The first test:

~~02/28~~ 02/28

07/21

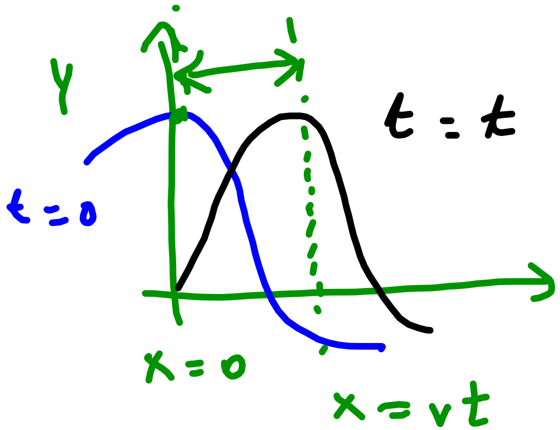
practice test

- ray optics
  - reflection
  - refraction
  - ray diagrams
  - lenses
- waves
- interference/diffraction
- Doppler effect

Simple harmonic wave

$k = \frac{2\pi}{\lambda}$ wavenumber $y(x, t=0) = A\cos kx$

To describe a wave moving with speed v we replace x with $x - vt$



$$y(x,t) = A\cos(k(x-vt))$$

$y(x,t) = A\cos[k(x - vt)] = A\cos(kx - \omega t)$

traveling wave in $+x$ direction

- we want to write a wave as a function of

ω not v

$\omega = \frac{2\pi}{T} = 2\pi f$ angular frequency

$v = \frac{\lambda}{T} = \frac{\lambda}{2\pi}\omega = \frac{\omega}{k}$ (with $\frac{\lambda}{2\pi} = 1/k$)

$k = \frac{2\pi}{\lambda}$

$$v = \frac{\omega}{k}$$

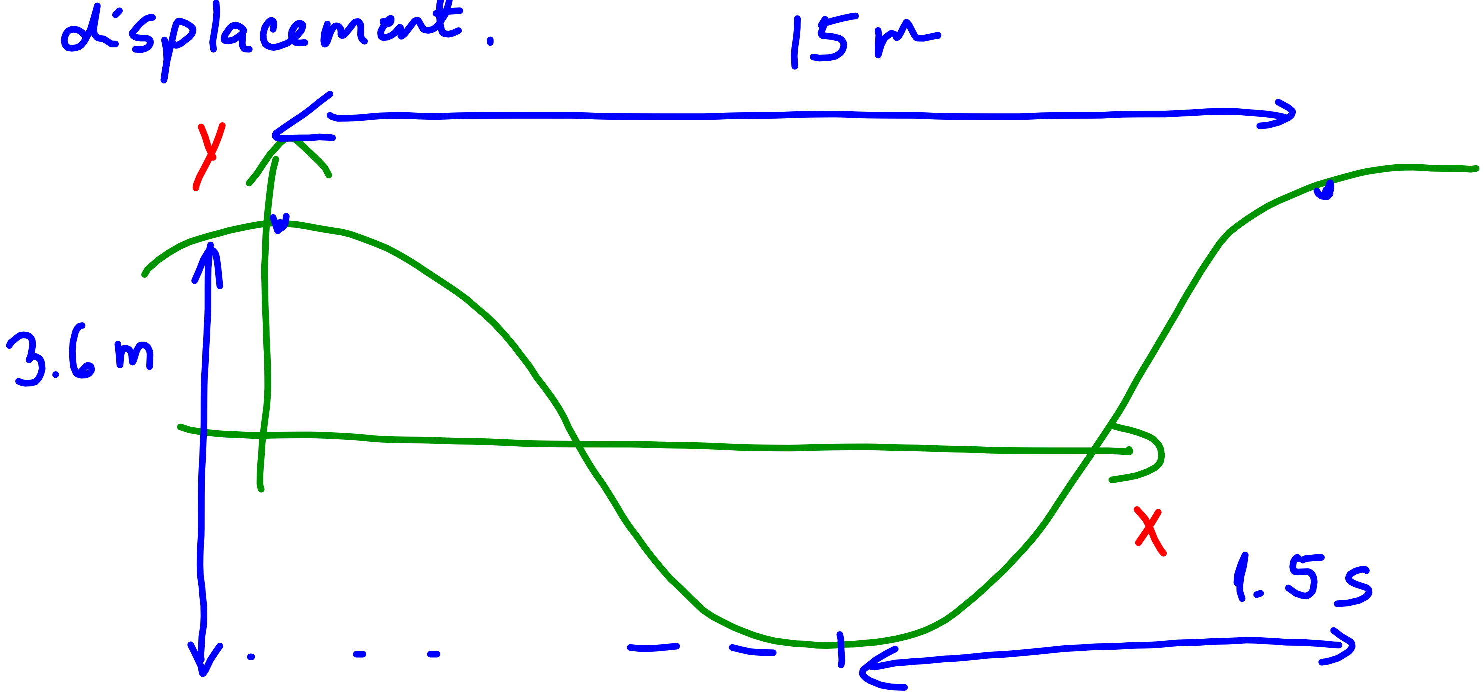
$$y(x,t) = A\cos(kx - \omega t) \qquad + x \text{ direction}$$

$$y(x,t) = A\cos(kx + \omega t) \qquad - x \text{ direction}$$

Example

A surfer is on the wave sinusoidal in shape with crests 15 m apart. The distance from trough to crest of the wave is 3.6 m and it takes surfer

1.5 s to cover that distance. Find the wave speed and write wave equation for displacement.



$A = 1.8 m$

$T = 3 s$

$\lambda = 15 m$

$k = ?$ $\quad k = \frac{2\pi}{\lambda}$

$\omega = ?$ $\quad \omega = \frac{2\pi}{T}$

$v = \frac{\lambda}{T} = 5 m/s$

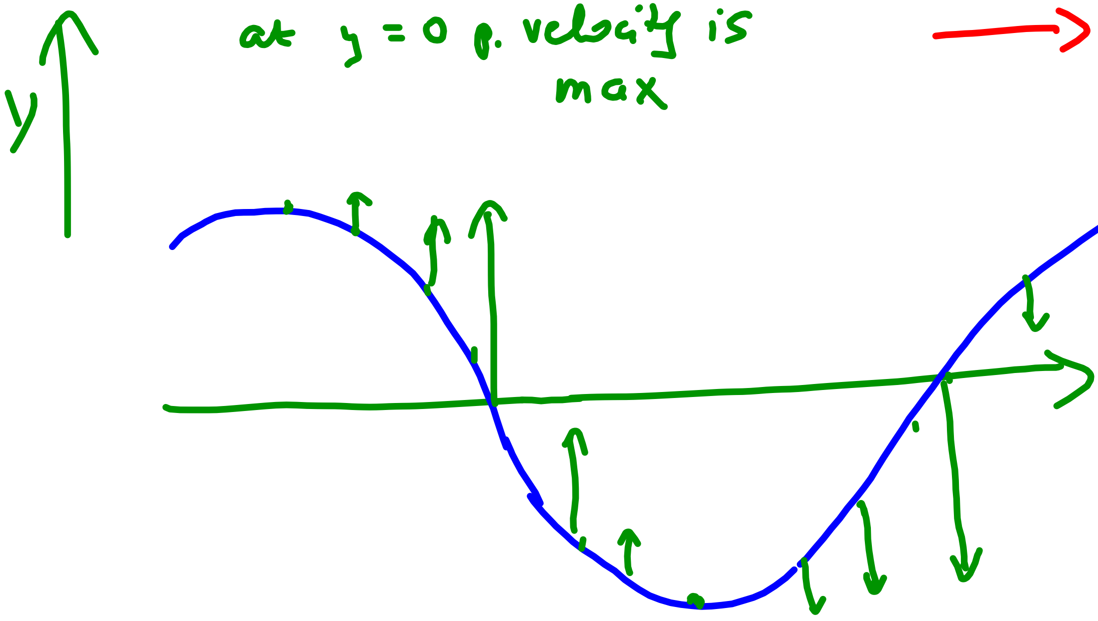
$y(x,t) = A \cos(kx - \omega t)$

$= 1.8 m \cos(0,42 x - 2.1 t)$

Waves on the string (Phet applet)

at y = 0 p. velocity is max

→ v velocity of the wave



green arrows velocity of a particle on a string

at turning points $v_p = 0$

- transverse wave

$$y(x,t) = A\cos(kx - \omega t)$$

$$v_y(x,t) = \frac{dy(x,t)}{dt}$$

$$v_y = -A(-\omega)\sin(kx - \omega t) = A\omega\sin(kx - \omega t)$$

$$v_{y_{max}} = A\omega$$

$$a_y = \frac{dv_y}{dt} = -A\omega^2\cos(kx - \omega t)$$

$\cos(kx - \omega t) = 1$

$a_y = -\omega^2 A$ & $v = \frac{\omega}{k}$ (wave)

particle

$a_y = -v^2 k^2 A$

we can relate particle's acceleration & speed of the wave

- larger wave speed => particles oscillating more quickly

- wave speed of the wave on the spring:

$$v = \sqrt{\frac{T}{\mu}}$$

T tension on the spring

μ linear density

$$\left(\Sigma \vec{F} = m\vec{a} \right)$$

Wave equation

- Newton's 2nd law

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2} \qquad \text{1-D}$$

$\Uparrow$ speed of the wave!

Verify that $y(x,t) = A\sin(kx - \omega t)$ satisfies the wave equation.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial y}{\partial x} = kA\cos(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(kx - \omega t)$$

$$\frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$$

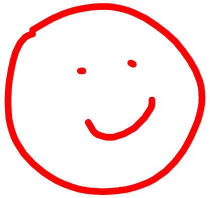
$$\frac{\partial^2 y}{\partial t^2} = A\omega^2\sin(kx - \omega t)$$

$$+k^2 A \sin(kx - \omega t) = \frac{1}{v^2}\left(+\omega^2 A\right)\sin(kx - \omega t)$$

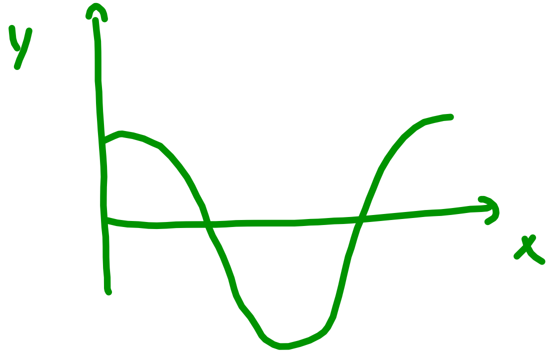
$$k^2 = \frac{1}{v^2}\,\omega^2$$

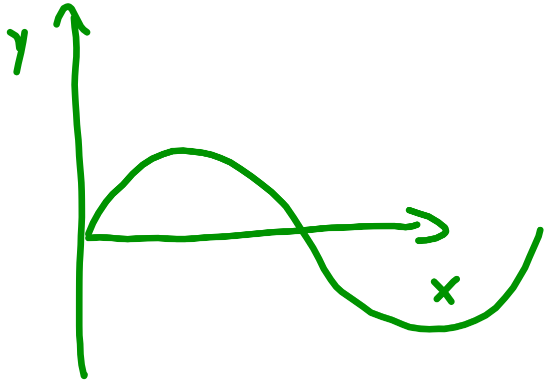
$$v^2 = \frac{\omega^2}{k^2}$$

$$v = \frac{\omega}{k}$$ ✓



Phase of the wave $y = A\cos(kx - \omega t + \phi)$



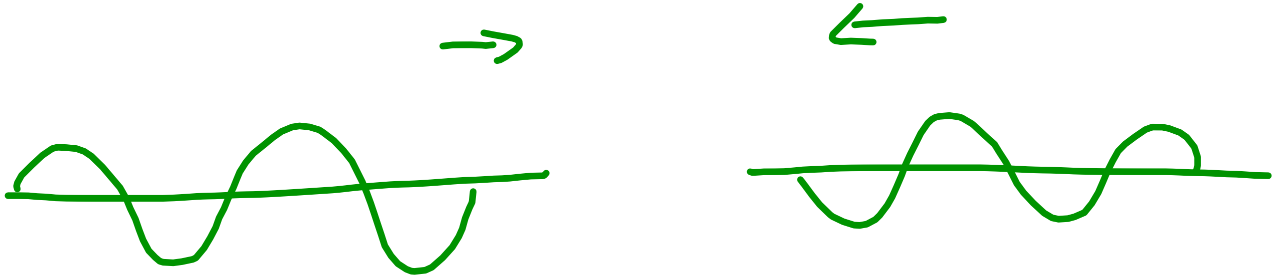


$-\frac{\pi}{2}$ difference in phase

$y = A\cos(kx - \omega t)$

$$y(x,t) = A\cos\left(kx - \omega t - \frac{\pi}{2}\right)$$
$$= A\sin(kx - \omega t)$$

Interference

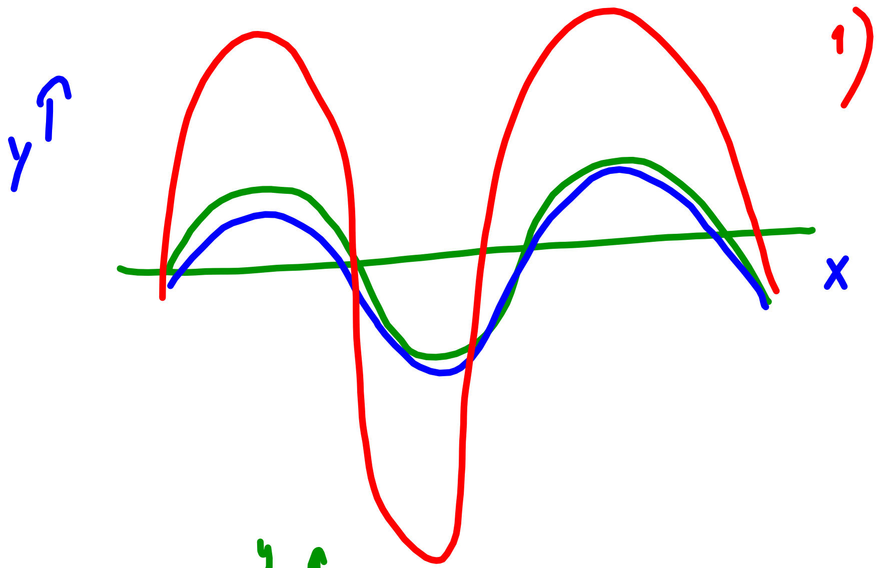


2 waves meet each other

- they interfere
- their displacements y(x) add up

superposition principle

2 opposite situations of interference:



1) constructive interference

the crests coincide & so do the troughs the resulting wave is for a moment twice as big

2) destructive interference

the waves cancel each other