Atmospheric Convection

PHYS 536 Fall 2015

Problem Assignment # 2

due 09-11-15

1. **Atmospheric Convection exercises** (20 points)

   Emanual Exercises 2.1-2.4.

2. **Boussinesq equations in terms of potential temperature** (6 points)

   (a) Starting with the definition of buoyancy (equation in box on page 6), combined with equation 1.2.5; show that buoyancy can be expressed in terms of the potential temperature, θ:
   $$B = g\left(\frac{\theta - \overline{\theta}}{\overline{\theta}}\right)$$
   in the Boussinesq approximation.

   (b) From this, derive the heat equation (2.6.2) from the first law of thermodynamics for an ideal gas (1.3.11).

3. **Laminar plumes from a point source** (6 points)

   We derive a similarity solution for laminar plumes from a point source by assuming algebraic dependence of vertical velocity, w, buoyancy, B, and plume radius, R, on the altitude, z (equations 2.6.4-2.6.6). Once the actual z dependence is determined (2.6.10), a dimensional analysis must be performed to determine the dependence on the parameters in the problem (F, ν, and κ). Combine the confirmed z dependence with an dimensional analysis to derive

$$w = \frac{F^{1/2}}{\nu^{1/2}} \times \text{func}\left(\frac{rF^{1/4}}{z^{1/2}\nu^{3/4}}, \sigma\right), \tag{1}$$

$$B = \frac{F}{\nu z} \times \text{func}\left(\frac{rF^{1/4}}{z^{1/2}\nu^{3/4}}, \sigma\right), \tag{2}$$

$$R = \frac{z^{1/2}\nu^{3/4}}{F^{1/4}} \times \text{func}(\sigma), \tag{3}$$

   where $\sigma = \nu/\kappa$ is the Prandtl number.