Conservation of energy

Conservative force "gives back", ex: gravity force, spring, electric

If you start at the same point that you come back to & the work you did is zero => conservative force

Nonconservative force – doesn't "give back"

If you start & come back from the same point & work is not zero – nonconserv. force

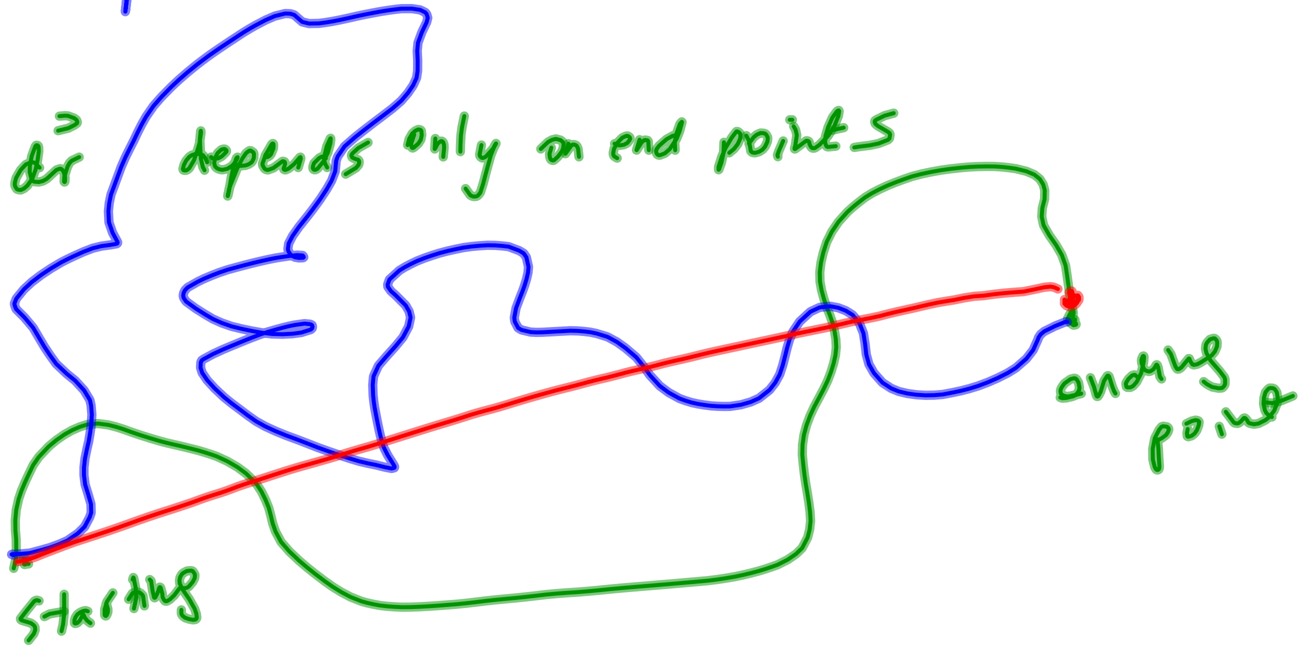
When the total work done by a force F acting on an object over closed path is zero, the force is conservative

$W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$ $W = \int_{x_1}^{x_2} F\,dx$ $\int_{x_1}^{x_1} \Rightarrow \oint$

$$\boxed{\oint \vec{F} \cdot d\vec{r} = 0}$$

The work done by a conservative force while moving between 2 points is independent of the path taken

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$$ depends only on end points



Potential energy PE

Work done against conservative force is somehow stored & you can get it back through KE kinetic energy ⇒ that's what PE is all about

The change ΔU_{AB} in PE associated with a conservative force is the negative of the work done by that force as it acts from point A to B

$$\underbrace{\Delta U_{AB}}_{\text{potential difference}} = -\int_A^B \vec{F} \cdot d\vec{r}$$

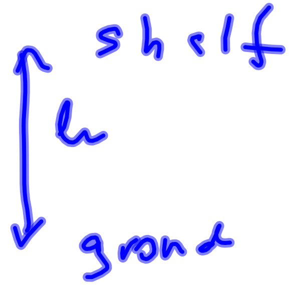
If $\vec{F}$ is parallel to displacement $\Delta\vec{r}$ & 1 D

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$$

If $F(x)$ is constant $\quad \Delta U = -F(x_2 - x_1)$

Gravitational PE

$\Delta U = mg\,\Delta y$ or $\Delta U = mgh$



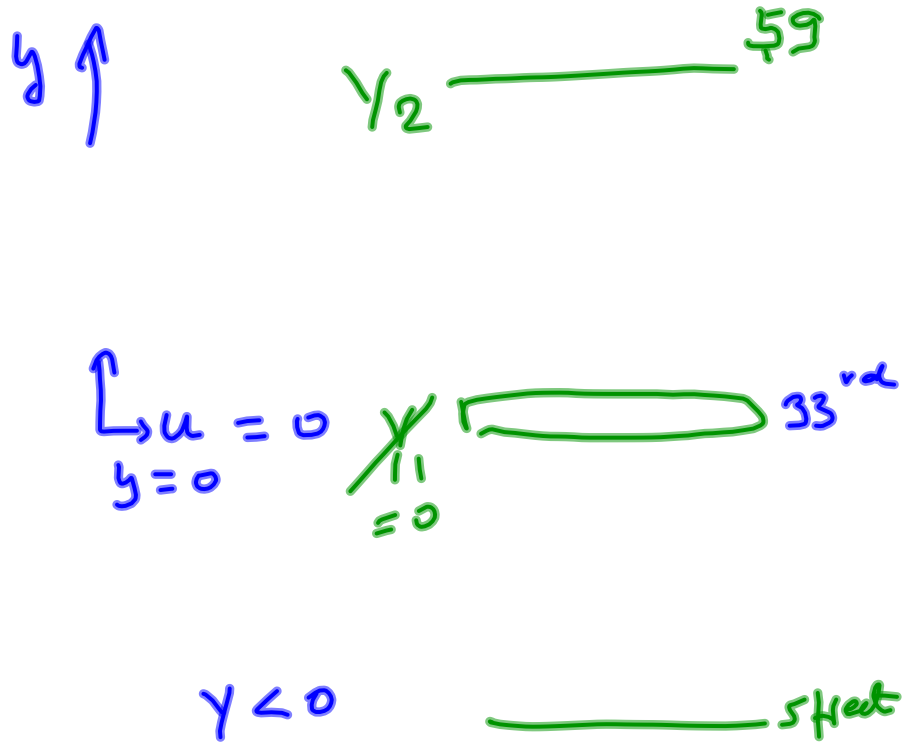
If $U = 0$ at floor then $U = mgh$ on a shelf

$U = 0$ shelf $U = -mgh$ on a floor

Ex 55 kg engineer leaves her office on the 33rd floor of a skyscraper & takes an elevator to 59th floor. Later she descends to street level (0). If the engineer takes her office as zero of PE & the distance between 2 floors is 3.5 m what is PE

a) in her office
b) 59th floor
c) street level

$U = mg\Delta y$



a) zero

b) $U_{55} = mg(y_2 - y_1)$

$= mg \cdot y_2$

$y_2 = 26 \cdot 3.5\,m$

$U_{59} = 55 \cdot 9.8 \cdot 26 \cdot 3.5 = 49\,kJ$

c) $U_{street} = mg\Delta y = mg(y_2 - \cancel{y_1}) =$

is negative

$\cong -60\,kJ$

Elastic PE

PE of arbitrary stretch or compression

$F = -kx$

$$\underbrace{\Delta U = -\int_{x_1}^{x_2} F(x)\,dx}= k\int_{x_1}^{x_2} x\,dx = k \cdot \frac{x^2}{2}\Bigg|_{x_1}^{x_2}$$

$$= \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

$x_1 = 0 \quad U = 0$

$$\boxed{U = \frac{1}{2}kx^2}$$

Ex

Ropes used in rock climbing are "springy".

A particular rope exerts a force given by

$F = -kx + bx^2$ where $k = 233$ N/m, $b = 4.1$ N/m^2.

x is the stretch. Find PE stored in the rope

when it has been stretched 2.62 m taking

$U = 0$ at $x = 0$.

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$$

$U_0 = 0$, $x_0 = 0$, $x = 2.62\ m$,

$(U_1 = ?)$

$$\Delta U = U - U_0 = -\int_0^{2.62} F(x) \cdot dx = \boxed{775.12\ J}$$

$$= -\int_0^{2.62} \left(-kx + bx^2\right) dx$$

$$= -\left(\frac{-kx^2}{2} + \frac{bx^3}{3}\right)\Bigg|_0^{2.62} = 799.70 - 24.57$$

Conservation of Mechanical Energy

$W_{net} = \Delta K$

$W_c = -\Delta U$

$W_{net} = W_c + W_{nc}$

$\Delta K = -\Delta U + W_{nc}$

$\Delta K + \Delta U = \underline{W_{nc}}$

c - conservative

nc - nonconservative

If $W_{nc} = 0$

$\Delta K + \Delta U = 0$

initial KE K_0
PE U_0 } const.

$K - K_0 + U - U_0 = 0$

$$\boxed{K + U = K_0 + U_0}$$

$$K + U = \text{constant}$$
$$= K_0 + U_0$$

law of conservation of mechanical energy