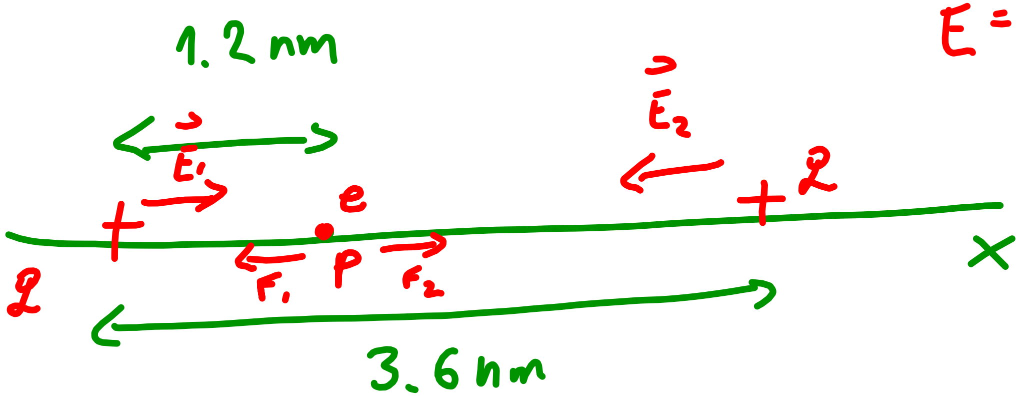
Electric field

Ex. 2 protons are 3.6 nm apart. Find the electric field at a point between them, 1.2 nm from one of the protons. Then find the force on an electron at this point.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$



$$r_1 = 1.2 \cdot 10^{-9}\ m$$

$$r_2 = 2.4 \cdot 10^{-9}\ m$$

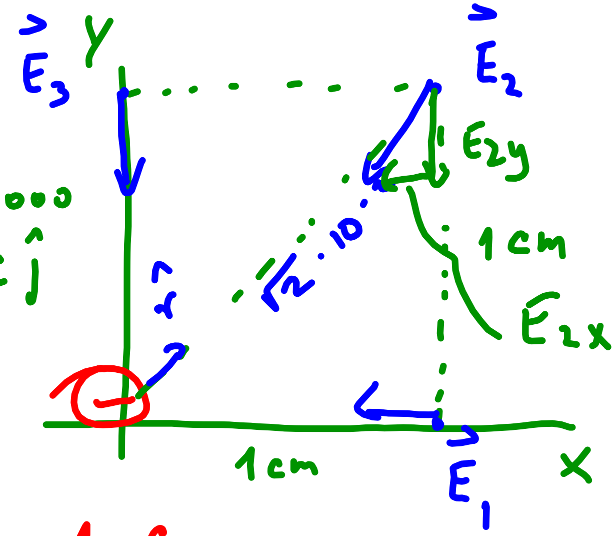
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r_1^2}\hat{i} + \frac{Q}{r_2^2}\cdot(-\hat{i})\right]$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left(\frac{\hat{i}}{r_1^2} - \frac{\hat{i}}{r_2^2}\right) = 750\,\hat{i}\ \text{MN/C}$$

$M \quad 10^6$

$$\vec{F} = q\vec{E} = -e\vec{E} = -0.12\,\hat{i}\ \text{nN}$$

e



$\vec{E}_3 = -90000$ N/C $\hat{j}$

$q = -1\,nC$

$x \quad \hat{i}$

$y \quad \hat{j}$

$z \quad \hat{k}$

$\hat{r}$ from charge toward the point

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}\hat{r}$$

$$= 9\cdot 10^9\, Nm^2/C^2 \cdot \frac{-1\cdot 10^{-9}\, C}{(0.01\,m)^2}\hat{r} =$$

$$= -90000\, N/C\, \hat{i}$$

q + or − already says $\hat{r}$ direction

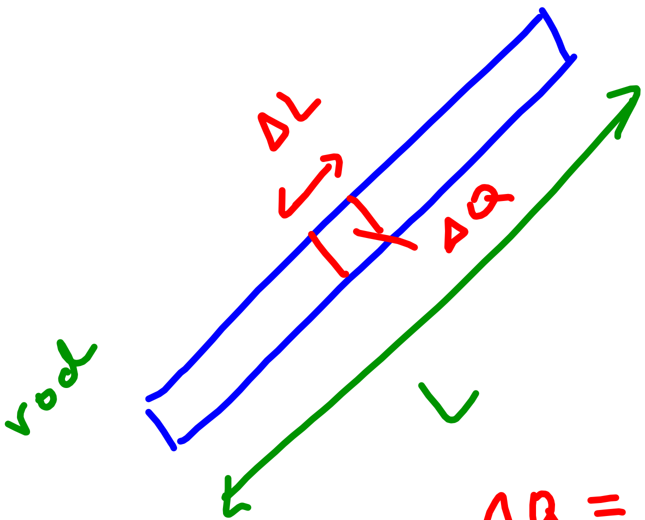
$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2^{\,2}}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r_2^{\,2}}\left(-\cos 45^\circ\,\hat{i} - \sin 45^\circ\,\hat{j}\right)$$

$$|\vec{E}_2| = 45\,000\ \text{N/C}$$

$$r_2 = 0.0141\ \text{m}$$

$$\vec{E}_2 = -45\,000\ \text{N/C}\cos 45^\circ\,\hat{i} - 45\,000\ \text{N/C}\sin 45\,\hat{j}$$

The electric field of continous charge distribution



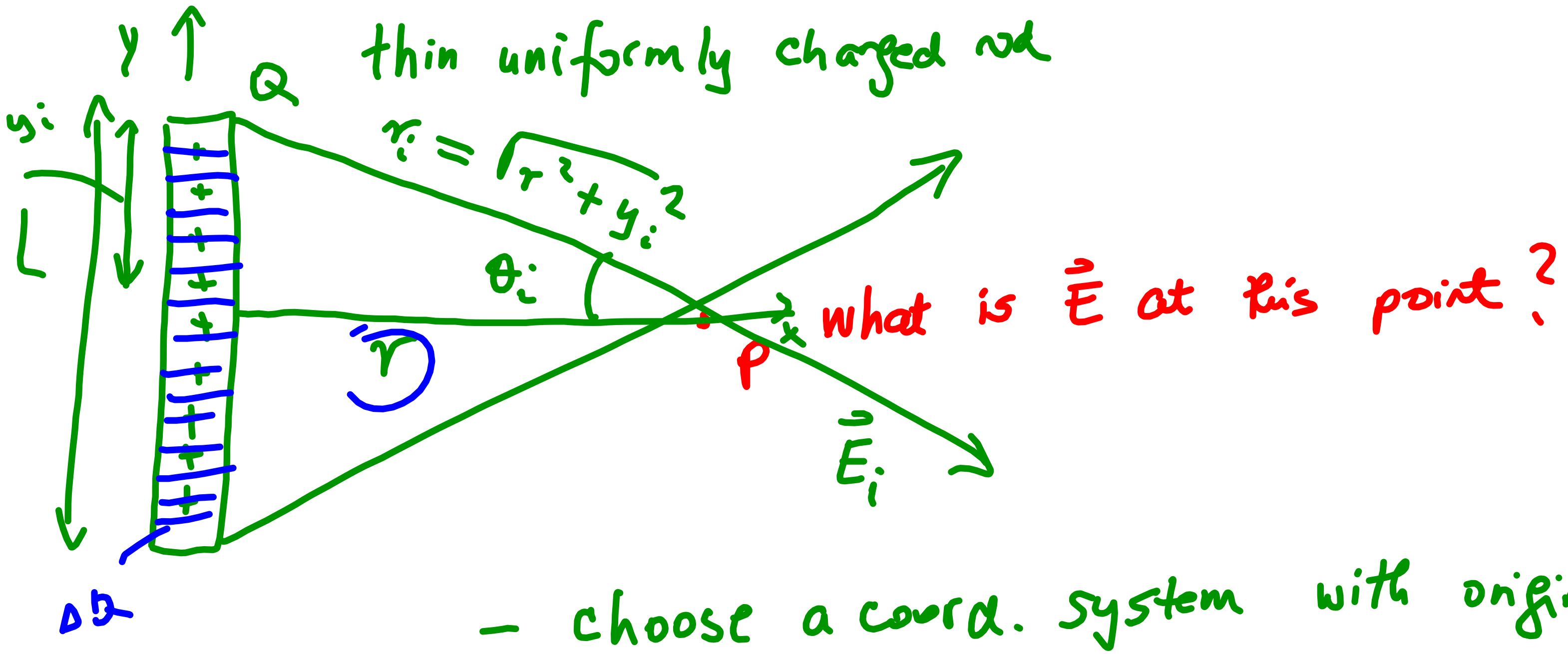
linear charge density

$$\lambda = \frac{Q}{L} \quad [C/m]$$

amount of charge per 1 m of length

$$\Delta Q = \lambda \Delta L$$

thin uniformly charged rod



- choose a coord. system with origin at the center of the rod
- divide rod into N small segments of length Δy & charge $\Delta Q = \lambda \Delta y$

$$\Delta Q = \lambda \Delta y$$

- choose a segment i & draw the field vector for that segment

we use symmetry to cancel out all E_ys

Each of the segments of charge can be modeled as a point charge!

$$(E_i)_x = E_i \cos\theta_i = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{r_i^2}\cos\theta_i$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{r^2+y_i^2}\cdot\frac{r}{\sqrt{r^2+y_i^2}} = \frac{1}{4\pi\varepsilon_0}\frac{r\,\Delta Q}{(r^2+y_i^2)^{3/2}}$$

$$E_x = \sum_{i=1}^{N}(E_i)_x = \frac{1}{4\pi\varepsilon_0}\sum_i^N \frac{r\,\Delta Q}{(r^2+y_i^2)^{3/2}}$$

$$\boxed{\Delta Q = \lambda\,\Delta y = \frac{Q}{L}\Delta y}$$

$$E_x = \frac{Q/L}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{r\,\Delta y}{(r^2 + y_i^2)^{3/2}}$$

$\Delta y \rightarrow dy$

$y_i \rightarrow y$

$N \rightarrow \infty$

each segment becomes infinitesimal length

$\Delta y \rightarrow dy$

y_i becomes continous integration variable y

$$\sum_{i=1}^{N} \Longrightarrow \int_{-L/2}^{L/2}$$

r is a constant

$$E_x = \frac{Q/L}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{r\,dy}{(r^2+y^2)^{3/2}}$$

$$\int \frac{dy}{(r^2+y^2)^{3/2}} = \frac{y}{r^2(r^2+y^2)^{1/2}}$$

$$E_x = \frac{Q/L}{4\pi\varepsilon_0} \frac{\cancel{r}\, y}{r^{\cancel{2}}(r^2+y^2)^{1/2}} \Bigg|_{-L/2}^{L/2}$$

$$= \frac{Q/\cancel{L}}{4\pi\varepsilon_0 r} \left[\frac{\cancel{L}/2}{(r^2+L^2/4)^{1/2}} - \frac{-\cancel{L}/2}{(r^2+L^2/4)^{1/2}} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{r\sqrt{r^2+L^2/4}} = E_{rod}$$

$$E_{rod} = E_x = \frac{1}{4\pi\varepsilon_0}\,\frac{|Q|}{r\sqrt{r^2 + L^2/4}}$$

What if $L \to \infty$ infinite line of charge

$$E_{rod} = \frac{1}{4\pi\varepsilon_0}\,\frac{|Q|}{r\cdot L/2}\,\frac{1}{\sqrt{1 + r^2/{L^2/4}}} \rightarrow 0$$

$$= \frac{1}{\cancel{4}\pi\varepsilon_0}\,\frac{\cancel{2}|\lambda|}{r}$$

$$\lambda = \frac{Q}{L}$$

$$E = \frac{|\lambda|}{2\pi\varepsilon_0 r}$$

Charge $E \sim \frac{1}{r^2}$

infinite rod $E \sim \frac{1}{r}$