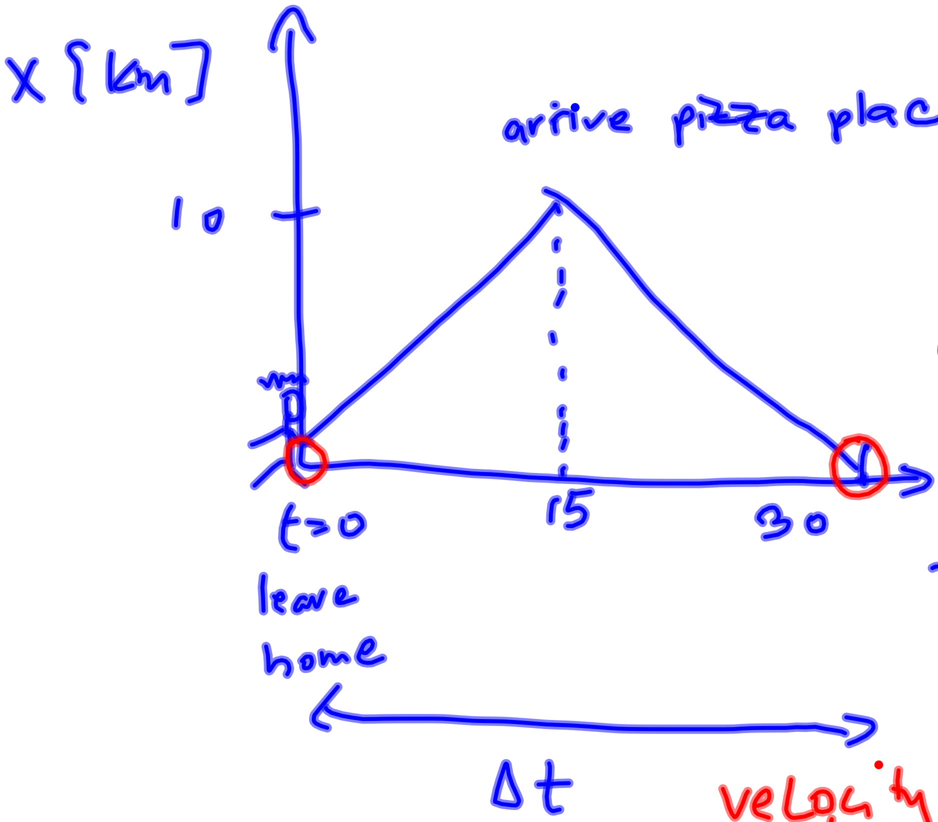


average speed $= \frac{\text{distance}}{\text{time}}$

$= \frac{20\,\text{km}}{30\,\text{min}}$

$= 40\,\text{km/h}$

average velocity

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} =$$

$= 0\,\text{km/h}$

velocity is a vector - direction matters

More closely
- it matters whether we go N, S, E or W

Displacement Δx therefore includes not only HOW FAR but IN WHAT DIRECTION as well.

- for motion in straight line -> we need + & -
- coordinate system (1-D) isn't physically real; it's a convenience & it is us who choose an origin & in which direction is + or -

Example

① Jun Z made a trip from Socorro to Santa Fe in 2h

$$\bar{v} = \frac{x_2 - x_1}{\Delta t} = \frac{200 - 0}{2\,h} = 100\,km/h$$

120

②

X from Socorro to SF in 30 minutes

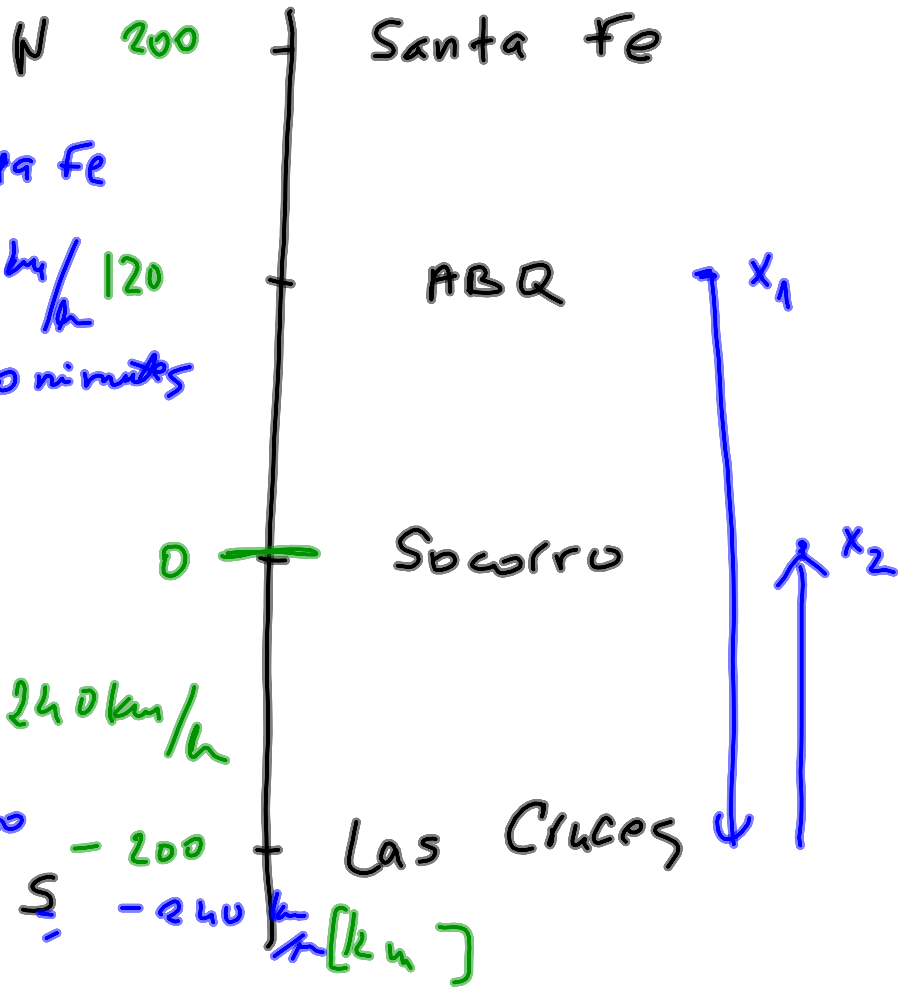
③ $$\bar{v} = \frac{200}{30\,min} = 400\,km/h$$

Student Y drove from ABQ to Socorro in 30 minutes

④ $$\bar{v} = \frac{0 - 120\,km}{30\,min} = -240\,km/h$$

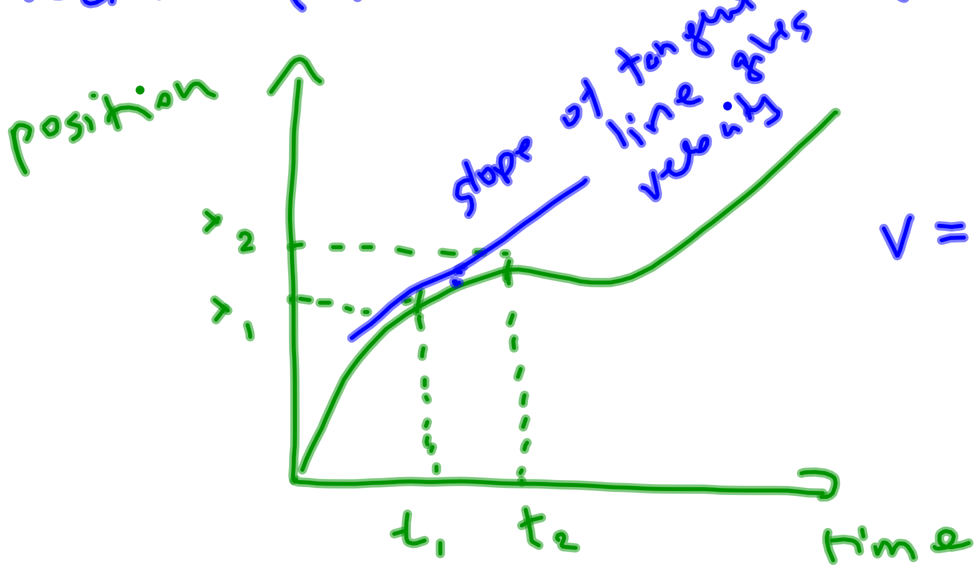
ABQ to LC & Socorro 30 min

$$\bar{V} = \frac{0 - 120}{30} = -240\,km/h\,[km]$$



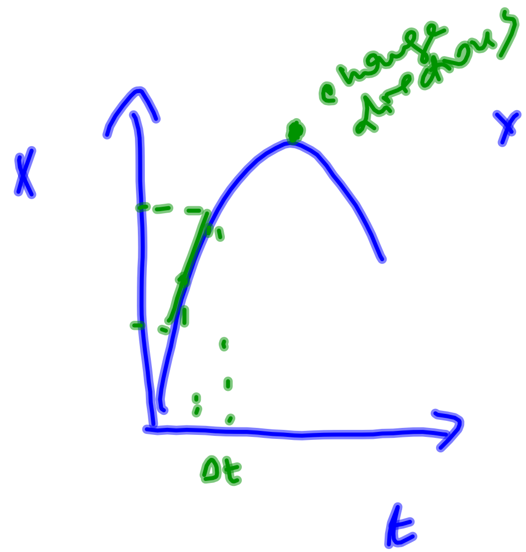
Instantaneous Velocity => velocity

- magnitude of velocity is speed
- vector (∀ vector has a magnitude & direction)



$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\boxed{v = \frac{dx}{dt}}$$

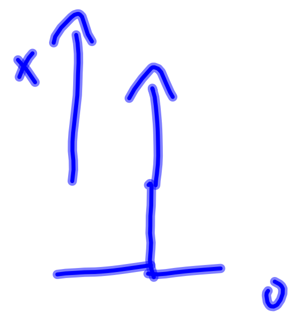


Example 1

The altitude of a space shuttle in the first half minute of its ascent is $x = bt^2$ where $b = 2.9\ m/s^2$. Find instantaneous velocity at $t = 20s$.

$x = bt^2$
$b = 2.9\ m/s^2$
$t = 20\ s$

$v(t = 20s) = ?$



$$v = \frac{dx}{dt} = \frac{d}{dt}\left(bt^2\right) = b\frac{d}{dt}\left(t^2\right) =$$

$$= b \cdot 2t = 2bt$$

$$v(t = 20\,s) = 2 \cdot 2.9\,\frac{m}{s^{\cancel{2}}} \cdot 20\,\cancel{s} =$$

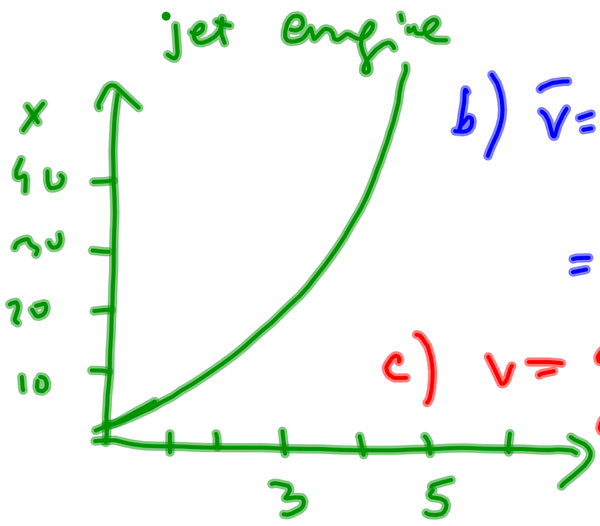
$$= 116\ m/s$$

Example 2

$$\boxed{x = At^2 + B}$$

$A = 2\,m/s^2$

$B = 1\,m$



a) Determine displacement of the engine during time interval $t_1 = 3\,s$ to $t_2 = 5\,s$

$t = 0 \quad x = 1$

$t = 1 \quad x = 3$

$\boxed{t_1 = 3 \quad x_1 = 19}$

$\boxed{\begin{array}{l} t_2 = 5 \\ x_2 = 51 \end{array}}$

$\Delta x = 51 - 19 = 32\,m$

b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{32\,m}{2\,s}$

$= 16\,m/s$

c) $v = \dfrac{dx}{dt} =$

$= \dfrac{d}{dt}\left(At^2 + B\right)$

$$\frac{d}{dt}\left(At^2 + B\right) = A \cdot 2t$$

derivative of constant (B) is zero!

$$v = 2 \cdot A \cdot t = 2 \cdot 2 \frac{m}{s^2} \cdot t = 4t$$

$$v(t = 2\,s) = 4 \cdot 2 = 8\ m/s$$

Acceleration

When velocity changes in time an object undergoes acceleration. Similarly to v =>
the average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

where Δv is change in velocity

bar is notation for average

Instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad [m/s^2]$$

$\longrightarrow$ +   a + means speeding up

           −   slowing down

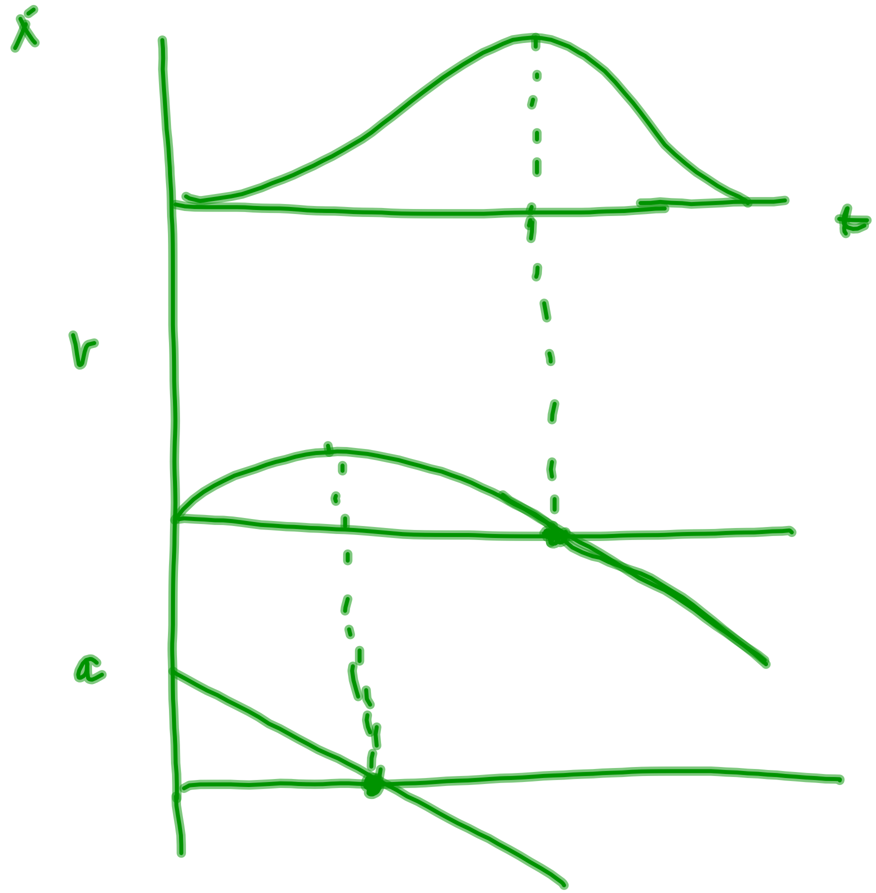
$a = 0$ cte velocity

If

$$a = \frac{dv}{dt} \qquad v = \frac{dx}{dt}$$

then we can relate a & x

$$a = \frac{d}{dt}\left[\frac{dx}{dt}\right] = \frac{d^2x}{dt^2}$$

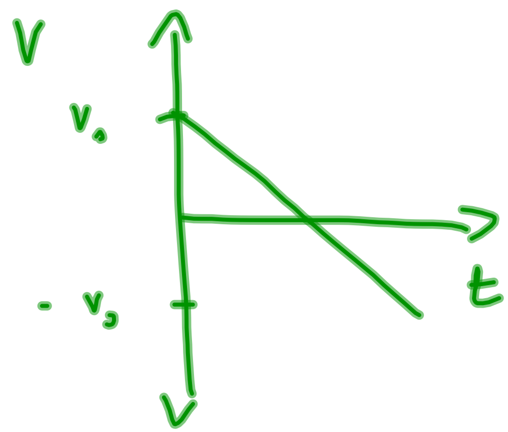


where x peaks
V = 0

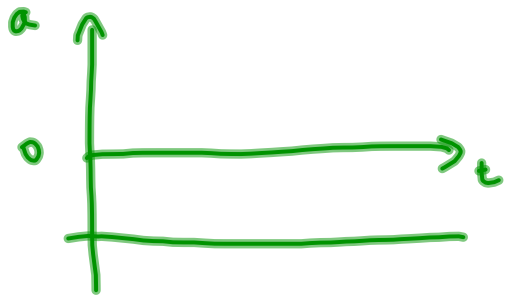
Where v peaks
a = 0

a = ctc

If



just before the peak v is positive, after negative



v is steadily decreasing

a = cte

& $-$

Special case of constant acceleration

- description of motion has a simple form when $a = cte$

Suppose an object starts at $t=0$ with some initial velocity v_0 & constant acceleration a.

Because a doesn't change :

$$\bar{a} = a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Let's call

$$\boxed{\begin{array}{ll} v_2 = v & v_1 = v_0 \\ t_2 = t & t_1 = 0\ s \end{array}}$$

$$\boxed{a = \frac{v - v_0}{t}}$$

or

$$at = v - v_0$$

$$\boxed{v = v_0 + at}$$

What about displacement?

$$\boxed{v = v_0 + at}$$

If $a = cte$ that means that v changes at steady rate $\left(a = \frac{dv}{dt} \right)$

$$\Rightarrow \quad \bar{v} = \frac{v + v_0}{2}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t}$$

$t = 0 \qquad t = t$

$t_0 = 0 \qquad t_2 = t$

$v_1 = v_0 \qquad v_2 = v$

$x_1 = x_0 \qquad x_2 = x$

$$\bar{v} = \frac{x - x_0}{t} \quad \Big| \cdot t \qquad \bar{v}t = x - x_0$$

$$x = x_0 + \bar{v}t \qquad \bar{v} = \frac{1}{2}(v_0 + v)$$

$$X = x_0 + \frac{1}{2}v_0 t + \frac{1}{2}vt \qquad \boxed{v = v_0 + at} \quad (1)$$

$$X = x_0 + \frac{1}{2}v_0 t + \frac{1}{2}(v_0 + at) \cdot t =$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2} \quad (2)$$

2nd eqn

$$\boxed{\begin{aligned} v &= v_0 + at \\ x &= x_0 + v_0 t + \frac{1}{2} a t^2 \end{aligned}}$$

$a = cte$!