Integrals

Because $v = \frac{dx}{dt}$

$a = \frac{dv}{dt}$

we can go backwards

$dv = a \cdot dt \quad | \int$

$$\int_{v_0}^{v} dv = \int_{t=0}^{t} a \cdot dt$$

$$\int_{v_0}^{v} dv = \int_{t=0}^{t} a\,dt$$

$$\left(v \Big|_{v_0}^{v} \right)$$

a is constant

$$v - v_0 = a\,t \Big|_0^t$$

$$v - v_0 = a \cdot t$$

$$\boxed{v = v_0 + at}$$

t^3

$\frac{t^4}{4}$

$v = \frac{dx}{dt}$

$v = v_0 + at$

$dx = v\,dt$

$x\Big|_{x_0}^{x} = v_0 t\Big|_0^t + a\frac{t^2}{2}\Big|_0^t$

$dx = (v_0 + at)\,dt$

$x - x_0 = v_0(t-0) + \frac{a}{2}(t^2 - 0^2)$

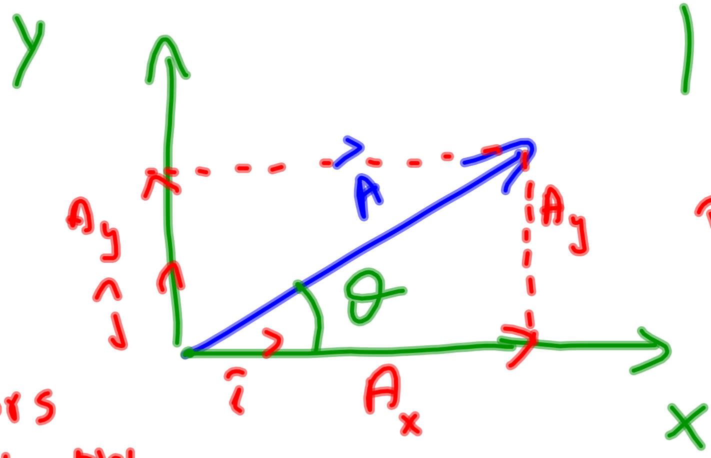
$dx = v_0\,dt + at\,dt$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$\int_{x_0}^{x} dx = v_0\int_{t=0}^{t} dt + a\int_{t=0}^{t} t\,dt$

Vectors (google phet)

2D



$\hat{i}, \hat{j}$ unit vectors in x & y direction

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$(c^2 = a^2 + b^2)$$

$$\boxed{\vec{A} = A_x \hat{i} + A_y \hat{j}}$$

θ theta angle

$$\sin\theta = \frac{A_y}{A} \qquad \cos\theta = \frac{A_x}{A} \qquad \tan\theta = \frac{A_y}{A_x}$$

Example

$$\vec{A} = \underline{\qquad}\,\hat{i} + \underline{\qquad}\,\hat{j} \text{ km}$$

You drive to a city 160 km from home going 35° north of east. Express your new position in unit vector notation using EW/NS coordinate system.

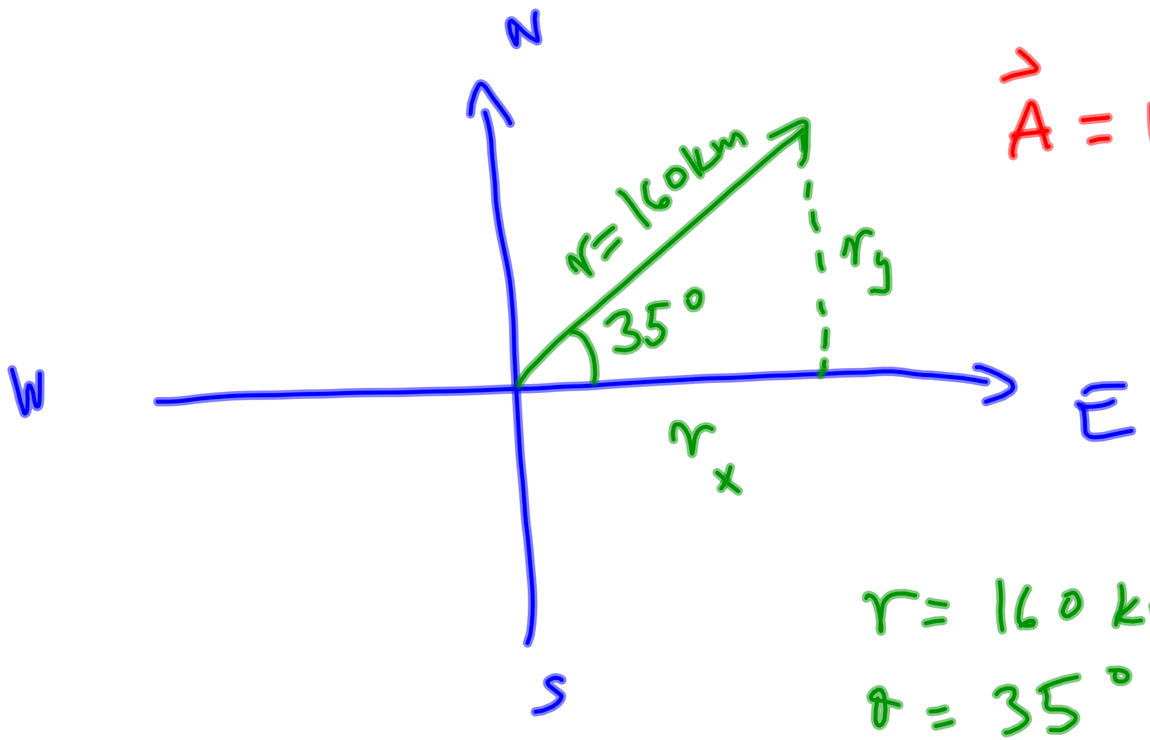
$\hat{i}$ in x direction

$\hat{j}$ y

N

W E

S



$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$r = 160$ km

$\theta = 35°$

$\vec{r} = ?$

$$\vec{r} = r_x\hat{i} + r_y\hat{j}$$

$$\vec{\text{☺}} = \text{☺}_x \hat{i} + \text{☺}_y \hat{j} + \text{☺} \hat{k}$$

$r_x = 131 \text{ km}$ $\qquad$ $r_x = r\cos\theta$

$r_y = 92 \text{ km}$ $\qquad$ $r_y = r\sin\theta$

$$\vec{r} = 131 \hat{i} + 92 \hat{j} \text{ km}$$

$$\vec{r} = 131 \text{ km } \hat{i} + 92 \text{ km } \hat{j}$$

Velocity & acceleration in 2 dimensions

$$\overline{\vec{V}} = \frac{\Delta \vec{r}}{\Delta t}$$

average velocity vector

$\vec{r}$ displacement vector

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

instantaneous velocity vector

$$\vec{r} = r_x \hat{i} + r_y \hat{j}$$
$$= x \hat{i} + y \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x\hat{i} + y\hat{j}\right) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$v_x \qquad v_y$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{\text{☺}} = \vec{\text{☺}}_x\,\hat{i} + \vec{\text{☺}}_y\,\hat{j}$$

$$\overline{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t}$$ average acceleration vector

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$ instantaneous a vector

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_x \hat{i} + v_y \hat{j}\right) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\boxed{\vec{v} = v_x \hat{i} + v_y \hat{j}} \quad \boxed{\vec{a} = a_x \hat{i} + a_y \hat{j}}$$

$$\left(\vec{r} = x\hat{i} + y\hat{j} \right)$$

Relative motion

- you stroll down the aisle of a plane toward the front at 4 km/h ; the plane is moving relative to the ground at 1000 km/h

=> you are then moving relative to the ground at 1004 km/h

velocity relative to what =>

This WHAT is called FRAME OF REFERENCE

- the same idea applies to 2D but it can get complicated as $\vec{v}$ is a vector

$$\boxed{\vec{v} = \vec{v}\,' + \vec{V}}$$

relative velocity

relative to ground

relative to air

wind blowing at the plane