Terminal velocity

Drag force : resistive force to the motion of the object $F_D(v)$

depends on velocity

We use the simplest drag force

small obj.
low speeds $F_D = -b \cdot v$ b - constant that depends on the viscosity of the fluid
high viscosity $\sim v^2$ & size & shape of an object

$$\boxed{v_T = \frac{mg}{b}} \Longleftarrow mg - bv_T = 0$$

$\vec{F}_D = -bv$

$\Sigma \vec{F} = m\vec{a}$

y $\downarrow$ $\quad m\vec{g} \quad \downarrow \vec{v} \quad \downarrow \vec{a}$

$\Sigma F_y = m a_y$

$$\boxed{mg - bv = m \cdot \frac{dv}{dt}}$$

$a_y = \frac{dv}{dt}$

At $t = 0$, $v = 0$, $\frac{dv}{dt} = g$

- object falls, v increases $\Rightarrow$ F_D increases, $a \downarrow$
- at some point velocity is so large and so is F_D that we reach force balance $\Rightarrow \frac{dv}{dt} = a = 0$

Gravity

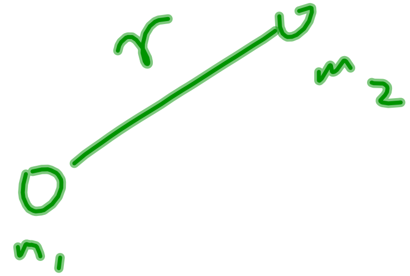
Earth itself exerts the gravitational force on objects.

- no contact

Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses & inversely proportional to the square of the distance between them.

Gravitational force

magnitude $\boxed{F = G \frac{m_1 \cdot m_2}{r^2}}$



inverse square law for gravitational force

G universal constant measured experimentally
has the same value for all objects

$$G = 6.67 \cdot 10^{-11}\ Nm^2/kg^2$$

Ex.

A 50 kg person & a 70 kg person are sitting on a bench close to each other. Estimate the magnitude of gravitational force that each exerts on the other.

$r = 0.5\,m \quad m_1 = 50\,kg \quad m_2 = 70\,kg$

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \cdot 10^{-11}\ \frac{Nm^2}{kg^2}$$

$$F = \frac{6.67 \cdot 10^{-11} \frac{N\cancel{m^2}}{\cancel{kg^2}} \cdot 50\,\cancel{kg} \cdot 70\,\cancel{kg}}{0.5^2\ \cancel{m^2}}$$

$$\cong 10^{-6}\ N$$

Find g at Earth's surface a)
on Everest (8850 m) b)
c) at 380 km altitude space station

- neglect forces such as friction etc

Write Newton 2nd law

$$\sum \vec{F} = m\vec{a}$$

$$G\frac{\cancel{m}\cdot M_E}{r^2} = \cancel{m}\cdot a$$

m some arbitrary mass of an object r distance from $\boxed{M_E \text{ earth}}$

$$R = 6.37\cdot 10^6\ m$$

a) $a = G \cdot \frac{M_E}{R_E^2} = 9.81\ m/s^2 = g$ ☺

b) $r = R_E + 8850$

$a = 9.77\ m/s^2$

c) $r = R_E + 380 \cdot 10^3$

$a = 8.74\ m/s^2$ $\quad M_E = 5.972 \cdot 10^{24}\ kg$

Satellites

- put to orbit by accelerating it to a sufficiently high tangental speed
- centripetal acceleration $a = \frac{v^2}{r}$

where $r = R_E + h$

$\Sigma F = ma$

$G \frac{m_s M_E}{r^2} = m_s \cdot \frac{v^2}{r}$

Geosynchronous satellite - above the equator

What is the height above Earth's surface that satellite must have? What is its speed?

$$G\frac{\cancel{m_s} M_E}{r^2} = \cancel{m_s}\frac{v^2}{r}$$

$$\boxed{v = \frac{2\pi r}{T}}$$

$v = 3070\ m/s$

the same for earth & satellite

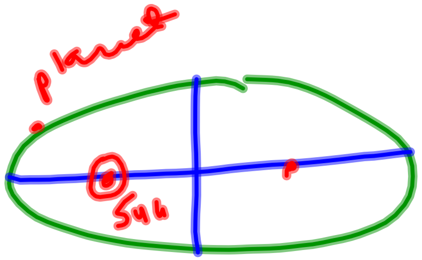
$$G\frac{M_E}{r^2} = \frac{4\pi^2 r^{\cancel{2}}}{T^{\cancel{2}}}$$

$$r^3 = \frac{G m_E T}{4\pi^2}$$

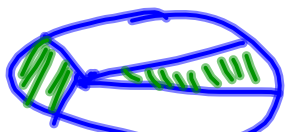
$r = 4.23 \cdot 10^7\ m$ $\quad r = R_E + h$

$h = 36000\ km$

Kepler's laws



1. Elliptical path of planets

2. 

Each planet moves so that imaginary line drawn from the Sun to the planet sweeps out equal areas in equal time periods

3. $$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^2}$$

Gravitational field

- we place mass m test mass m at some point at some distance

$G\frac{M \cdot m}{r^2}$

- Gravitational field is then

$$\vec{g} = \frac{\vec{F}}{m} \quad [N/kg]$$

$$\vec{g} = -\frac{GM}{r^2}\hat{r}$$