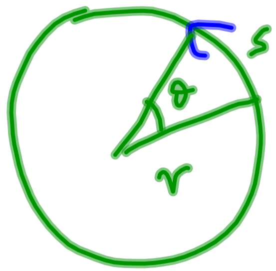
Intro to rotational motion

	Linear	Angular
Position	x	θ
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$



$s = \theta \cdot r$

If $a = \text{const}$

$$\bar{v} = \frac{1}{2}(v_0 + v)$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

If $\alpha = \text{constant}$

$$\bar{\omega} = \frac{1}{2}(\omega_0 + \omega)$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

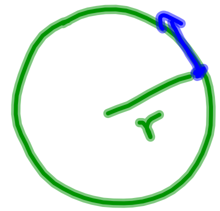
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$s = \theta \cdot r$ $\qquad$ r is constant

$$v = \frac{ds}{dt} = r \frac{d\theta}{dt} = r\omega$$

linear velocity $\qquad \omega \qquad$ angular speed

Ex. A wind turbine's blades are 28 m long and rotate at 21 rpm (revolution per minute). Find the angular speed of the blades in radians/s & determine the linear speed of the tip of the blade.



$r = 28 m$

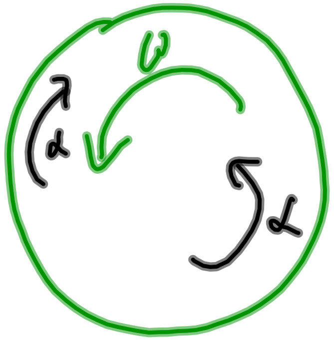
1 revolution = 2π radians

$$\omega = 21 \text{ rpm} = \frac{21 \cdot 2\pi \text{ rad}}{60 s} = 2.2 \text{ rad/s}$$

$$v = \omega \cdot r = 2.2 \cdot 28 = 62 \text{ m/s}$$

counterclockwise $\omega > 0$

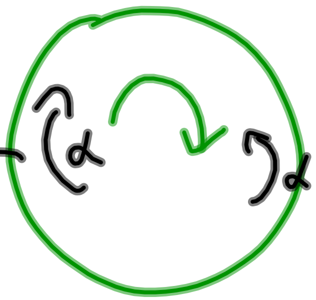
slowing down
$\alpha < 0$



speeding up $\alpha > 0$

clockwise $\omega < 0$

$\alpha < 0$
speeding up



slowing down
$\alpha > 0$

$v_r = 0$ $\quad\quad$ $a_r = \omega^2 r$

$v_t = \omega r$ $\quad\quad$ $a_t = r\alpha$

$\theta, \; \omega = \frac{d\theta}{dt}, \; \alpha = \frac{d\omega}{dt}$