Oscillations (continued)

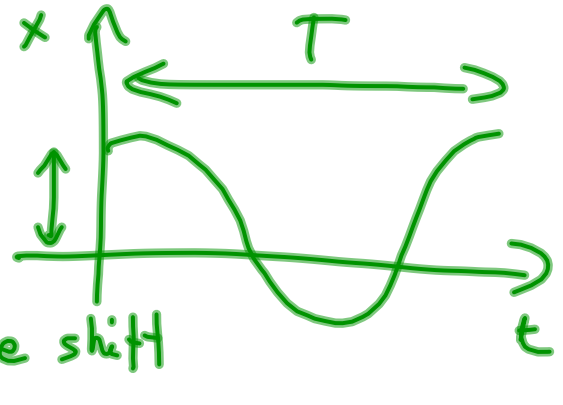
A, T, f, ω

Simple harmonic oscillation (SHO, SHM)
motion

$x(t) = A\cos\omega t$

$A\cos(\omega t + \phi)$

↑ phase shift



Ex. SHM

$F = -kx$

$\Sigma F = ma$

$-kx = m \cdot \frac{d^2x}{dt^2}$

$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$

$\omega^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{k}{m}}$

guess

$x(t) = A\cos\omega t$

$\frac{dx}{dt} = -A\omega \sin\omega t$

$\frac{d^2x}{dt^2} = -A\omega^2 \cos\omega t$

$-A\omega^2\cos\omega t + \frac{k}{m}A\cos\omega t = 0$

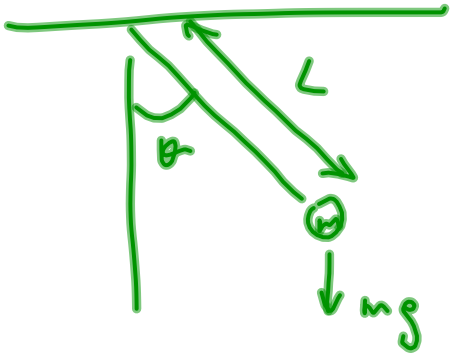
$\omega = \sqrt{\frac{k}{m}}$ depends on k & m

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Pendelum ex.

$k = \frac{mg}{\ell}$

$\omega = \sqrt{\frac{mg}{m\ell}} = \sqrt{\frac{g}{\ell}}$



$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$

Energy

PE $U = \frac{1}{2}kx^2$

KE $K = \frac{1}{2}mv^2$

$x = A\cos\omega t$

$v = \frac{dx}{dt} = -A\omega\sin\omega t$

$U = \frac{1}{2}kA^2\cos^2\omega t$

$K = \frac{1}{2}mA^2\omega^2\sin^2\omega t$

$E = U + K$

$= \frac{1}{2}A^2\left(k\cos^2\omega t + m\omega^2\sin^2\omega t\right)$

$m \cdot \frac{k}{m}$

$\omega^2 = \frac{k}{m}$

$k\left(\cos^2\omega t + \sin^2\omega t\right)$

1

energy is conserved

$E = \frac{1}{2}kA^2$

Damped harmonic motion

$F_d = -bv$ b constant

damping / friction force

$\Sigma F = ma$

$$-kx - bv = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

ODE

guess

$x(t) = A e^{-\text{☺}t} \cos(\omega t + \phi)$ plug in ODE

$x(t) = A e^{-bt/2m} \cos(\omega t + \phi)$

$\text{☺} = \frac{b}{2m}$

Driven oscillations

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos \omega_d t$$

F_0: cte

ω_d: frequency of driven oscillation

guess: $x(t) = A\cos(\omega_d t + \phi)$

$$A(\omega_d) = \frac{F_0}{m\sqrt{(\omega_d^2 - \omega_0^2)^2 + \frac{b\omega_d^2}{m^2}}}$$

$\omega_0 = \sqrt{\frac{k}{m}}$

→ if b is small

$\omega_d \sim \omega_0$ $1/0 \rightarrow \infty$ RESONANCE

Ex

$A = 2\,m$

$T = 3\,s$

- wrik down SHM equation $x(t) = A\cos\omega t$
- calculate speed, max speed, acceleration, energy ...

$\omega = \frac{2\pi}{T} = \frac{2\pi}{3}$ $\quad x(t) = 2m\cos\left(\frac{2\pi}{3}t\right)$

$a = \frac{dv}{dt}$ $\quad v = \frac{dx}{dt} = -2\cdot\frac{2\pi}{3}\sin\left(\frac{2\pi}{3}t\right)$

$u = \frac{1}{2}kx^2$ $\quad K = \frac{1}{2}mv^2$ $\quad v_{max} = -\frac{4\pi}{3}$