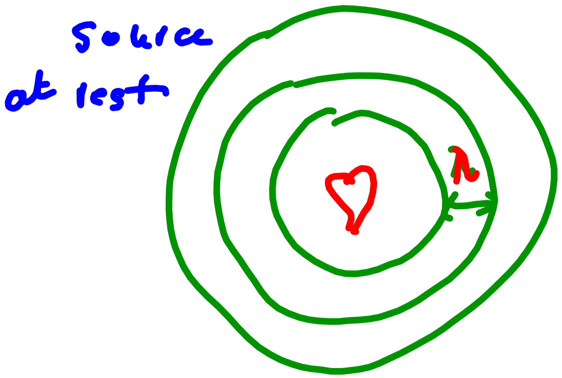
Doppler effect

An interesting effect that occurs when the wave source is moving relative to the observer or the other way around.

Pitch of ambulance siren drops
police
as it passes you by.

If the source is at rest & v is the speed of sound (wave speed) in the medium, we get uniform radiation of waves

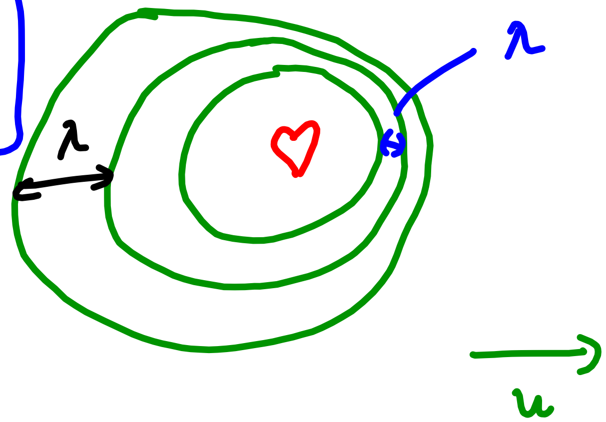


But the source moves, wave crests are bundled in the direction in which the source is moving ; λ decreases.

moving source

λ bigger
v the same
f lower

B



A

λ smaller
v is the same
f higher

v does not change

$$v = \lambda \cdot f$$

λ changes
f has to change

This change is called Doppler effect after the Austrian physicist (19th century)

Let's analyse

λ wavelength when source is stationary

λ' - " - moving at speed u

through a medium where the wave speed is $v = 343$ m/s

The stationary source – period T, one λ in T

The moving source will do 2 things:

- emit λ
- move the distance ut in the same time period

Therefore the distance between wavecrests as seen by an observer in front of the moving source is $\lambda' = \lambda - ut$

$\lambda' = \lambda - ut$ ← time → T

$\lambda' = \lambda - uT$

$T = \frac{\lambda}{v}$

$v = \frac{\lambda}{T}$

$\lambda' = \lambda - \frac{u}{v}\lambda$

$\lambda'_a = \lambda\left(1 - \frac{u}{v}\right)$

Source approaching the observer

λ stationary source

λ' moving -"-

u speed of the police car

$v = 343$ m/s

$\lambda'_{rt} = \lambda\left(1 + \frac{u}{v}\right)$ source is receding

Written in terms of frequency:

$\lambda = \frac{v}{f}$ $\quad$ $\lambda' = \frac{v}{f'}$

f' frequency of waves from moving source

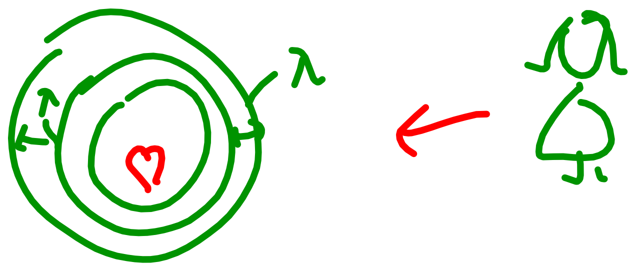
$\frac{v}{f'} = \frac{v}{f}\left(1 + \frac{u}{v}\right) \quad \Big| \cdot \frac{f'}{f}\frac{1}{v}$

$f = f'\left(1 + \frac{u}{v}\right)$

$$f' = \frac{f}{1 + u/v}$$ for receding source

$$f' = \frac{f}{1 - u/v}$$ approaching source

A Doppler shift in f also occurs when a moving observer approaches a stationary source



- an observer passes wave crests more often than she would have if she were at rest → λ smaller, f higher

$f' = f\left(1 + \frac{u}{v}\right)$ observer is approaching a source

$f' = f\left(1 - \frac{u}{v}\right)$ observer is moving away from the source

Example

A police siren has frequency $f = 550$ Hz as the car approaches you; 450 Hz after it passes. How fast is the police car traveling?

$$f_a' = \frac{f}{1 - \frac{u}{v}} \qquad f_r' = \frac{f}{1 + \frac{u}{v}}$$

$u = ?$

$f_a' = 550\ Hz$

$f_r' = 450\ Hz$

$f = f_r'\left(1 + \frac{u}{v}\right)$

$u = 34.3\ m/s$

$f = f_a'\left(1 - \frac{u}{v}\right)$

not half way → $f = 495\ Hz$

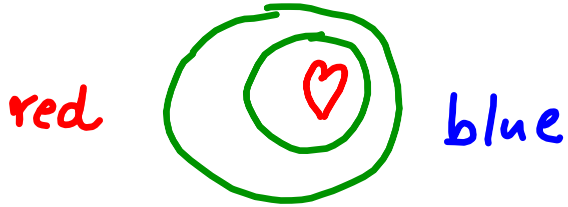
The Doppler effect for light waves

- the same thing happens as a source of light is receding from you: λ_r is longer than λ
- one main difference; sound propagates through medium & wave speed is relative to the medium BUT electromagnetic waves don't need a medium

- we use Einstein theory

$$\lambda_r = \sqrt{\frac{1 + u/c}{1 - u/c}}$$

$$\lambda_a = \sqrt{\frac{1 - u/c}{1 + u/c}}$$



Doppler effect & shift in light → Big Bang theory