Rotational motion (continued)

x, v, a

θ, ω, α

$$KE = \frac{1}{2} m v^2$$

Pure rotation, $v = \omega r$

$$K = \frac{1}{2} m \omega^2 r^2$$

For many particles system

$$K = \frac{1}{2} m_1 \omega^2 r_1^2 + \frac{1}{2} m_2 \omega^2 r_2^2 + \ldots$$

$$= \frac{1}{2} \Big(\underbrace{\sum_i m_i r_i^2}_{*} \Big) \omega^2$$

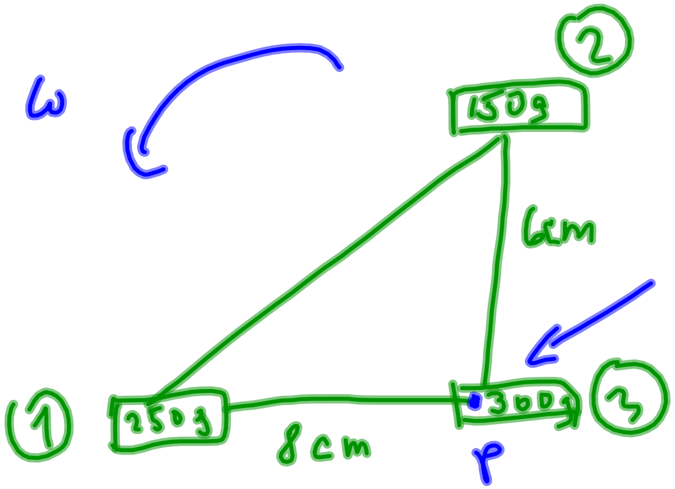
$$I = \sum m_i r_i^2$$

Moment of inertia

$[kg\,m^2]$

$$K = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

$$\boxed{K = \frac{1}{2} I \omega^2}$$



rotate about P

$K_{rot} = 100\,mJ$

$\omega = ?$

$I = \sum m_i r_i^2 =$

$= 0.25\,kg \cdot 0.08^2\,m^2 +$

$0.15\,kg \cdot 0.06^2\,m +$

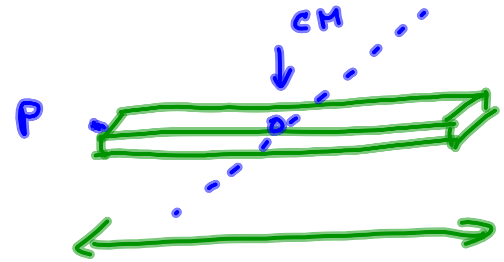
$0.3\,kg \cdot 0^2 = 2.14 \cdot 10^{-3}\,kg\,m^2$

$K_{rot} = \frac{1}{2} I \omega^2$

$\omega = \sqrt{\frac{2K}{I}}$

$\omega = 9.67\,rad/s$

Moments of inertia



	Moment of inertia I
thin rod about the center	$\frac{1}{12}ML^2$
thin rod about the end	$\frac{1}{3}ML^2$
cylinder about center	$\frac{1}{2}MR^2$
hoop	MR^2
solid sphere	$\frac{2}{5}MR^2$

Parallel axis theorem

$$I = I_{CM} + Md^2$$

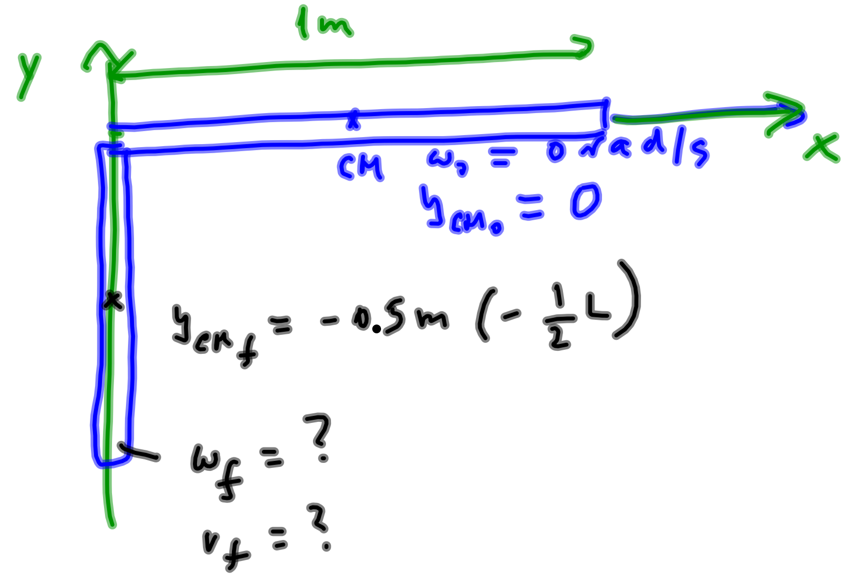
$$I = \frac{1}{12}ML^2 + M\cdot\left(\frac{L}{2}\right)^2$$

$$= \left(\frac{1}{12} + \frac{1}{4}\right)ML^2$$

$$= \frac{1}{3}ML^2$$

Ex

A 1m long 200 g rod is hinged at one end & connected to the wall (held horizontally). After it is released calculate the speed of the tip of the rod.



Conservation of energy

$$E = K + U$$

$$K_0 + U_0 = K + U$$

$$K_{rot} = \frac{1}{2} I \omega^2$$

$$U = Mg\,y_{cm}$$

initial = final

$$\cancel{\frac{1}{2} I \omega_0^2} + \cancel{M g y_{cm_0}} = \frac{1}{2} I \omega_f^2 + M g y_{cm_f}$$

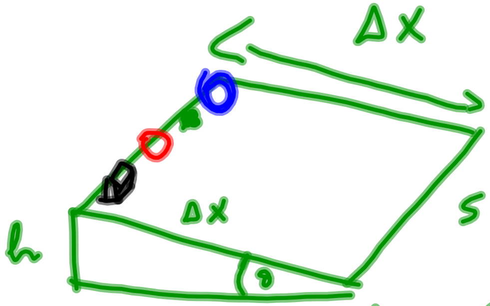
$$\frac{1}{2} I \omega_f^2 + M g y_{cm_f} = 0$$

$$I = \frac{1}{3} M L^2$$

$$\frac{1}{\cancel{2}} \cdot \frac{1}{3} \cancel{M} L^{\cancel{2}} \omega_f^2 + \cancel{M} g \cdot \left(- \frac{1}{\cancel{2}} \cancel{L} \right) = 0$$

$$\omega_f = \sqrt{\frac{3g}{L}} \qquad v = \omega L = \sqrt{3gL}$$

$$= 5.4 \ m/s$$



$\sin\theta = \frac{h}{\Delta x}$

$U_o = Mgh$

$K_o = 0$

$U_f = 0$

$K_f = ?$

$\boxed{U_o = K_f}$

		I_{cm}
cube	M	0
sphere	M	$\frac{2}{5}MR^2$
cylinder	M	$\frac{1}{2}MR^2$
hoop	M	MR^2

Rolling motion = translation + rotation

$$KE_{rolling} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

xMR^2

$\omega = \frac{V_{cm}}{R}$

$$I_{cm} = x M R^2$$

remember $v_{cm}^2 = 2 a_{cm} \Delta x$

$$v_{cm} = \sqrt{\frac{2gh}{1+x}}$$

$$a_{cm} = \frac{g \sin\theta}{1+x}$$

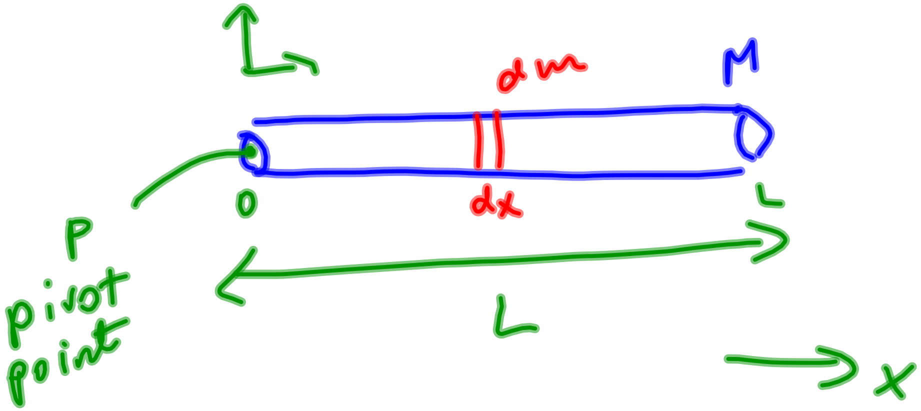
larger x → smaller v

smaller x → larger v

Winners : cube sphere cylinder hoop

1 2 3 4

Calculating moment of inertia



$$y \sim 0$$

$$\frac{dm}{M} = \frac{dx}{L}$$

$$dm = \frac{M}{L}\,dx$$

$$I = \int r^2\,dm$$

$$I = \int x^2\,dm = \int x^2 \frac{M}{L}\,dx = \frac{M}{L}\int_0^L x^2\,dx = \frac{M}{L}\,\frac{x^3}{3}\Big|_0^L = \frac{1}{3} M L^2$$