Geophysical Fluid Dynamics

PHYS 527 Fall 2016

Problem Assignment # 3

due 09-16-16

1. **Energetics of a Boussinesq fluid** (2 points)

   Starting with the simple Boussinesq equations (2.108), derive an energy equation for the Boussinesq system (2.112).

2. **Hydrostatic approximation** (8 points)

   (a) Using the scaling arguments in section 2.7.2, show how hydrostasy is equivalent to a small aspect ratio approximation.

   (b) Is the hydrostatic approximation a good one in the ocean? In the atmosphere? Explain.

   (c) Vallis problem 2.16.

3. **Pressure coordinates** (4 points)

   Starting from the ideal gas primitive equations in heigh coordinates (2.144), derive the analogous equations in pressure coordinates (2.153).

4. **Buoyancy frequency** (5 points)

   (a) We derived the buoyancy frequency in terms of vertical density gradients (equation 2.219). Obtain an expression in terms of vertical potential temperature gradients for an ideal gas (equation 2. 225).

   (b) Vallis problem 2.18

5. **Gravity waves** (4 points)

   Consider a Boussinesq fluid initially at rest, with N constant. The linearized momentum, mass continuity, and thermodynamic equations are given by 2.245 and 2.246. Combine these equations to obtain a single equation in terms of w' (equation 2.247). Assume a plane wave solution and derive the dispersion relation (2.248).

6. **Ekman layer** (2 points)

   What does the Ekman number measure? What determines the thickness of the Ekman layer?