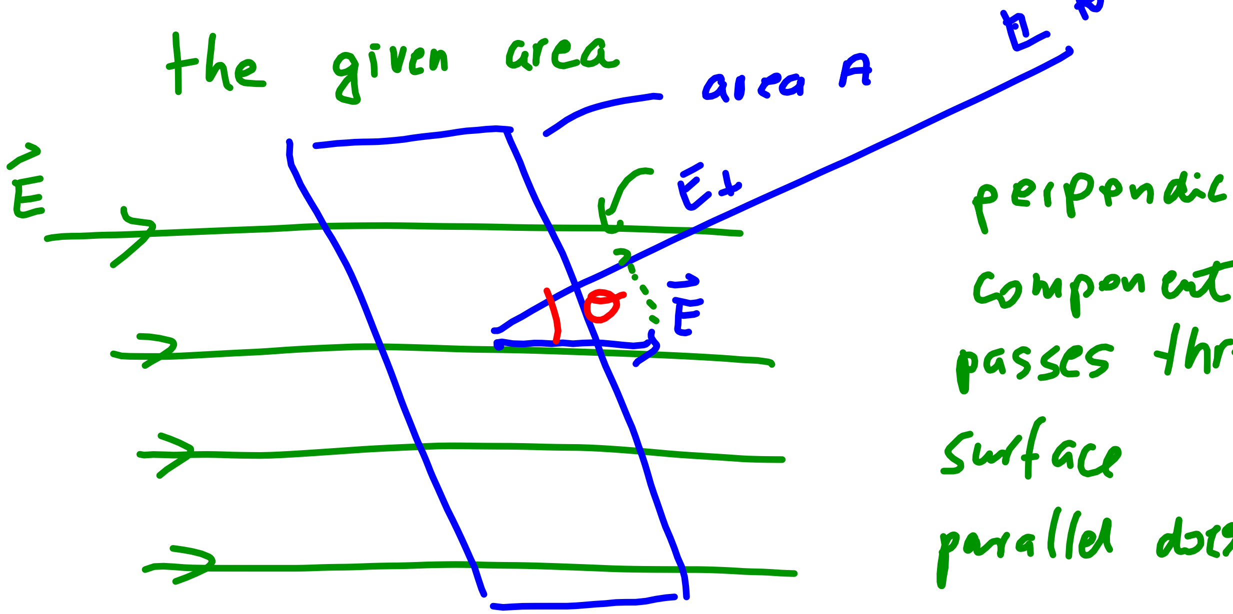
mass flux $\simeq$ outflow - inflow

net flux

Electric flux

ϕ_E electric field passing through the given area

uniform electric field



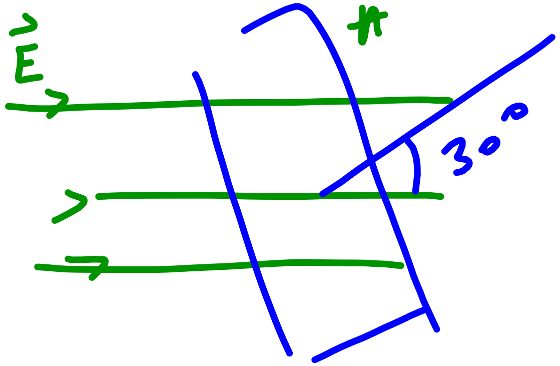
perpendicular component passes through surface

parallel doesn't

$$E_\perp = \vec{E}\cos\theta \qquad A_\perp = A\cos\theta$$

$$\Phi_E = E_\perp A = \vec{E} A \cos\theta$$
$$= \vec{E} \cdot \vec{A}$$

Ex. Calculate the electric flux through the rectangle that is 10 cm x 20cm if the electric field is uniform $E = 200\,N/C$ & angle is 30°

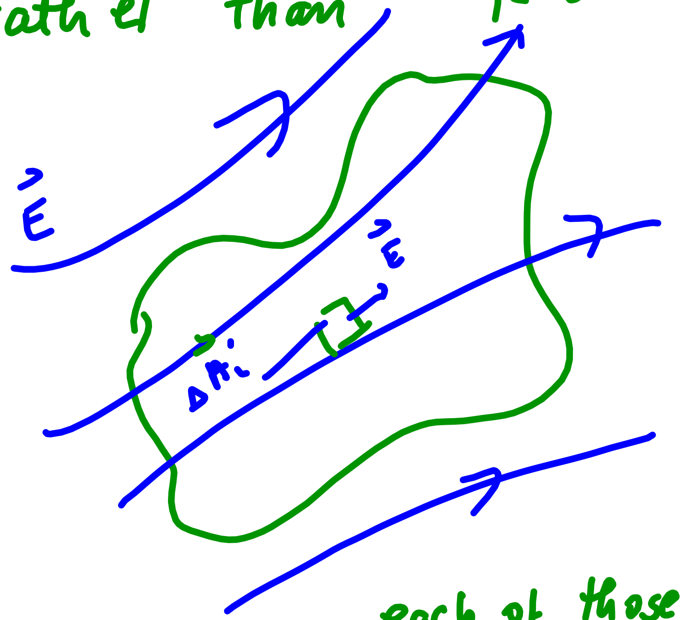


$$\Phi_E = EA\cos\theta$$

$$= 200\,N/C \cdot 0.2 \cdot 0.1\,m^2 \cdot \cos 30^\circ$$

$$= 3.5\,Nm^2/C$$

Imagine now that the surface is curved rather than flat



$\vec{E}$ is not uniform

- we need to divide the chosen surface into n small elements with surface areas $\Delta A_1, \Delta A_2 \ldots$

- each of those elements are small enough that can be considered as flat & E uniform

$$\phi_E \cong \sum_{i=1}^{N} \vec{E}_i \cdot \Delta\vec{A}_i$$

$$\Delta A_i \to 0 \qquad \Sigma \to \int$$

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

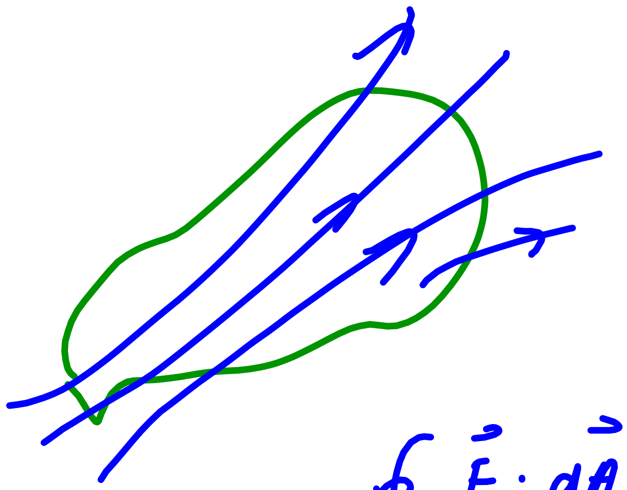
total flux through a closed surface of any shape that completely encloses a volume

Flux entering the enclosed volume is negative.

Flux exiting the . . . . . is positive.

Net flux $\Phi_E = \oint \vec{E} \cdot d\vec{A}$

positive if net flux is coming out

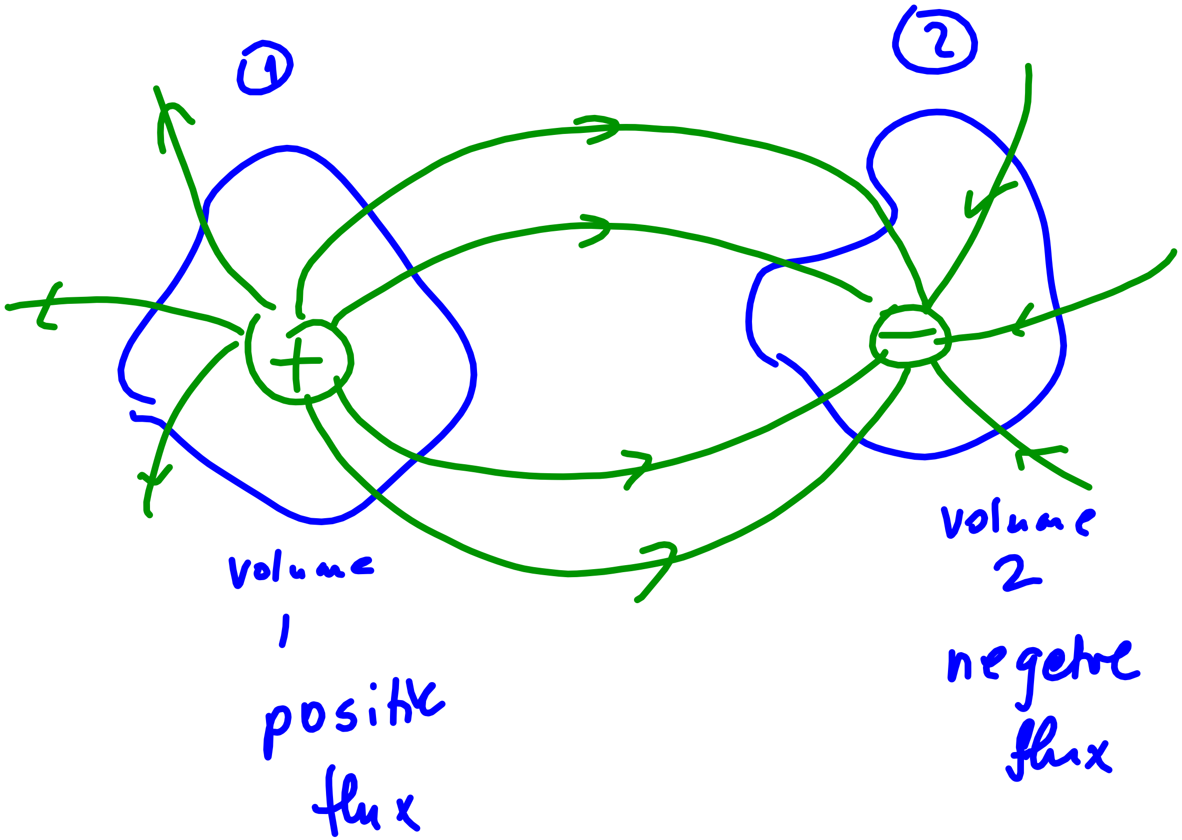


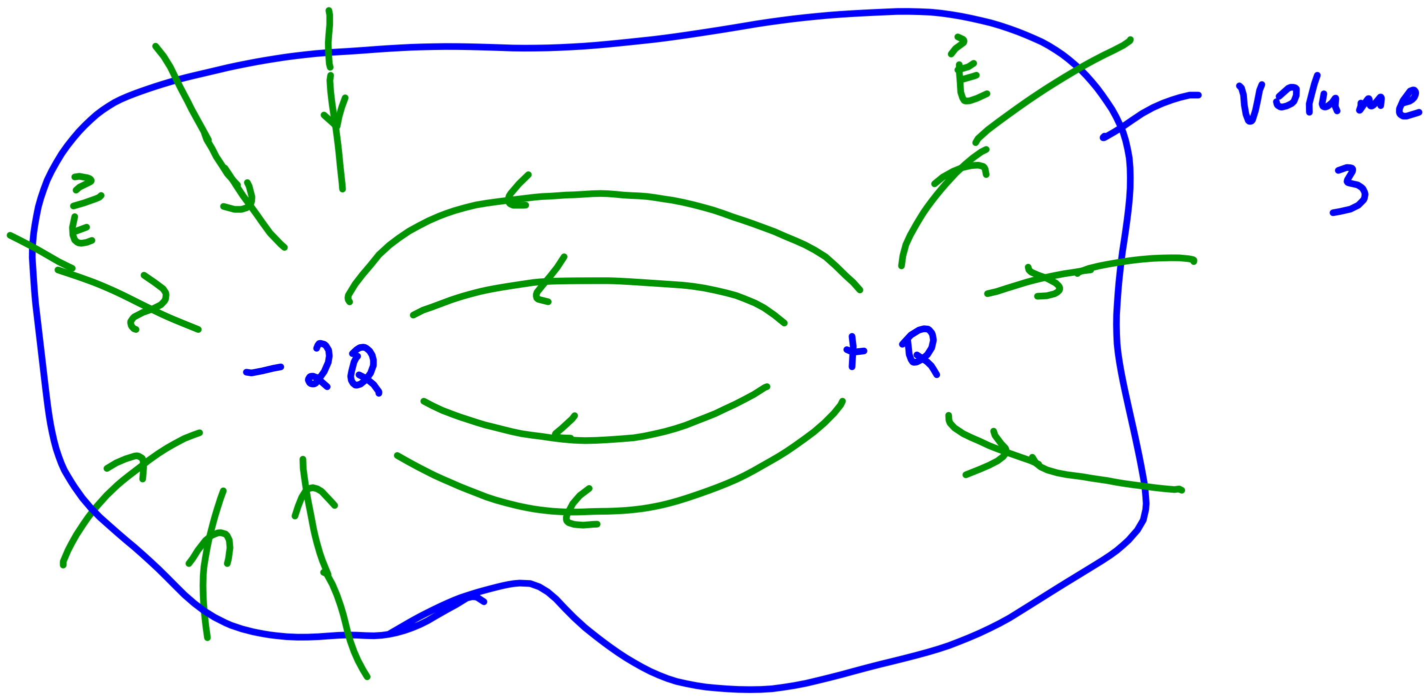
flux in = flux out

flux is 0

$\oint \vec{E} \cdot d\vec{A} \neq 0$ if one or more lines start or end within the surface

Since the electric field lines start or stop only on electric charges, the flux will be nonzero only if the surface encloses a net charge.


1
2
+
−
volume 1
positive flux
volume 2
negative flux



flux

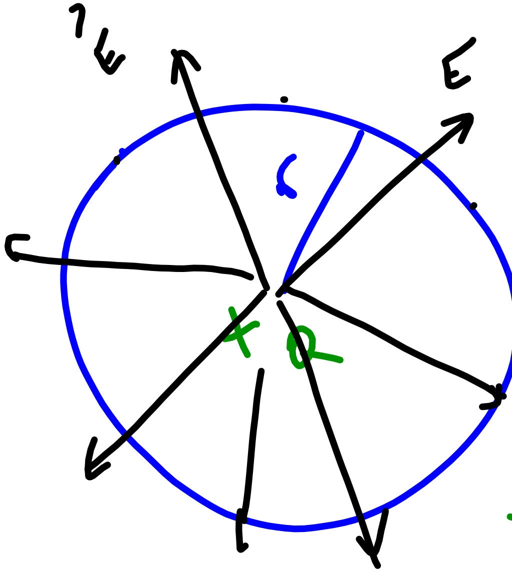
more lines $\vec{E}$ coming in than going out $\Rightarrow$ flux is negative

The value of ϕ_E depends on the charge enclosed by the surface & that is what Gauss's law is all about!

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$Q_{enclosed}$ – net charge

Point charge



Choose an imaginary sphere of radius r (gaussian sphere) with a center where the charge is

sphere - $\vec{E}$ will have the same magnitude at any point of sphere

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$\oint dA = 4\pi r^2$ for the sphere

$\vec{E} \cdot d\vec{A}$ parallel $\cos\theta = 1$ $(\theta = 0°)$

$Q_{enclosed} = Q$

$$\oint E\,dA = E \oint dA = \boxed{E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

If we have more than one charge

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

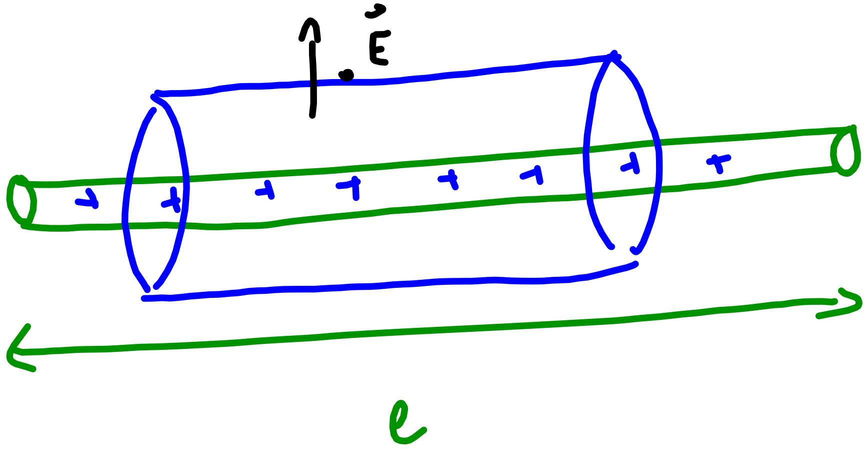
$$\left.\begin{aligned} \vec{E} &= \sum \vec{E}_i \\ Q_{enclosed} &= \sum Q_i \end{aligned}\right\} \quad \oint \left(\sum E_i \right) \cdot d\vec{A} = \sum \frac{Q_i}{\varepsilon_0} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss's law holds even for fields produced by changing magnetic field

Long uniform line of charge

Calculate $\vec{E}$ near but outside the wire far from its ends.

at the ends $\vec{E}$'s parallel to the ends



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$Q_{encl} = \lambda \ell$$

$$\left(\lambda = \frac{Q}{\ell} \right)$$

surface

$$Q_{encl} = \eta A$$

$$E \oint d\vec{A} = \frac{\lambda \ell}{\varepsilon_0}$$

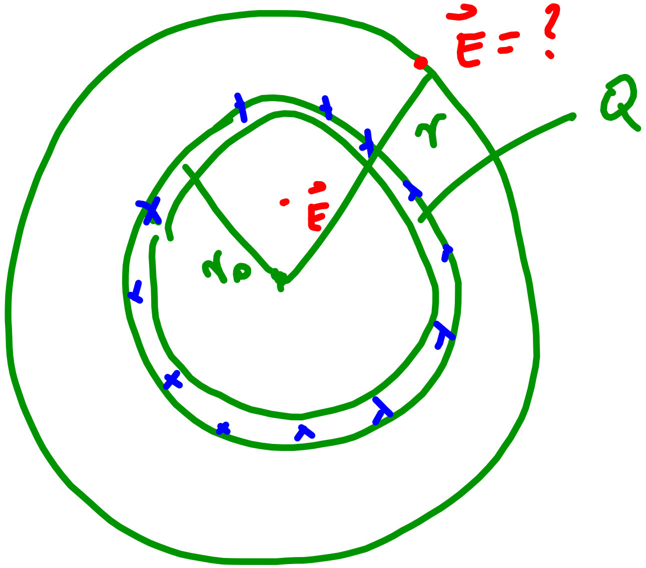
$$E \cdot 2\pi R \cancel{\ell} = \frac{\lambda \cancel{\ell}}{\varepsilon_0}$$

$$\oint d\vec{A} = 2\pi R \ell$$

for cylinder

$$\boxed{E = \frac{\lambda}{2\pi \varepsilon_0 R}}$$

Spherical conductor



thin spherical conductor (shell)

a) $\vec{E}$ outside the shell

$r > r_0$

b) inside the shell

$r < r_0$

a) gaussian surface - sphere

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$Q = \lambda \ell$$
$$= \eta A$$
$$= Q$$

$$E \underbrace{\oint d\vec{A}}_{4\pi r^2} = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad r > r_0$$

b) inside the shell $r < r_0$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = 0$$

$r < r_0$

$E = 0 \quad r < r_0$

Solid sphere problems – a) & b) applies