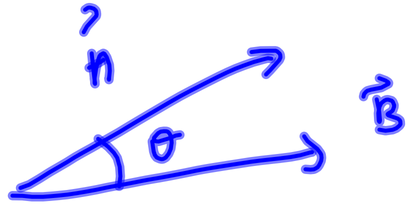
Continuing with work

Work is a scalar! But it is a product of 2 vectors.

- instead of $\Delta x \rightarrow \Delta\vec{r}$

Scalar product math overview



$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$
$$= AB\cos\theta$$

gives a scalar

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$$

$$W = \vec{F} \cdot \Delta\vec{r} = |\vec{F}||\Delta\vec{r}| \cos\theta = \underbrace{F\cos\theta}_{F_x} \Delta r$$

$$W = \vec{F} \cdot \Delta\vec{r} = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

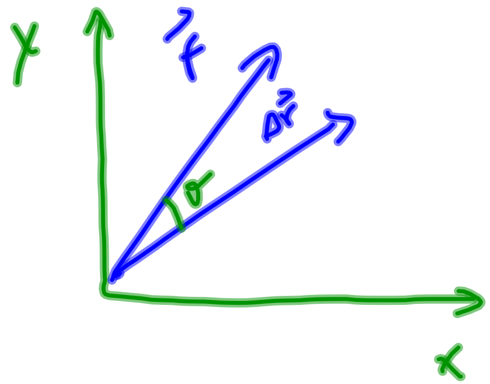
$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\Delta r = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}$$

Ex.

A tugboat pushes on a cruise ship with a force $\vec{F} = \underbrace{1.2}_{F_x}\hat{i} + \underbrace{2.3}_{F_y}\hat{j}$ MN moving the ship along a straight path with displacement $\Delta\vec{r} = \underbrace{380}_{\Delta x}\hat{i} + \underbrace{460}_{\Delta y}\hat{j}$ m

Find work & angle between $\vec{F}$ & $\Delta\vec{r}$



$W = \vec{F}\cdot\Delta\vec{r} = F_x\Delta x + F_y\Delta y$

$= 1510$ MJ

$W = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta$

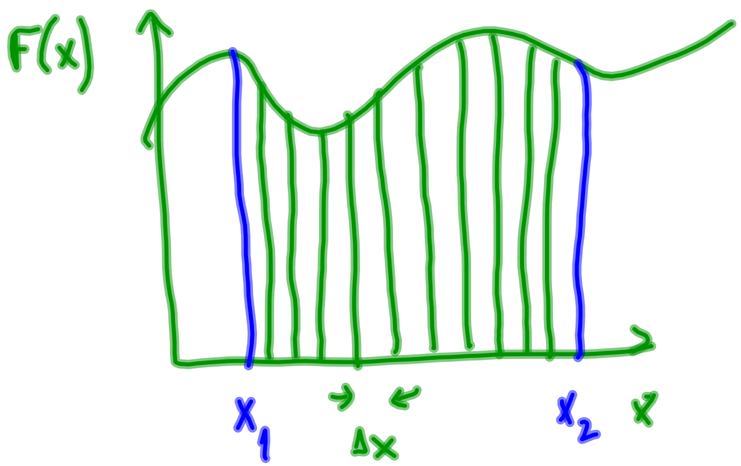
$$\cos\theta = \frac{W}{F\Delta r}$$

$F = ?$ $\quad F = |\vec{F}| = \sqrt{F_x^2 + F_y^2}$

$\Delta r = ?$ $\quad \Delta r = |\Delta\vec{r}| = \sqrt{\Delta x^2 + \Delta y^2}$

$\theta = 12°$ small angle, lots of work

Forces that vary



$$W = \sum_{i=1}^{N} W_i$$

$$= \sum_{i=1}^{N} F(x_i)\,\Delta x$$

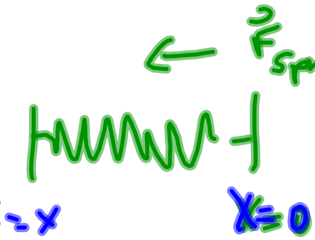
$$= \lim_{\Delta x \to 0} \sum_{i=1}^{N} F(x_i)\,\Delta x =$$

$$\int_{x_1}^{x_2} F(x)\,dx$$

definite integral
definition

$$\int_{x_1}^{x_2} x^n \, dx = \left. \frac{x^{n+1}}{n+1} \right|_{x_1}^{x_2} = \frac{x_2^{n+1}}{n+1} - \frac{x_1^{n+1}}{n+1}$$

Ex.



$F_{spring} = -kx$ $\quad F \neq$ constant

Imagine a force that you exert on a spring & calculate work

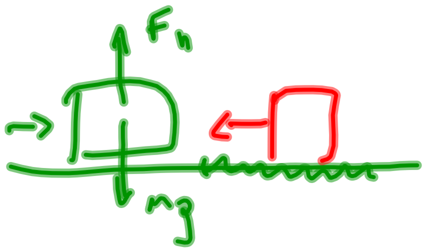
$F = +kx$

$$W = \int_0^x F \, dx = \int_0^x kx \, dx = k \int_0^x x \, dx =$$

$$W_{spring} = k \cdot \frac{x^2}{2} \Big|_0^x = \frac{k}{2}\left(x^2 - 0\right) = \frac{1}{2}kx^2$$

work against gravity $W = mgh$

Ex. Worker pushes a piano across level floor that becomes rough. $\mu_k = \mu_0 + ax^2$



$$F_{friction} = \mu_k F_n \qquad F_n = mg$$

$$= \mu_k mg$$

$$W = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} \big(\underbrace{\mu_0 + a x^2}_{\mu_k}\big) \cdot \underbrace{mg}_{F_n}\,dx =$$

$$= mg\left[\int_{x_1}^{x_2} \mu_0\,dx + a\int_{x_1}^{x_2} x^2\,dx\right] =$$

$$= mg\left[\mu_0 x\Big|_{x_1}^{x_2} + a\,\frac{x^3}{3}\Big|_{x_1}^{x_2}\right] =$$

$$= mg\left[\mu_0 x_2 - \mu_0 x_1 + \frac{1}{3} a x_2^3 - \frac{1}{3} a x_1^3\right]$$

Power

$$P = \frac{\Delta W}{\Delta t}$$ average power

[W]
Watts

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$ instantaneous power

If P = constant $\quad W = P\Delta t$

$P \neq$ constant $\quad W = \int P\,dt$

Ex. Each of the 500 floodlights at Yankee stadium uses electric energy at the rate of 1 kW. How much does it cost to run those lights during 4 h night game if electricity costs 9.5 ¢ / kWh

$$W = P\Delta t = 500\ kW \cdot 4h = 2000\ kWh$$

$\left[P \sim \frac{W}{t} \right]$ the cost $= 2000\ kWh \cdot \frac{9.5\ ¢}{kWh}$

$= 190\ \$$

KE **Kinetic energy** **Potential energy**

- moves
- stored

$$W_{net} = \int F_{net}\, dx$$

$$F_{net} = ma = m\frac{dv}{dt}$$

$$W_{net} = \int m \frac{dv}{dt} dx = m \int \frac{dx}{dt} \cdot dv$$

$$= m\int_{v_1}^{v_2} v \cdot dv = m\frac{v^2}{2} = m\frac{v_2^2}{2} - m\frac{v_1^2}{2} = K_2 - K_1 = \Delta K$$

$K = \frac{1}{2}mv^2$ $\Delta K = W_{net}$ work energy theorem