Standing waves

- Superimpose (add) 2 waves traveling in opposite directions & reflecting at the ends

① $y_1(x,t) = A\cos(kx - \omega t)$ + x direction

② $y_2(x,t) = -A\cos(kx + \omega t)$ (−A) it is reflected

Superimposed :

$$y(x,t) = y_1(x,t) + y_2(x,t) = A\left[\cos(kx-\omega t) - \cos(kx+\omega t)\right]$$

$$= -2A\sin\left[\frac{1}{2}(kx-\omega t+kx+\omega t)\right]\sin\left[\frac{1}{2}(kx-\omega t-kx-\omega t)\right]$$

$$= -2A\sin kx\,\sin(-\omega t) = \underbrace{2A\sin kx}_{\text{amplitude at a given } x}\;\underbrace{\sin\omega t}_{\text{oscillation in y direction}}$$

Because the string is clamped at both ends the amplitude at the ends must be zero.

$2A\sin(kx) = 0$ at $x = 0$

$2A\sin(kL) = 0$ at $x = L$

$kL = m\pi \qquad m = 1, 2, 3 \ldots$

mode number

$k = \frac{2\pi}{\lambda}$

$L = \frac{m\lambda}{2}$

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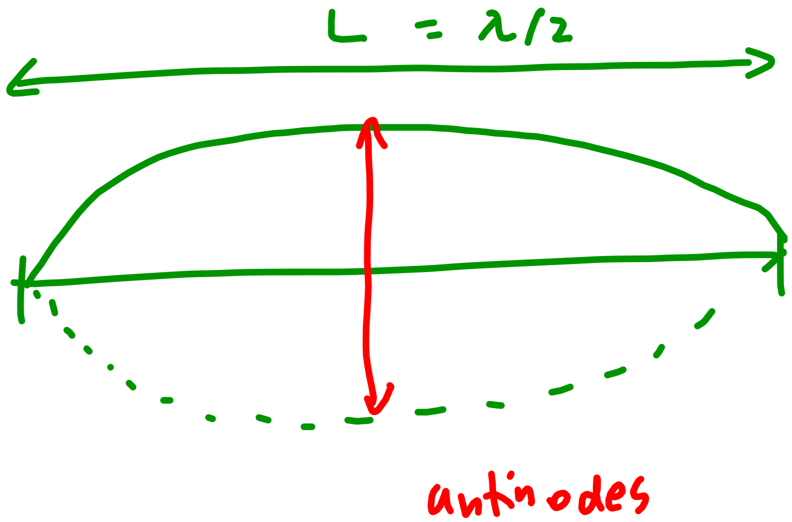
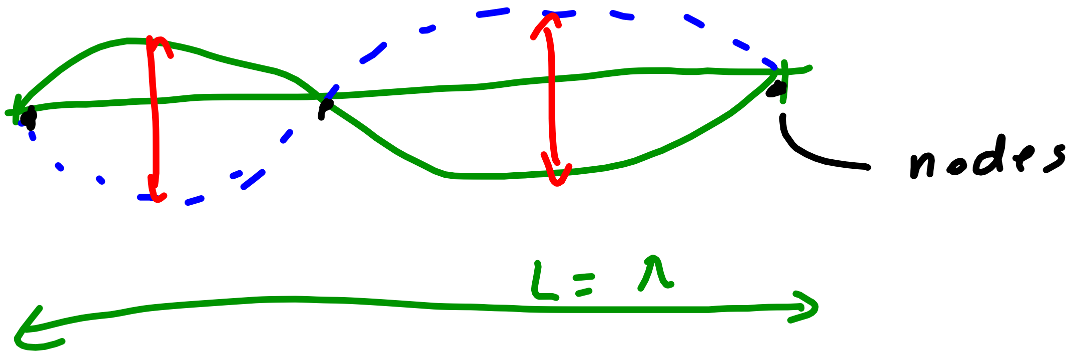


$m = 1$

$\lambda = 2L$

$m = 2$



Given a particular string length L we limit the allowed standing waves to a descrete set of wavelengths that we call harmonics. m = 1 gives a fundamental mode while the rest are called overtones.

Nodes: points where the string doesn't move

Antinodes: points where A is max

Freguency

$$v = f\lambda$$

standing wave with fixed ends: $\lambda = \frac{2L}{n}$

$$f = \frac{v}{\lambda} = \frac{v}{2L} n$$

f_1 is fundamental freguency $f_1 = v/2L$

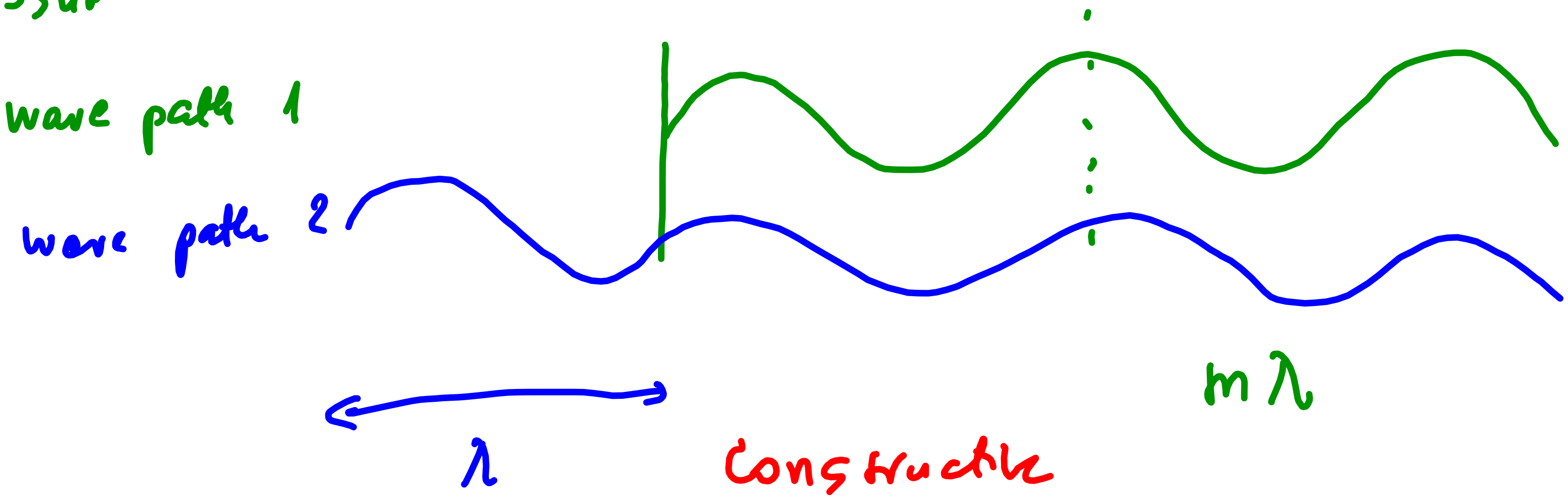
$f_n = n \cdot f_1$ harmonics

Electromagnetic waves including light produce constructive & destructive interference. Electric and magnetic fields obey superposition principle.

What is "waving"? Electromagnetic field

Consider light waves that originate at a single source, travel 2 paths and then rejoin

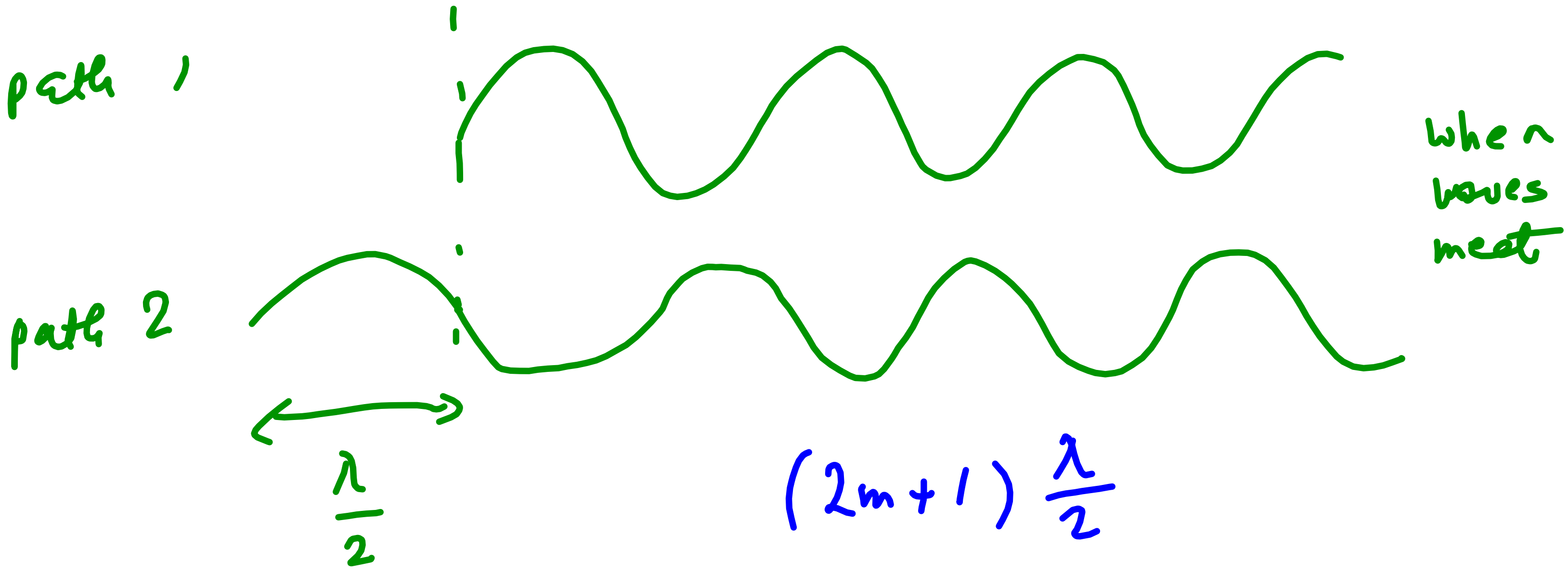
Assume: 1st case scenario



Constructive interference

2nd scenario

destructive interference



$$(2m+1)\frac{\lambda}{2}$$

$$\left(m+\frac{1}{2}\right)\lambda$$

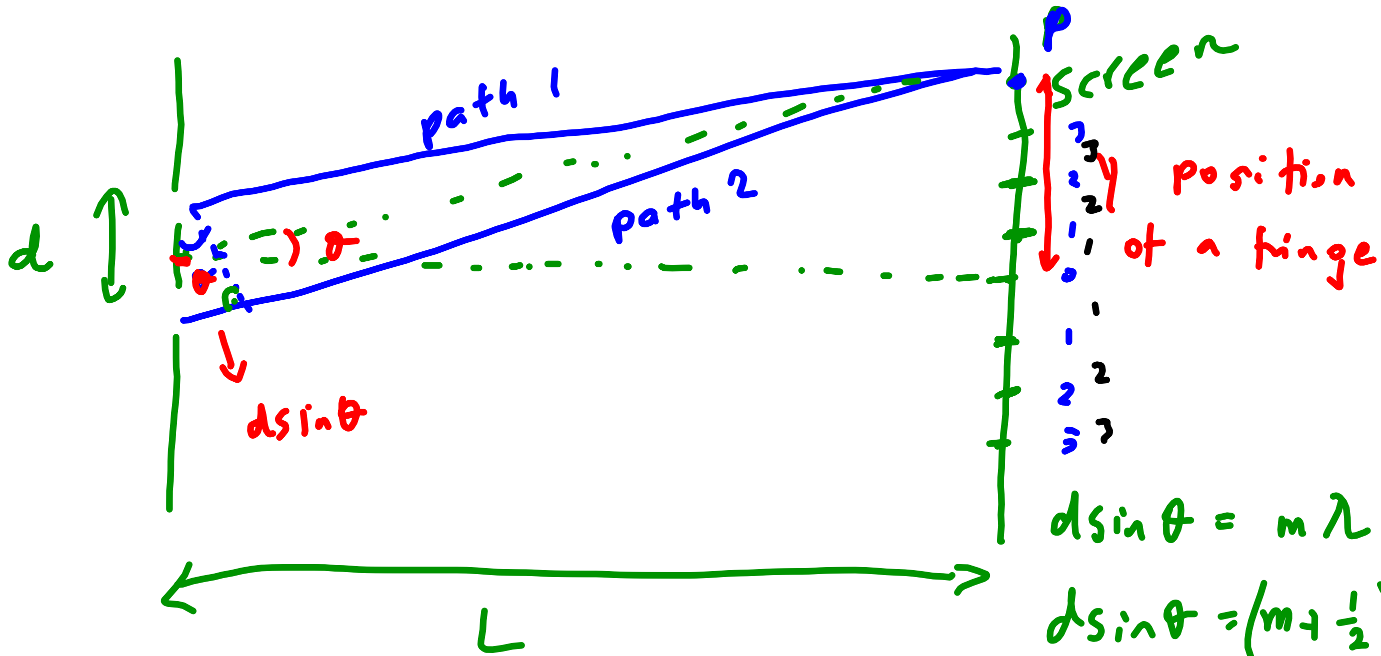
In between - interfering waves superimpose, interfere, change amplitudes ...

Double slit interference

- 1801 Young confirmed that light is a wave
- the light passes through a pair of narrow closely spaced slits

- constructive interference produce bright bands
- destructive — " — dark bands

} interference fringes



$d \sin\theta = m\lambda$

$d \sin\theta = (m + \frac{1}{2})\lambda$

$$d \sin\theta = m\lambda \qquad m = 0, 1, 2, \ldots$$

bright fringes - constructive interference

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

dark fringes

destructive interference

$$\tan\theta = \frac{y}{L}$$

typically $\lambda << d$

θ very small

$$\sin\theta \simeq \tan\theta = \frac{y}{L}$$

$$y_{bright} = m\frac{L\lambda}{d}$$ for bright fringes

$$y_{dark} = \left(m + \frac{1}{2}\right)\frac{L\lambda}{d}$$ for dark fringes

- increasing d → decrease in y & closer fringes

$$\Delta y = y_{m+1} - y_m = (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d}$$

$$= \frac{\lambda L}{d}$$

∀ bright fringes pair has the same Δy

Diffraction grating

- multiple slit (instead of 2) is called diffraction grating
(interference of N overlapped waves)

  - constructive interference

$$d \sin\theta_m = m\lambda$$

$$y_m = L \tan\theta_m$$

2 slits 0.075 mm apart are located 1.5 m from the screen. Laser shines => produces pattern whose 3rd order bright fringe is 3.8 cm from the screen center. Find λ

$d = 0.075$ mm

$L = 1.5$ m

$m = 3$

$y_{bright_3} = 3.8$ cm

$\lambda = ?$

$$y_{bright} = m \frac{\lambda L}{d}$$

$= 633$ nm

red light

$\lambda << d$