Remember

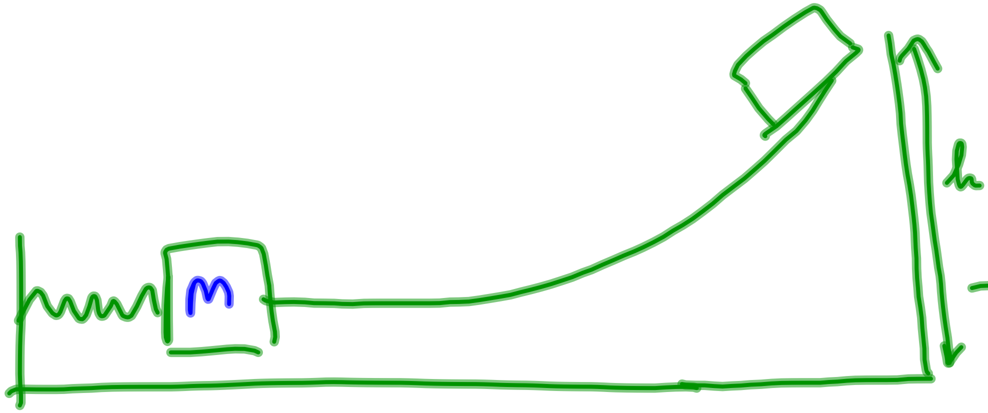
Conservation of energy

$$K + U = K_0 + U_0$$

$$= \text{constant}$$

Ex.

The spring has $k = 140$ N/m. A 50 kg block is placed against the spring which is compressed 11 cm. When the block is released, how high up the slope does it rise? Neglect friction.



$k = 140\ N/m$

$m = 50\ kg$

$x = 11\ cm = 0.11\ m$

$h = ?$

Initial state

$K = KE = 0$

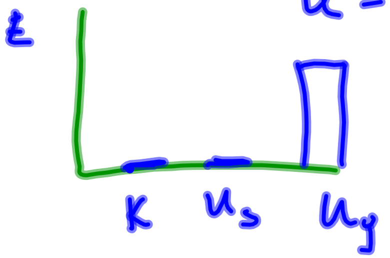
$U_{spring} = PE = \frac{1}{2}kx^2$

Final state

$K = 0$

$U_{spring} = 0$

$U = mgh$



$$\cancel{K_0} + U_0 = \cancel{K_f} + U$$

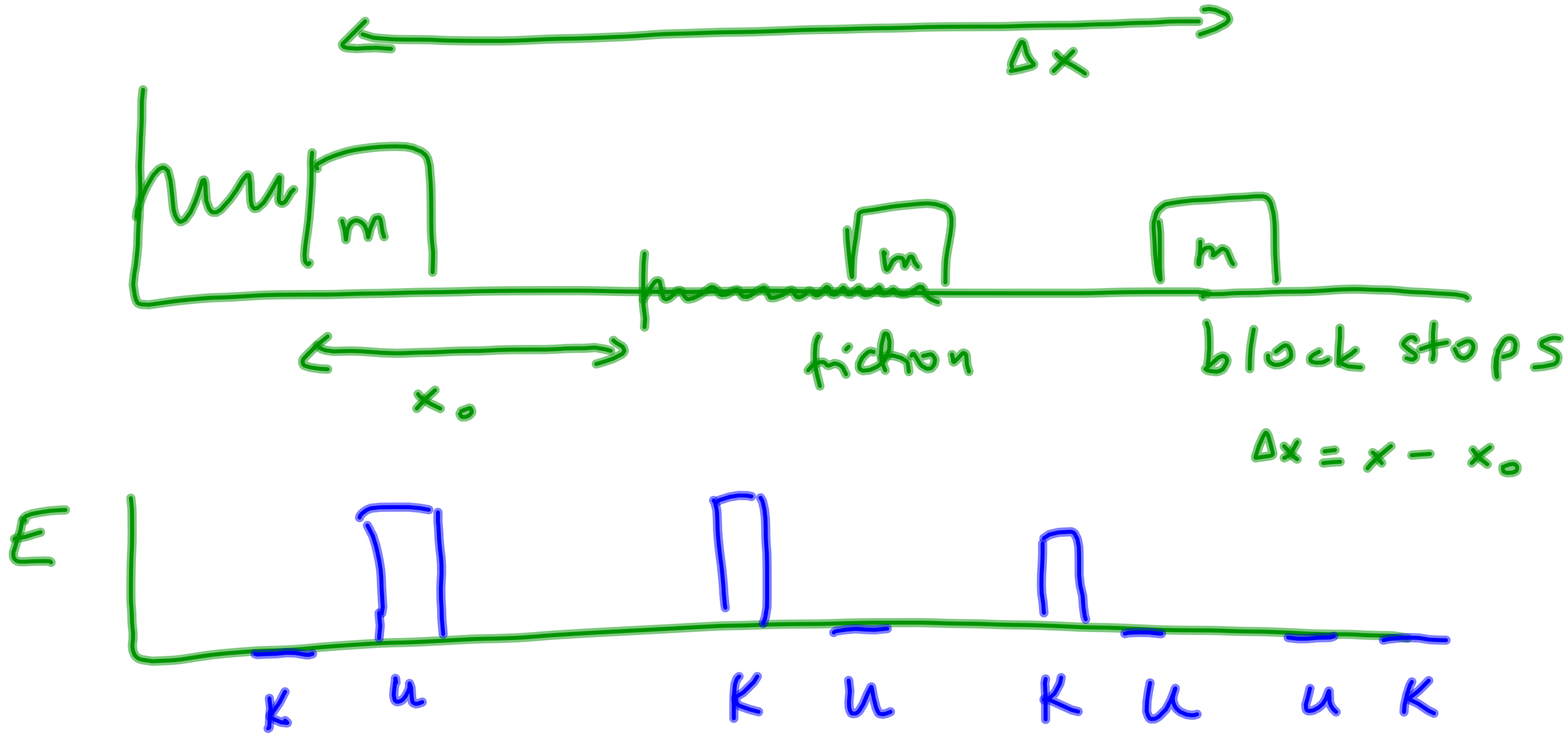
$$\frac{1}{2}kx^2 = mgh$$

$$h = \frac{1}{2}\frac{kx^2}{mg} = 1.7\text{ mm}$$

Nonconservative forces

Ex.

A block of mass m launched from a spring of constant k that's initially compressed a distance x_0. After leaving the spring, the block slides on a horizontal surface with frictional coefficient μ. Find an expression for the distance the block slides before coming to rest.


Δx
m
m
m
x₀
friction
block stops
Δx = x - x₀
E
K
u
K
u
K
u
u
K

$$\Delta K + \Delta U = W_{nc}$$ — work done by non conservative force

$U_0 = \frac{1}{2} k x_0^2$

$U = 0$

$K_0 = 0$

$K = 0$

$$W_{nc} = \underbrace{-\mu m g}_{F_x} \underbrace{\Delta x}_{\text{distance } \Delta x}$$

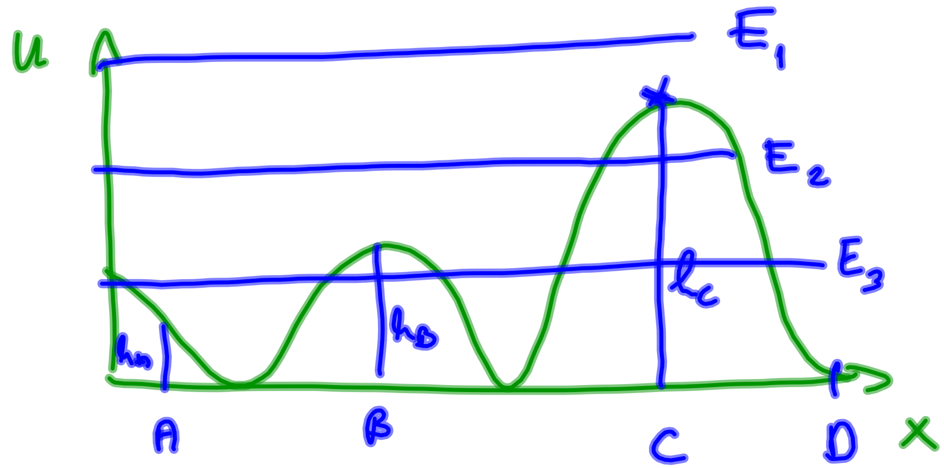
$$(\cancel{K} - \cancel{K_0}) + (\cancel{U} - U_0) = W_{nc}$$

$$-\frac{1}{2} k x_0^2 = -\mu m g \Delta x$$

$$\Delta x = \frac{k x_0^2}{2 \mu m g}$$

Potential energy curves

- roller-coaster track



How fast must a car be coasting at point A to reach a point D?

Conservation of energy

$$mgh_C < \frac{1}{2}mv_A^2 + mgh_A$$

- need to clear peak C

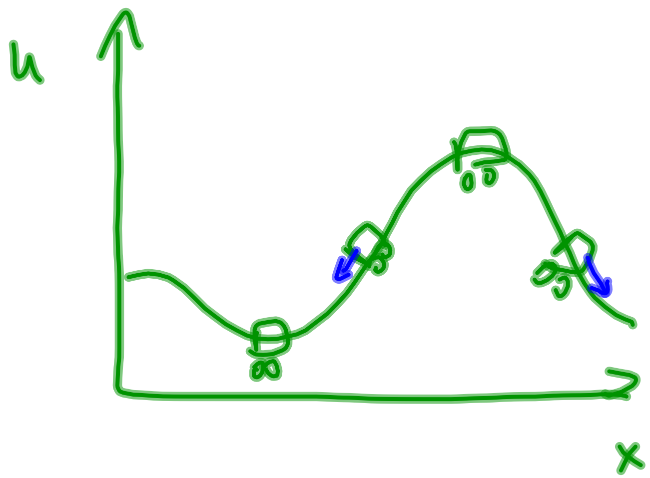
$$v_A > \sqrt{2g(h_C - h_A)}$$

If $\frac{1}{2}mv_A^2 + mgh_A = mgh_C$ * turning point

Turning points - where the car runs back & forth between 2 points.

If a car can't reach a point we say it's trapped in a potential well between its turning points.

Forces & PE



- at peaks & valleys there is no force

Force is a slope of PE!

$$F_x = -\frac{du}{dx}$$

$$\Delta u = -F_x \Delta x$$

$$F_x = -\frac{\Delta u}{\Delta x} = -\lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{du}{dx}$$

Systems of Particles

- so far rigid objects - baseballs, cars, planets
- we move to many particle systems

Center of mass - an average position of all the mass making up the system

- obeys Newton's 2^{nd} law

$$\vec{F}_{net} = m\vec{a} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2}$$

- let's have N particles

$$\vec{F}_i = m_i \vec{a}_i = m_i \frac{d^2\vec{r}_i}{dt^2} = \frac{d^2(m_i \vec{r}_i)}{dt^2}$$

$$\underline{\vec{F}_{total_{net}}} = \sum_{i=1}^{N} \vec{F}_i = \sum_{i=1}^{N} \frac{d^2(m_i \vec{r}_i)}{dt^2} = \underline{\frac{d^2}{dt^2}\left(\sum_{i=1}^{N} m_i \vec{r}_i\right)}$$

$$\sum_{i=1}^{N} m_i = M$$

$$\vec{F}_{total} = \frac{d^2}{dt^2} \sum_{i=1}^{N} m_i \vec{r}_i \quad \Big| \cdot \frac{\sum m_i}{\sum m_i} \qquad \sum_{i=1}^{N} m_i = M$$

$$= M \frac{d^2}{dt^2} \boxed{\frac{\left(\sum_{i=1}^{N} m_i \vec{r}_i\right)}{M}} \qquad \vec{r}_{cm} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M}$$

center of mass

$$\vec{F}_{total} = M \frac{d^2 \vec{r}_{cm}}{dt^2} = M \vec{a}_{cm}$$