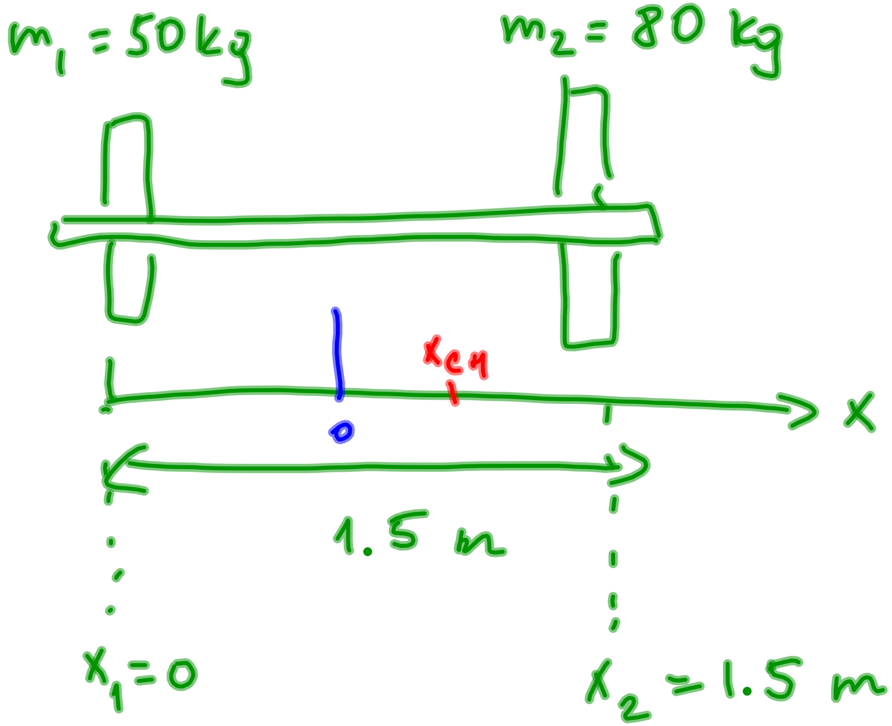
CM in 1D & 2 bodies

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Ex. Find the center of mass of a barbell consisting of 50kg and 80 kg weights at the opposite ends of a 1.5 m long bar of negligible mass.



$$x_{cm} = \frac{m_1 \cancel{x_1} + m_2 x_2}{m_1 + m_2}$$

$$x_{cm} = \frac{m_2 x_2}{m_1 + m_2}$$

$$= \frac{80\,kg \cdot 1.5\,m}{(80+50)\,kg} =$$

$$= 0.92\,m$$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{M} \qquad M = \sum m_i$$

$i = 2$

$\vec{r} = x \quad 1D$

$$\vec{x}_{cm} = \frac{\sum_{i=1}^{2} m_i x_i}{M} =$$

$$= \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Momentum

$$\vec{v}_{cm} = \frac{\sum m_i \vec{r}_i}{M}$$

$$\vec{F}_{net\,ext} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\boxed{m\vec{v}}\right) = \frac{d\vec{P}}{dt}$$

$$\vec{p} = m\vec{v}$$

$$\vec{P} = \sum p_i = \sum m_i \vec{v}_i$$

$$\vec{P} = \sum m_i \frac{d\vec{r}_i}{dt} = \frac{d}{dt}\left(\sum m_i \vec{r}_i\right) = \frac{d}{dt}\left(M\vec{r}_{cm}\right)$$

$$\vec{P} = M\frac{d\vec{r}_{cm}}{dt} \quad \vec{v}_{cm} = M\vec{v}_{cm}$$

Collisions

- brief intense interaction between objects
- we assume that momentum $\vec{p}$ is conserved during the collision $\Rightarrow$
  that means that we can relate before & after collision states

Collision forces are large but they are internal so they cancel out.

However the force of collisions alters the motion of individual colliding particles.

By How much? → it depends on magnitude of the force & how long it was applied

If $\overline{\vec{F}}$ is the average force acting on a particle during the collision that lasts Δt :

$$\overline{\vec{F}} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\Delta \vec{p} = \overline{\vec{F}} \Delta t \equiv \vec{J} \quad \text{impulse} \quad [Ns]$$

If $\vec{F} \neq$ constant $\quad \vec{J} = \int_{\text{collision time}} \vec{F}(t)\, dt$

Collisions – elastic

KE is conserved

– inelastic

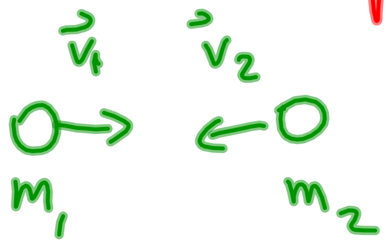
KE is not conserved

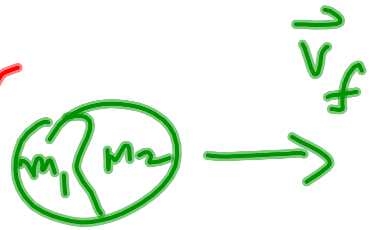
Momentum is always conserved

Totally inelastic collision

– objects stick together after colliding

$$\vec{p}_{before} = \vec{p}_{after}$$



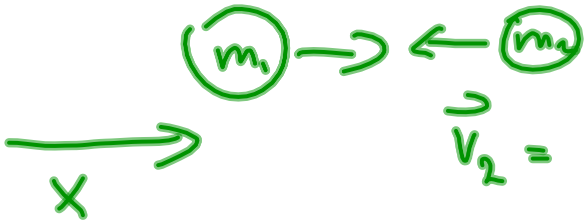


befre

final

$$\boxed{m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}_f}$$

$\vec{v}_1 = 2\,m/s$



$\vec{v}_2 = 1\,m/s$

$\vec{v}_f$

$\vec{v}_f = ?$

$m_1 = 500\,kg$

$m_2 = 200\,kg$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_f$$

$$500\,kg \cdot 2\,m/s - 200\,kg \cdot 1\,m/s = 700\,kg \cdot \vec{v}_f$$

$$800 = 700\,v_f$$

$$\vec{v}_f = \frac{800}{700}\,m/s$$

$$= \frac{8}{7}\,m/s$$

$$= 1.14\,m/s$$

Elastic collision

i initial

f final

$p_{before} = p_{after}$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$KE_{before} = KE_{after}$ only elastic colisions

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$KE = \frac{1}{2}mv^2$